Counting Fuzzy Subgroups of $\overset{n}{\underset{i=1}{\otimes}} Z_2$ by Lattice Subgroups

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ABSTRACT

In this paper, we compute the number of fuzzy subgroups of an abelian group $\overset{n}{\underset{i=1}{\otimes}} Z_2$ when n=1, 2, 3 and 4 by using the subgroups lattice of it .Also we construct the diagram of subgroups lattice of $\overset{n}{\underset{i=1}{\otimes}} Z_2$, n=1, 2, 3 and 4.

حساب الزمر الجزئية الضبابية للزمرة $\overset{n}{\underset{i=1}{\otimes}}$ Z_2 بواسطة زمرها الجزئية المشبكة

الخلاصة

في هذا البحث قمنا بحساب عدد الزمرالجزئية الضبابية للزمرة الابدالية $\overset{n}{\underset{i=1}{\otimes}} Z_2$ عندما n=1,2,3,4 باستخدام زمرها الجزئية المشبكة .كما قمنا بأنشاء مخطط الزمرالجزئية المشبكة للزمرة $\overset{n}{\underset{i=1}{\otimes}}$ Z_2 عندما n= 1,2,3,4 .

INTRODUCTION

The study of fuzzy algebraic structures was started with the introduction of the concept of fuzzy subgroups by Rosenfeld in 1971 [5]. Without any equivalence relation on fuzzy subgroups of group *G*, the number of fuzzy subgroups is infinite, even for the trivial group {e}. Some authors have used the equivalence relation of fuzzy sets to study the equivalence of fuzzy subgroups ([1], [2], [3], and [9]). All of them have treated the particular case of finite Abelian group. It is interesting to count the number of fuzzy subgroups of nonabelian groups and construct them. Laszlo in [1] has studied the construction of fuzzy subgroup of a group of order one to six. Sulaiman and Abd Ghafur in [6] have counted the number of fuzzy subgroups of nonabelian symmetric groups S_2, S_3 and alternating group A_4. In the other paper, they [8] have counted the number of fuzzy subgroups of group defined by a presentation. In this paper we compute the number of fuzzy subgroups of an abelian group $\overset{n}{\underset{i=1}{\otimes}} Z_2$, for all n=1, 2, 3 and 4 by using the subgroups lattice, and we construct the diagram of subgroups lattice of it.

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BASIC DEFINITIONS AND RESULTS OF FUZZY SUBGROUPS

We recall some definitions and results that will be used later.

Definition 2.1.[4] A partial ordered on a nonempty set P is a binary relation $\leq$ on P that is reflexive, antisymmetric and transitive. The pair $< P, \leq >$ is called a partially ordered set or poset. Poset $< P, \leq >$ is totally ordered if every x, y $\in$ P are comparable, that is $x \leq y$ or $y \leq x$. A nonempty subset S of P is a chain in P if S is totally ordered by $\leq$.

Definition 2.2.[4] Let $< P, \leq >$ be a poset and let $S \subseteq P$. An upper bound for S is an element $x \in P$ for which $s \leq x$, $\forall s \in S$. The least upper bound of S is called the supremum or join of S. A lower bound for S is an element $x \in P$ for which $x \leq s$, $\forall s \in S$. The greatest lower bound of S is called the infimum or meet of S. Poset $<$ P, $\leq >$ is called a lattice if every pair x, y elements of P has a supremum and an infimum.

Note that the set of all of subgroups G under the"subgroup" relation is a lattice. This lattice is called the lattice subgroup of G.

Definition 2.3. [5] Let X is a nonempty set. A fuzzy set of X is a function μ from X into [0, 1].

Definition 2.4. [5] A fuzzy subset μ of a group G is called a fuzzy subgroup of G if:

i. $\mu(xy) \geq \min \{\mu(x) , \mu(y)\}$, $\forall x, y \in G$ and

ii. $\mu(x^{-1}) = \mu(x), \forall x \in G$.

Example. Let G = S_3 be the symmetric group of degree 3.

Define g: G → [0, 1] as follows:

$$g(x)= \begin{cases} 1 & \text{if } x=e \\ 0.5 & \text{if } x= (123), (132) \\ 0 & \text{otherwise} \end{cases}$$

where e is the identity element of S_3. It can be easily verified that g is a fuzzy subgroup of S3.

Theorem 2.5.[5] Let e denote the identity element of G. If μ is a fuzzy subgroup of G, then $\mu(e) \geq \mu(x)$, $\forall x \in G$.

Theorem 2.6.[7] Function $\mu : G \rightarrow [0, 1]$ is a fuzzy subgroup of G if there is a chain

$P_1 < P_2 < ... < P_n = G$ in subgroups lattice of G such that μ can be written as

$$\mu(x) = \begin{cases} \theta_1, & x \in P_1 \\ \theta_2, & x \in P_2 \setminus P_1 \\ \vdots \\ \theta_n, & x \in P_n \setminus P_{n-1} \end{cases}$$

Where θ_i is element of [0, 1] and $\theta_i > \theta_j$ if $i > j$.

Example. Consider the group G = Z_{12}. Define function μ as follows:

$$\mu(x) = \begin{cases} 1, & x \in \{0, 2, 4, 6, 8, 10\} \\ 1/2, & x \in \{1, 3, 5, 7, 9, 11\}, \end{cases}$$

Note that $P_1(\mu) = \{0, 2, 4, 6, 8, 10\}$ and $P_2(\mu) = Z_{12}$ both are subgroup of Z_{12}. According to Theorem 2.6, μ is a fuzzy subgroup of Z_{12}.

Definition 2.7. **[7]** Let μ, λ be fuzzy subgroups of G of the form

$$\mu(x) = \begin{cases} \theta_1, x \in P_1 \\ \theta_2, x \in P_2 \setminus P_1 \\ \vdots \\ \theta_n, x \in P_n \setminus P_{n-1} \end{cases}, \qquad \lambda(x) = \begin{cases} \delta_1, & x \in M_1 \\ \delta_2, & x \in M_2 \setminus M_1 \\ \vdots \\ \delta_m, & x \in M_m \setminus M_{m-1} \end{cases}$$

Then we say that μ and λ are equivalent and write $\mu \sim \lambda$ if

(1) m = n and

(2) Pi = Mi, $\forall$ i $\in$ {1, 2, m}.

Two fuzzy subgroups of G are said to be different if they are not equivalent.

Lemma 2.8. [6] The number of fuzzy subgroups of G is equal to the number of chain on the lattice subgroups of G.

THE NUMBER OF FUZZY SUBGROUPS OF $\overset{n}{\underset{i=1}{\otimes}} Z_2$ IF N=1, 2, 3 AND 4

In this section, we give a guiding principle to determine the number of fuzzy subgroups of $\overset{n}{\underset{i=1}{\otimes}} Z_2$ when n=1, 2, 3 and 4. We denote the number of fuzzy subgroups of group $\overset{n}{\underset{i=1}{\otimes}} Z_2$ by o (F_G) and the identity element by 1.

The Number of Fuzzy Subgroups of Z2

Let $Z_2 = \{1, x\}$. The two subgroups of Z2 are I= {1} and Z_2. Therefore, we can construct a fuzzy subgroup μ of Z_2 with length of μ equal to 1 or 2. The fuzzy subgroup of Z2 with length 1 is $\mu(x) = \theta_1$, $\forall x \in Z_2$.

While the fuzzy subgroup of Z_2 with length 2 is

$$\mu(x) = \begin{cases} \theta_1 & , x \in \{1\} \\ \theta_2 & , x \in Z_2 \setminus \{1\}, \text{ where } \theta_1 \text{ and } \theta_2 \in [0, 1]. \end{cases}$$

Thus, o $(F_{Z2}) = 2$.

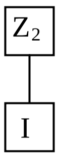

Figure (1) Diagram of poset subgroups of Z_2.

The Number of Fuzzy Subgroups of $Z_2 \times Z_2$

Let $Z_2 \times Z_2 = \{(1,1) , (x,1) ,(1,y) ,(x,y) \}$.There are five subgroups of $Z_2 \times Z_2$, namely
$Z_2 \times Z_2$,$I =\{ (1,1)\}$, $H_1= \{ (1,1), (x,1) \}$,$H_2= \{ (1,1) ,(1,y) \}$and $H_3 =\{ (1,1) , (x,y) \}$.
Let S(G) denotes the set of all subgroups of group G. The diagram of poset $< S(Z_2 \times Z_2),< >$is as follows:

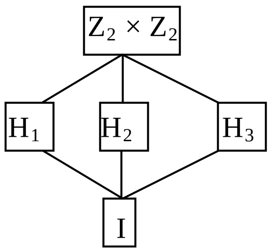

Figure (2) Diagram of poset subgroups of $Z_2 \times Z_2$.

We will compute the number of fuzzy subgroups of $Z_2 \times Z_2$.
Let μ be a fuzzy subgroup of $Z_2 \times Z_2$. We will identify μ according to $P_1(\mu)$. Every subgroup of $Z_2 \times Z_2$ can be chosen to be $P_1(\mu)$.

If $P_1(\mu) = Z_2 \times Z_2$, we only have one fuzzy subgroup of $Z_2 \times Z_2$, that is

$$\mu_1(x) = \theta_1, \ \forall\, x \in Z_2 \times Z_2.$$

If $P_1(\mu) = H_1$, then the only option we have is $Z_2 \times Z_2= P_2(\mu)$. Therefore we only have one fuzzy subgroup of $Z_2 \times Z_2$ namely

$$\mu_2(x) = \begin{cases} \theta_1 & , x \in H_1 \\ \theta_2 & , x \in Z_2 \times Z_2 \setminus H_1. \end{cases}$$

Similarly, we have one fuzzy subgroups for $P_1(\mu) = H_2$, $P_1(\mu) = H_3$, namely

$$\mu_3(x) = \begin{cases} \theta_1, & x \in H_2 \\ \theta_2, & x \in Z_2 \times Z_2 \setminus H_2 \end{cases} , \qquad \mu_4(x) = \begin{cases} \theta_1, & x \in H_3 \\ \theta_2, & x \in Z_2 \times Z_2 \setminus H_3 \end{cases}$$

Respectively.

Finally if $P_1(\mu)= I$, then by observing the poset of $Z_2 \times Z_2$ see Figure (2) we may construct fuzzy subgroups of $Z_2 \times Z_2$ of length 2, If the length is 2, then $P_2(\mu)= Z_2 \times Z_2$ and we can choose one out of three to be $P_2(\mu)$, namely H_1, H_2, and H_3.

Thus, there are four fuzzy subgroups that can be constructed with $P_1(\mu) = I$. Thus the total number of fuzzy subgroups of $Z_2 \times Z_2$ is eight.

The Number of Fuzzy Subgroups of $\overset{3}{\underset{i=1}{\otimes}} Z_2$ ($Z_2\times Z_2\times Z_2$)

Let $Z_2\times Z_2\times Z_2$ = {(1,1,1) , (x,1,1) ,(1,y,1), (1,1,z) , (x,1,z), (x,y,1), (1,y,z), (x,y,z) }.
There are sixteen subgroups of $Z_2\times Z_2\times Z_2$, namely
$Z_2\times Z_2\times Z_2$, I ={(1,1,1)}, K_1 ={(1,1,1),(x,1,1)} , K_2 ={(1,1,1) ,(1,y,1)} , K_3 ={ (1,1,1) , (1,1,z)} , K_4={(1,1,1) ,(x,y,1)} , K_5= {(1,1,1) ,(x,1,z)} ,K_6= {(1,1,1) ,(1,y,z)}, K_7 ={(1,1,1) ,(x,y,z)}, H_1={(1,1,1), (x,1,1), (1,y,1), (x,y,1)} ,H_2 ={(1,1,1), (x,1,1), (1,1,z), (x,1,z)} ,H_3={(1,1,1), (1,1,z), (1,y,1), (1,y,z)} ,H_4={(1,1,1), (x,y,1), (1,1,z), (x,y,z) } ,H_5={(1,1,1), (x,1,z), (1,y,1), (x,y,z) } ,H_6={(1,1,1), (1,y,z), (x,1,1), (x,y,z) } and
H_7={(1,1,1), (x,y,1), (1,y,z), (x,1,z)} .

The diagram of poset subgroups < S ($\overset{3}{\underset{i=1}{\otimes}} Z_2$), < > is as follows:

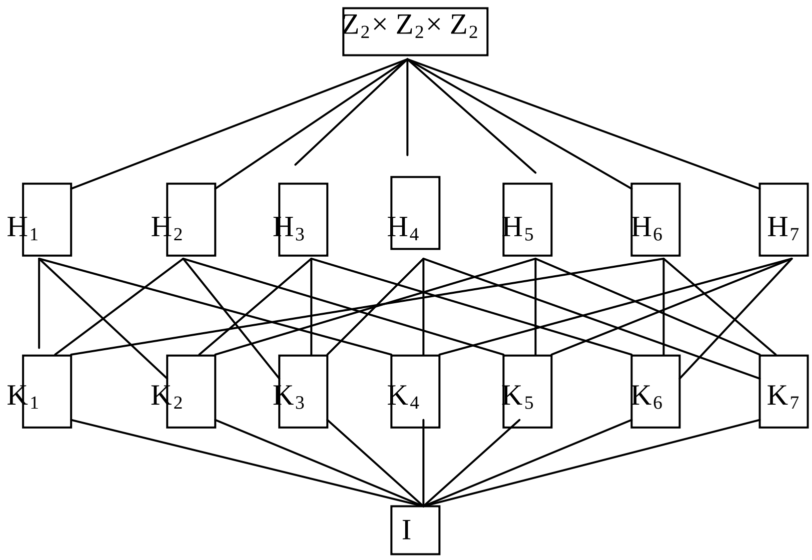

Figure (3) Diagram of poset subgroups of $\overset{3}{\underset{i=1}{\otimes}} Z_2$.

We will count the number of the fuzzy subgroups of $\overset{3}{\underset{i=1}{\otimes}} Z_2$ by observing that diagram and using Lemma 2.8. We can see that the maximal chain on that lattice consists of four subgroups of $\overset{3}{\underset{i=1}{\otimes}} Z_2$. Therefore, the fuzzy subgroup μ of $\overset{3}{\underset{i=1}{\otimes}} Z_2$ has length 1,2,3 or 4.

Let μ be a fuzzy subgroup of $\overset{3}{\underset{i=1}{\otimes}} Z_2$. We will identify μ according to $P_1(\mu)$. Every subgroup of $\overset{3}{\underset{i=1}{\otimes}} Z_2$ can be chosen to be $P_1(\mu)$. If $P_1(\mu) = \overset{3}{\underset{i=1}{\otimes}} Z_2$, we only have one fuzzy subgroup of $\overset{3}{\underset{i=1}{\otimes}} Z_2$, that is

$$\mu(x) = \theta_1 \quad , \forall x \in \overset{3}{\underset{i=1}{\otimes}} Z_2 .$$

If $P_1(\mu) = H_1$, then the only option we have is $\overset{3}{\underset{i=1}{\otimes}} Z_2 = P_2(\mu)$. Therefore we only have one fuzzy subgroup of $\overset{3}{\underset{i=1}{\otimes}} Z_2$ namely

$$\mu(x) = \begin{cases} \theta_1, & x \in H_1 \\ \theta_2, & x \in \overset{3}{\underset{i=1}{\otimes}} Z_2 \setminus H_1. \end{cases}$$

Similarly, we have one fuzzy subgroups for $P_1(\mu)= H_2$, $P_1(\mu)= H_3$, $P_1(\mu)= H_4$, $P_1(\mu)= H_5$, $P_1(\mu)= H_6$ and $P_1(\mu)= H_7$.

If $P_1(\mu)= K_1$, then we have four chains, those are $K_1 < \overset{3}{\underset{i=1}{\otimes}} Z_2$, $K_1 < H_1 < \overset{3}{\underset{i=1}{\otimes}} Z_2$, $K_1 < H_2 < \overset{3}{\underset{i=1}{\otimes}} Z_2$ and $K_1 < H_6 < \overset{3}{\underset{i=1}{\otimes}} Z_2$ .Therefore, we get four fuzzy subgroups of $\overset{3}{\underset{i=1}{\otimes}} Z_2$ with $P_1(\mu)= K_1$, those are

$$\mu(x) = \begin{cases} \theta_1, & x \in K_1 \\ \theta_2, & x \in \overset{3}{\underset{i=1}{\otimes}} Z_2 \setminus K_1 \end{cases} , \qquad \mu(x) = \begin{cases} \theta_1, & x \in K_1 \\ \theta_2, & x \in H_1 \setminus K_1 \\ \theta_3, & x \in \overset{3}{\underset{i=1}{\otimes}} Z_2 \setminus H_1 \end{cases}$$

$$\mu(x) = \begin{cases} \theta_1, & x \in K_1 \\ \theta_2, & x \in H_2 \setminus K_1 \\ \theta_3, & x \in \overset{3}{\underset{i=1}{\otimes}} Z_2 \setminus H_2 \end{cases} \quad \text{and} \quad \mu(x) = \begin{cases} \theta_1, & x \in K_1 \\ \theta_2, & x \in H_6 \setminus K_1 \\ \theta_3, & x \in \overset{3}{\underset{i=1}{\otimes}} Z_2 \setminus H_6 \end{cases}$$

Similarly, we have four fuzzy subgroups for $P_1(\mu)= K_2$, $P_1(\mu)= K_3$, $P_1(\mu)= K_4$, $P_1(\mu)= K_5$, $P_1(\mu)= K_6$ and $P_1(\mu)= K_7$.

Finally if $P_1(\mu) = I$, we have 36 subgroups fuzzy. Thus, the total number of fuzzy subgroups of $\overset{3}{\underset{i=1}{\otimes}} Z_2$ is (1 + 28+7 +36 = 72).

The Number of Fuzzy Subgroups of $\overset{4}{\underset{i=1}{\otimes}} Z_2$ ($Z_2 \times Z_2 \times Z_2 \times Z_2$)

Let $Z_2 \times Z_2 \times Z_2 \times Z_2$ = {(1,1,1,1) , (x,1,1,1) ,(1,y,1,1), (1,1,z,1) ,(1,1,1,h), (x,1,z,1), (x,y,1,1), (1,y,z,1),(1,1,z,h),(1,y,1,h),(x,1,1,h), (x,y,z,1), (1,y,z,h), (x,1,z,h), (x,y,1,h), (x,y,z,h) }.

There are 48 subgroups of $\overset{4}{\underset{i=1}{\otimes}} Z_2$, namely $Z_2 \times Z_2 \times Z_2 \times Z_2$, I = {(1, 1, 1, 1)}, H_1 ={(1,1,1,1),(x,1,1,1)} ,H_2= {(1,1,1,1) ,(1,y,1,1)} ,H_3 ={ (1,1,1,1) , (1,1,z,1)} , H_4= {(1,1,1,1) ,(1,1,1,h) } ,H_5= {(1,1,1,1), (x,y,1,1)} ,H_6= {(1,1,1,1) ,(x,1,z,1)} , H_7= {(1,1,1,1) ,(1,y,1,h)} ,H_8= {(1,1,1,1) ,(1,y,z,1)}, H_9= {(1,1,1,1) ,(1,1,z,h)}, H_{10}={(1,1,1,1),(x,1,1,h)},H_{11}={(1,1,1,1),(x,y,z,1)},H_{12}={(1,1,1,1),(x,y,1,h)},H_{13}= {(1,1,1,1),(x,1,z,h)},H_{14}={(1,1,1,1),(1,y,z,h)},H_{15}={(1,1,1,1),(x,y,z,h)},K_1={(1,1,1 ,1), (x,1,1,1), (1,y,1,1), (x,y,1,1)},K_2={(1,1,1,1), (x,1,1,1), (1,1,z,1), (x,1,z,1)} , K_3={(1,1,1,1), (x,1,1,1), (1,1,1,h), (x,1,1,h)}, K_4={(1,1,1,1), (1,1,1,h), (1,1,z,1), (1,1,z,h) } , K_5={(1,1,1,1), (1,y,1,1), (1,1,1,h), (1,y,1,h)}, K_6={(1,1,1,1), (1,y,1,1), (1,1,z,1), (1,y,z,1) }, K_7={(1,1,1,1), (x,y,z,1), (1,y,z,1), (x,1,1,1)}, K_8={(1,1,1,1), (x,y,z,1), (x,1,z,1), (1,y,1,1)}, K_9={(1,1,1,1), (x,y,z,1), (x,y,1,1), (1,1,z,1)}, K_{10}={(1,1,1,1), (x,y,1,h), (x,y,1,1), (1,1,1,h)}, K_{11}={(1,1,1,1), (x,y,1,h), (1,y,1,1), (x,1,1,h)}, K_{12}={(1,1,1,1), (x,y,1,h), (x,1,1,1,), (1,y,1,h)}, K_{13}={(1,1,1,1), (x,1,z,h), (x,1,1,1), (1,1,z,h)}, K_{14}={(1,1,1,1), (x,1,z,h), (1,1,z,1),(x,1,1,h)}, K_{15}={(1,1,1,1), (x,1,z,h), (1,1,1,h), (x,1,z,1)}, K_{16}={(1,1,1,1), (1,y,z,h), (1,y,1,1), (1,1,z,h)}, K_{17}={(1,1,1,1), (1,y,z,h), (1,1,z,1), (1,y,1,h)}, K_{18}={(1,1,1,1), (1,y,z,h), (1,1,1,h), (1,y,z,1)}, K_{19}={(1,1,1,1), (x,y,z,h), (x,y,1,1), (1,1,z,h)}, K_{20}={(1,1,1,1), (x,y,z,h), (1,y,z,1), (x,1,1,h)}, K_{21}={(1,1,1,1), (x,y,z,h), (x,1,z,1), (1,y,1,h)}, K_{22}={(1,1,1,1), (x,y,z,h), (x,1,1,1), (1,y,z,h)}, K_{23}={(1,1,1,1), (x,y,z,h), (1,y,1,1), (x,1,z,h)}, K_{24}={(1,1,1,1), (x,y,z,h), (1,1,z,1), (x,y,1,h)}, K_{25}={(1,1,1,1), (x,y,z,h), (1,1,1,h), (x,y,z,1)},

G_1={(1,1,1,1), (x,y,z,1), (x,1,1,1), (1,y,1,1), (1,1,z,1), (1,y,z,1), (x,1,z,1), (x,y,1,1)},

G_2={(1,1,1,1), (x,y,1,h),(x,1,1,1), (1,y,1,1), (1,1,1,h), (1,y,1,h), (x,1,1,h), (x,y,1,1)},

G_3={(1,1,1,1), (x,1,z,h), (x,1,1,1), (1,1,1,h), (1,1,z,1), (1,1,z,h), (x,1,1,h), (x,1,z,1)},

G_4={(1,1,1,1), (1,y,z,h),(1, y,1,1), (1,1, z,1), (1,1,1,h),(1,y,z,1), (1,y,1,h),(1,1,z, h)},

G_5={(1,1,1,1), (x,y,z,h), (x,y,z,1), (1,1,1,h), (1,y,z,h), (x,1,1,1), (x,1,1,h), (1,y,z,1)}, and

G_6={(1,1,1,1), (x,y,z,h), (x,y,1,h), (1,1,z,1), (x,1,z,h), (1,y,1,1), (1,y,z,1), (x,1,1,h)}

.The diagram of poset subgroups < S($\overset{4}{\underset{i=1}{\otimes}} Z_2$), < > is as follows:

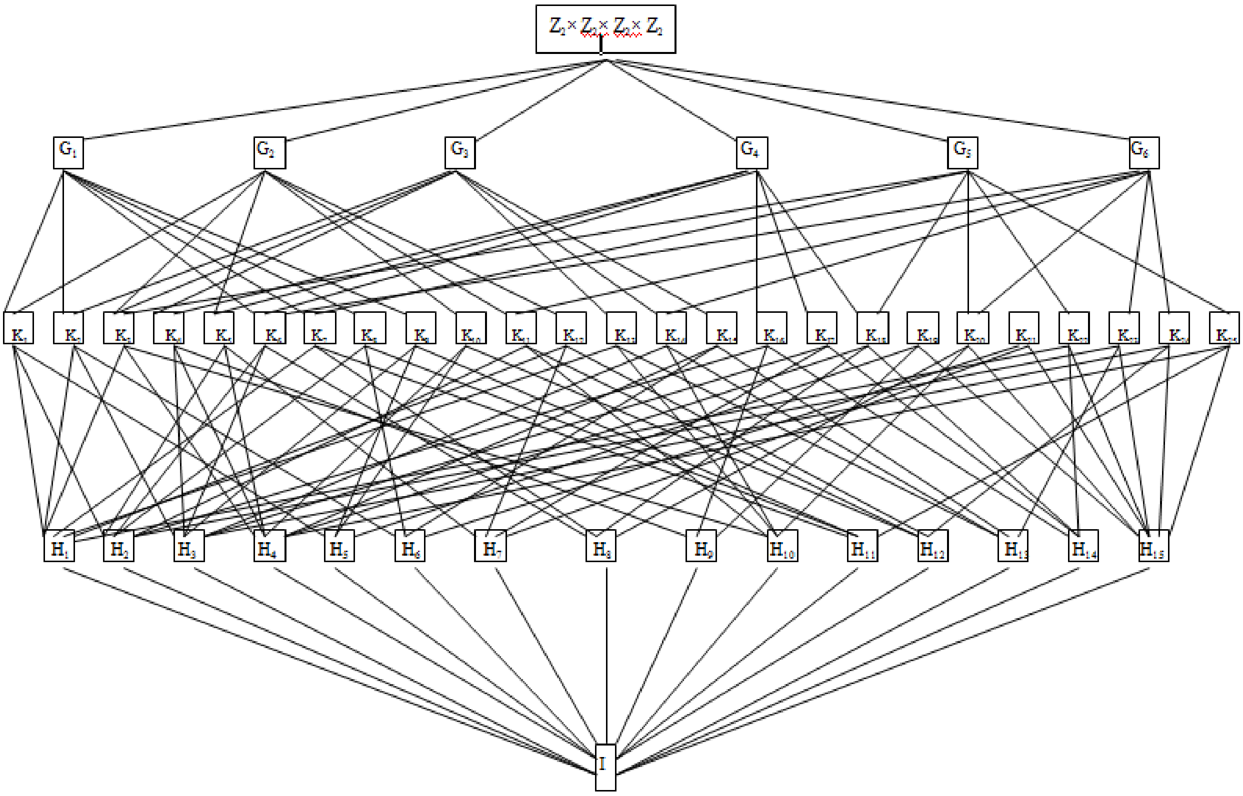

Figure (4) Diagram of poset subgroups of ($Z_2 \times Z_2 \times Z_2 \times Z_2$).

We will count the number of the fuzzy subgroups of $\overset{4}{\underset{i=1}{\otimes}} Z_2$ by observing its diagram and using Lemma 2.8.We can see that the maximal chain on that lattice consists of five subgroups of $\overset{4}{\underset{i=1}{\otimes}} Z_2$. Therefore, the fuzzy subgroup μ of $\overset{4}{\underset{i=1}{\otimes}} Z_2$ has length 1,2,3,4 or 5.

Let μ be a fuzzy subgroup of $\overset{4}{\underset{i=1}{\otimes}} Z_2$. We will identify μ according to $P_1(\mu)$. Every subgroup of $\overset{4}{\underset{i=1}{\otimes}} Z_2$ can be chosen to be $P_1(\mu)$. If $P_1(\mu) = \overset{4}{\underset{i=1}{\otimes}} Z_2$, we only have one fuzzy subgroup of $\overset{4}{\underset{i=1}{\otimes}} Z_2$, that is

$$\mu(x) = \theta_1 \quad , \forall x \in \overset{4}{\underset{i=1}{\otimes}} Z_2 .$$

If $P_1(\mu) = G_1$, then the only option we have is $\overset{4}{\underset{i=1}{\otimes}} Z_2 = P_2(\mu)$. Therefore we only have one fuzzy subgroup of $\overset{4}{\underset{i=1}{\otimes}} Z_2$ namely

$$\mu(x) = \begin{cases} \theta_1, & x \in G_1 \\ \theta_2, & x \in \overset{4}{\underset{i=1}{\otimes}} Z_2 \setminus G_1. \end{cases}$$

Similarly, we have one fuzzy subgroups for $P_1(\mu)= G_2$, $P_1(\mu)= G_3$, $P_1(\mu)= G_4$, $P_1(\mu)= G_5$, $P_1(\mu)= G_6$, $P_1(\mu)= H_{19}$ and $P_1(\mu)= H_{21}$.

If $P_1(\mu)= K_1$, then we have three chains, those are $K_1 < \overset{4}{\underset{i=1}{\otimes}} Z_2$, $K_1 < G_1 < \overset{4}{\underset{i=1}{\otimes}} Z_2$ and $K_1 < G_2 < \overset{4}{\underset{i=1}{\otimes}} Z_2$ .Therefore, we get three fuzzy subgroups of $\overset{4}{\underset{i=1}{\otimes}} Z_2$ with $P_1(\mu)= K_1$, those are

$$\mu(x) = \begin{cases} \theta_1, & x \in K_1 \\ \theta_2, & x \in \overset{4}{\underset{i=1}{\otimes}} Z_2 \setminus K_1 \end{cases} , \qquad \mu(x) = \begin{cases} \theta_1, & x \in K_1 \\ \theta_2, & x \in G_1 \setminus K_1 \\ \theta_3, & x \in \overset{4}{\underset{i=1}{\otimes}} Z_2 \setminus G_1 \end{cases}$$

and

$$\mu(x) = \begin{cases} \theta_1, & x \in K_1 \\ \theta_2, & x \in G_2 \setminus K_1 \\ \theta_3, & x \in \overset{4}{\underset{i=1}{\otimes}} Z_2 \setminus G_2 \end{cases}$$

Similarly, we have three fuzzy subgroups for $P_1(\mu)= K_2$, $P_1(\mu)= K_4$, $P_1(\mu)= K_5$, $P_1(\mu)= K_7$, $P_1(\mu)= K_{11}$, $P_1(\mu)= K_{14}$, $P_1(\mu)= K_{18}$ and $P_1(\mu)= K_{20}$.

If $P_1(\mu)= K_3$, then we have four chains, those are $K_3 < \overset{4}{\underset{i=1}{\otimes}} Z_2$, $K_3 < G_2 < \overset{4}{\underset{i=1}{\otimes}} Z_2$,$K_3 < G_3 < \overset{4}{\underset{i=1}{\otimes}} Z_2$ and $K_3 < G_5 < \overset{4}{\underset{i=1}{\otimes}} Z_2$. Therefore, we get four fuzzy subgroups of $\overset{4}{\underset{i=1}{\otimes}} Z_2$ with $P_1(\mu)= K_3$ and we have the same number for $P_1(\mu)= K_6$.

By similar method, we have

(1) Two fuzzy subgroups of $\overset{4}{\underset{i=1}{\otimes}} Z_2$ for $P_1(\mu)= K_8$, $P_1(\mu)= K_9$, $P_1(\mu)= K_{10}$, $P_1(\mu)= K_{12}$, $P_1(\mu)= K_{13}$, $P_1(\mu)= K_{15}$, $P_1(\mu)= K_{16}$, $P_1(\mu)= K_{17}$, $P_1(\mu)= K_{22}$, $P_1(\mu)= K_{23}$, $P_1(\mu)= K_{24}$ and $P_1(\mu)= K_{25}$.

(2) seven fuzzy subgroups of $\overset{4}{\underset{i=1}{\otimes}} Z_2$ for $P_1(\mu)= H_5$, $P_1(\mu)= H_6$, $P_1(\mu)= H_7$, $P_1(\mu)= H_9$, $P_1(\mu)= H_{11}$, $P_1(\mu)= H_{12}$, $P_1(\mu)= H_{13}$ and $P_1(\mu)= H_{14}$.

(3) nine fuzzy subgroups of $\overset{4}{\underset{i=1}{\otimes}} Z_2$ for $P_1(\mu)= H_8$ and $P_1(\mu)= H_9$.

(4) ten fuzzy subgroups of $\overset{4}{\underset{i=1}{\otimes}} Z_2$ for $P_1(\mu)= H_{15}$.

(5) twenty four fuzzy subgroups of $\overset{4}{\underset{i=1}{\otimes}} Z_2$ for $P_1(\mu)= H_1$, $P_1(\mu)= H_2$, $P_1(\mu)= H_3$ and $P_1(\mu)= H_4$.

Finally if $P_1(\mu)= I$, we have 248 subgroups fuzzy.

Thus, the total number of fuzzy subgroups of $\overset{4}{\underset{i=1}{\otimes}} Z_2$ is 496=2(248).

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