Relativistic Self-Focusing of Intense Laser Beam in Magnetized Plasma

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ABSTRACT

The relativistic self focusing of an intense laser beam, propagating in plasma with longitudinal static magnetic field, has been studied. The laser with Gaussian intensity distribution propagates in plasma. The extent of self focusing increases with increasing of magnetic field. Both the minimum beam width parameter and the self focusing length decrease with increase the magnetic field. The relativistic self focusing of laser beam is increased with the increase of plasma density and laser intensity, also the laser beam gets self-focused at minimum values and then diverges due to the natural diffraction by the plasma medium, self-focusing in a plasma can balance the natural diffraction and laser beam.

التركيز الذاتي لحزمة الليزرعالية الشدة في البلازما الممغنطة

الخلاصة

في هذا البحث تم دراسة التركيز (التبئير) الذاتي لحزمة الليزر عالية الشدة أثناء أنتشارها خلال البلازما الممغنطة حيث أن الليزر ذو النمط الكاوسي يتفاعل لاخطيا مع البلازما الممغنطة. أن عملية التركيز الذاتي لحزمة الليزر تزداد بزيادة شدة المجال المغناطيسي حيث أن كل من القيمة الصغرى لنصف القطر ومدى التركيز الذاتي لحزمة الليزر يقلان بزيادة شدة المجال المغناطيسي. أن عملية التركيز الذاتي لحزمة الليزر بتأثير عامل النسبية تزداد أيضا بزيادة كل من كثافة البلازما و شدة الليزر. عندما يتحقق التوازن بين التركيز الذاتي لحزمة الليزر وأنفراجها نتيجة للحيود الطبيعي داخل البلازما تستطيع حزمة الليزر الأنتشار بعملية التوجيه الذاتي داخل البلازما.

INTRODUCTION

Self-focusing of electromagnetic beams occupies a unique place in the field of the interaction of electromagnetic radiation with nonlinear media, because the change in the transverse irradiance distribution associated with self-focusing affects other nonlinear phenomena [1].

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Early studies of self-focusing of electromagnetic beams in nonlinear media, reviewed by Akhmanov *et al*. [2] were based on the assumption of quadratic nonlinearity of the dielectric function. Such studies predict a single focus in the steady state; the theory, however, breaks down near the focus. This theory is also not applicable to beams with high irradiance on account of the corresponding invalidity of the quadratic nonlinearity. Basic nonlinear physical mechanisms which play crucial role in self-focusing phenomenon are collisional, ponderomotive, relativistic, heating type as reported in the research work [3].

Relativistic and ponderomotive self-focusing of short-pulse radiation beams in plasmas has been investigated by a number of researchers [4-8]. It is further established that relativistic and ponderomotive nonlinear processes occur together [9]. Max [10] presented steady state solutions for the radiation beam structure due to poderomotive self-focusing in relativistic plasma. Sodha *et al* [11] studied steady state self-focusing in magnetoplasma. They considered two different situations in order to show explicitly the effects of nonlinearities arising through the heating of electrons and the poderomotive force. Numerical solutions have been attempted for arbitrary times, including the combined effect of relativistic, poderomotive and thermal self-focusing [10, 11]. Pukhov *et al* [12] studied the relativistic magnetic self-channeling of light in near-critical plasma by using three dimensional particle-in-cell simulations. Hassoon *et al.* studied the relativistic self-focusing of an intense laser beam, propagating in a plasma transverse to the ambient magnetic field, by using the slab model. The extent of self-focusing increases with increasing magnetic field [13].

In this paper, the effect of a longitudinal dc magnetic field on the self-focusing of a Gaussian intense laser beam propagating in underdense plasma has been studied. The laser beam is linearly polarized and propagates along the direction of the magnetic field. When such high laser intensity interacts nonlinearly with the plasma medium, it exerts sharp radial density gradient in the plasma, due to the relativistic electron mass nonlinearity. Then the pump laser beam undergoes relativistic self-focusing during its propagation through the plasma. The laser beam can propagate as a spatial solution when the self-focusing and the natural diffraction effects are balanced.
The magnetic field introduces cyclotron frequency on the plasma electrons which enhance the relativistic nonlinearity and the process of self-focusing of the pump laser beam.

In section 2 the relativistic self-focusing of intense laser beam is studied analytically and numerically. In section 3 the effect of the dc magnetic field on the relativistic self-focusing of intense laser beam is considered. Discussion of the results is presented in section 4.

RELATIVISTIC SELF-FOCUSING OF THE INTENSE LASER BEAM

Consider the propagation of an intense (high power) laser beam of frequency ω_0, wave number $k_0 = \left((\omega_0/c)\left\{1-\left(\omega_p^2/\omega_0^2\right)\right\}^{1/2}\right)$ in unmagnetized plasma along the z direction.

The Gaussian electromagnetic wave (laser beam) is linearly polarized $\vec{E}_0 = (\hat{y}E_0)$, where $\vec{E}_0$ is the electric field of the electromagnetic wave. The intensity distribution of the beam along the wave front at (Z=0) is given by:

$$E_0.E_0^*\big|_{z=0} = E_{00}^2 e^{-r^2/r_0^2} \qquad \dots (1)$$

Where r_0 is the initial beam width, r is the radial coordinate of cylindrical coordinate system, and E_0 is the axial amplitude. The dielectric constant of the plasma is given by,

$$\varepsilon_0 = 1 - \frac{\omega_p^2}{\omega_0^2} \qquad \dots (2)$$

Where $\omega_p = (4\pi n_e e^2/m)^{1/2}$ is the plasma frequency? Here n_e is the equilibrium electron plasma density, $-e$ and m are the electronic charge and mass. Substituting for relativistic mass $m_\gamma = (\gamma m_0)$ where m_0 is the electron rest mass, and the intensity-dependent dielectric constant of the plasma is

$$\varepsilon_{0\gamma} = 1 - \frac{\omega_p^2}{\gamma\omega_0^2} \qquad \dots (3)$$

where γ is the relativistic factor given by

$$\gamma \approx [1 + \alpha E_0 E_0^*]^{1/2} \qquad \dots (4)$$

Here, $\alpha = (e^2/m_0^2\omega_0^2 c^2)$.

The wave equation governing the electric field of the pump laser beam in plasma is

$$\nabla^2 \vec{E}_0 = \frac{1}{c^2}\frac{\partial^2 \vec{E}_0}{\partial t^2} + \frac{4\pi}{c^2}\frac{\partial \vec{J}}{\partial t} \qquad \dots (5)$$

Where $\vec{J}$ is the high-frequency current density and the term $\vec{\nabla}(\vec{\nabla}.\vec{E}_0)$ has been neglected as long as $(\omega_p^2/\omega_0^2)1/\varepsilon_0 \ln\varepsilon_0 << 1$, (Sodha, *et al.*)[14]. Substituting the value of $\vec{J}$ in the above equation and assuming, the variation of the electric field as

$$\vec{E}_0 = \vec{A}_0(x, y, z)\exp[-i(\omega_0 t - k_0 z)] \qquad \dots (6)$$

The following equation can be obtained,

$$-k_0^2 A_0 - 2ik_0\frac{\partial A_0}{\partial z} + \nabla_\perp^2 A_0 = -\frac{\omega_0^2}{c^2}\varepsilon_0 A_0 \qquad (7)$$

Here A_0 is complex function of space, further, assuming the variation of A_0 is [2]

$$A_0 = A_{00}(x,y,z)\exp[-ik_0 S_0(x,y,z)] \qquad \dots (8)$$

Where, A_{00} and S_0 are the real functions of space given by,

$$A_{00} = \frac{E_{00}}{f_0} e^{-r^2/2r_0^2 f_0^2} \qquad \dots (9)$$

the laser beam intensity, f_0 is the dimensionless beam width parameter and,

$$S_0 = \frac{r_0^2}{2f_0}\frac{df_0}{dz}. \qquad \dots (10)$$

By substituting Eq. (8) into Eq. (7) and separating the real and imaginary parts of the resulting equation, the following set of equations, are obtained.
The real part from equation (7) is given by

$$2\frac{\partial S_0}{\partial z} + \left(\frac{\partial S_0}{\partial r}\right)^2 = \frac{\omega_0^2}{c^2 k_0^2} + \frac{1}{k_0^2 A_{00}}\left(\frac{\partial^2 A_{00}}{\partial r^2} + \frac{1}{r}\frac{\partial A_{00}}{\partial r}\right). \qquad \dots (11)$$

And the imaginary part from Equation (7) is given by

$$\frac{\partial A_{00}^2}{\partial z} + A_{00}^2\left(\frac{\partial^2 S_0}{\partial r^2} + \frac{1}{r}\frac{\partial S_0}{\partial r}\right) + \frac{\partial A_{00}^2}{\partial r}\frac{\partial S_0}{\partial r} = 0 \qquad \dots (12)$$

By substituting Equations (9) and (10) into Equation (11) and equating the coefficient of r^2 on both sides of the resulting equation, the equation governing the beam width parameter f_0 is obtained:

$$\frac{d^2 f_0}{dz^2} = \frac{1}{k_0^2 r_0^4 f_0^3} - \frac{\omega_p^2 \alpha E_{00}^2}{2c^2 k_0^2 r_0^2 f_0^2} \qquad \dots (13)$$

In Equation (13), the first term on the right-hand side is the diffraction term and the second term is the nonlinear term due to relativistic nonlinearity. By choosing the parameters of the laser beam and plasma, the two terms on the right hand-hand side, balance each other exactly with $f_0 = 1$, and the laser beam propagates in self-guided mode.Following the boundary conditions for an initially plane wave front, $f_0|_{z=0} = 1$ and $df_0/dz = 0$, one can write Equation (13) in the dimensionless form

$$\frac{d^2 f_0}{d\zeta^2} = \frac{1}{f_0^3} - \frac{R}{f_0^2} \qquad \dots (14)$$

Where $\zeta = z/R_d$, here $R_d = k_0 r_0^2$ and $R = \frac{1}{2}\frac{\omega_p^2}{\omega_0^2} k_0^2 r_0^2 \alpha E_{00}^2$

A numerical computation of Equation (3-1-14), using Runge-Kutta method has been performed to solve the dimensionless beam width parameter f_0 with the distance of propagation for laser beam and numerical results are obtained with typical laser parameters; the vacuum wavelength of the laser beam is $(\lambda_0 = 1.053\,\mu m)$, the initial radius of the laser is $(r_0 = 10\,\mu m)$ for different laser beam intensities (αE_{00}^2) and at different plasma densities (ω_P^2/ω_0^2).

Figure (1) describes the variation of the dimensionless laser beam width parameter (f_0) with the normalized distance of propagation (ζ) for the following parameters: $\alpha E_{00}^2 = 0.3$, and $k_0^2 r_0^2 = 400$,at different plasma densities. For all values of plasma density (ω_P^2/ω_0^2) the laser beam gets self-focused at minimum values and then diverges due to the natural diffraction by the plasma medium. When the plasma density is increased from $0.04-0.12$ the laser beam gets self-focused at relatively less propagation distance and attain slowest minimum beam width value.

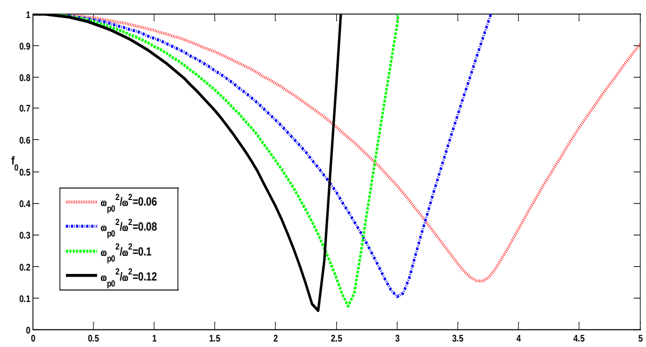

Figure (1) Variation of laser beam width parameter f_0 with normalized distance propagation $\zeta = (z/R_d)$ for different plasma density $(\omega_P^2/\omega_0^2 = 0.06, 0.08, 0.1, 0.12)$ at a pump laser intensity $\alpha E_{00}^2 = 0.3$, and $k_0^2 r_0^2 = 400$.

Figure (2) describes the variation of the dimensionless laser beam width parameter (f_0) with the normalized distance of propagation (ζ),for the following parameters: $\omega_p^2/\omega_0^2 = 0.06$ and $k_0^2 r_0^2 = 400$,at different values of laser pump intensities (αE_{00}^2). For all values of laser intensities, the laser beam gets self-focused at minimum values and then diverges due to the natural diffraction by the plasma medium. When the pump laser intensity is increased from $0.4-0.7$ the laser beam gets self-focused at relatively less propagation distance and attains lowest minimum beam width value.

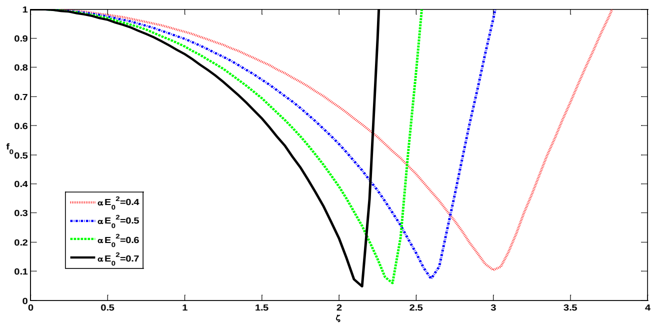

Figure (2) Variation of laser beam width parameter f_0 with normalized distance propagation $\zeta = (z/R_d)$ for different pump laser intensities $(\alpha E_{00}^2 = 0.4, 0.5, 0.6, 0.7)$, at a plasma density $(\omega_P^2/\omega_0^2 = 0.06)$ and $k_0^2 r_0^2 = 400$.

Figure (3) describes also the variation of the dimensionless laser beam width parameter (f_0) with the normalized distance of propagation (ζ), for the following parameters: $\alpha E_{00}^2 = 0.5$ and $\omega_p^2/\omega_0^2 = 0.06$, but for different values for the pump laser beam widths $(k_0^2 r_0^2)$. For all values the pump laser beam widths, the laser beam gets self-focused at minimum values and then diverges due to the natural diffraction by the plasma medium. When the pump laser beam widths is increased from 300 -700 the laser beam gets self-focused at relatively less propagation distance and attains lowest minimum beam width value.

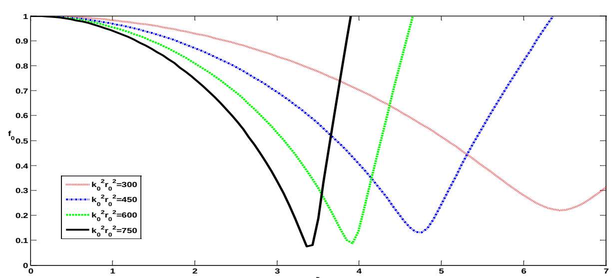

Figure (3) Variation beam width parameter f_0 with normalized distance propagation $\zeta = (z/R_d)$ for different laser beam widths $(k_0^2 r_0^2 = 300, 450, 600, 750)$ at pump laser intensity $(\alpha E_{00}^2 = 0.5)$, and plasma density $(\omega_P^2/\omega_0^2 = 0.06)$.

RELATIVISTIC SELF-FOCUSING OF LASER BEAM WITH MAGNETIC FIELD

Consider the propagation of same intense Gaussian laser beam of frequency ω_0 and wave number k_0 in a magnetoplasma along the direction of a static magnetic field $B_s \| \hat{z}$. The laser is linearly polarized $\vec{E}_0 = (\hat{y}E_0)$, where $\vec{E}_0$ is the electric field of the electromagnetic wave.

The wave number k_0 in the presence of external static magnetic field is given by,

$$k_0 = \left((\omega_0/c)\left\{1-\left(\omega_p^2/\omega_0^2\right)\right\}^{1/2}\right) \quad \dots (15)$$

Where $\omega_c = (eB_s/mc)$ is the electron cyclotron frequency, and the relativistic equation of motion is

$$\frac{\partial}{\partial t}(\gamma m\vec{v}_0) + \gamma m(\vec{v}_0 \cdot \nabla\vec{v}_0) = -e\left[\vec{E}_0 + \frac{1}{c}\vec{v}_0 \times \left(\vec{B}_0 + \vec{B}_s\right)\right] \quad \dots (16)$$

Applying the perturbation approximation, by neglecting the terms $(\vec{v}_0 \cdot \nabla\vec{v}_0)$ and $\vec{v}_0 \times \vec{B}_0$ as the products of small quantities, and using the linearization then plasma electron equation of motion reduces to,

$$im\gamma\omega\vec{v}_0 = e\left[\vec{E}_0 + \frac{1}{c}\left(\vec{v}_0 \times \vec{B}_s\right)\right] \quad \dots (17)$$

Further simplification

$$i\omega\vec{v}_0 = e\frac{\vec{E}_0}{\gamma m} + \vec{v}_0 \times \frac{\vec{\omega}_c}{\gamma} \quad \dots (18)$$

Giving $(\vec{v}_0)$ as

$$\vec{v}_0 = -\frac{e\left(i\omega\vec{E}_0 + \vec{E}_0 \times \vec{\omega}_c/\gamma\right)}{m\gamma\left(\omega_0^2 - \omega_c^2/\gamma^2\right)} \quad \dots (19)$$

Applying high intensity external static magnetic field makes the effective nonlinear dielectric (permittivity) tensor $(\underline{\underline{\varepsilon}} = \underline{\underline{I}} + 4\pi i\underline{\underline{\sigma}}/\omega_0)$ direction dependent, and the dielectric function of the plasma medium shows anisotropic behavior. The components of the effective nonlinear dielectric (permittivity) tensor in magneto plasma due to the relativistic mass nonlinearity with the effect of magnetic field for the mode of propagation can be written as

$$\varepsilon_{yy} = \varepsilon_{xx} = 1 - \frac{\omega_p^2/\gamma}{\omega_0^2 - \omega_c^2/\gamma^2} \qquad \text{… (20a)}$$

$$\varepsilon_{zz} = 1 - \frac{\omega_p^2}{\gamma\omega_0^2} \qquad \text{… (20b)}$$

$$\varepsilon_{yx} = -\varepsilon_{xy} = i\,\frac{\omega_p^2\omega_c/\gamma^2}{\omega\left(\omega_0^2 - \omega_c^2/\gamma^2\right)} \qquad \text{… (20c)}$$

$$\varepsilon_{xz} = \varepsilon_{zx} = \varepsilon_{yz} = \varepsilon_{zy} = 0 \qquad \text{… (20d)}$$

And the beam width parameter equation for nonlinear relativistic case of magneto plasma can be modified for the effective permittivity $\left(\varepsilon_{eff}\right)$.

$$\varepsilon_{eff} = 1 - \frac{\omega_p^2/\gamma}{\omega_0^2}\,\frac{\omega_0^2 - \omega_p^2/\gamma}{\omega_0^2 - \omega_p^2/\gamma - \omega_c^2/\gamma^2} \qquad \text{… (21)}$$

Figure (4) describes the variation of the dimensionless laser beam width parameter $\left(f_0\right)$ with the normalized distance of propagation $\left(\zeta\right)$ for the following parameters: $\alpha E_0^2 = 0.3$, and $k_0^2 r_0^2 = 400$, at different plasma densities with effect of electron cyclotron frequency (ω_c/ω_0=0.6). For all values of plasma density $\left(\omega_P^2/\omega_0^2\right)$ the laser beam gets self-focused at minimum values and then diverges due to the natural diffraction by the plasma medium. When the plasma density is increased from $0.04 - 0.12$ the laser beam gets self-focused and the magnetic field enhancement the focused at relatively less propagation distance and attains lowest minimum beam width value.

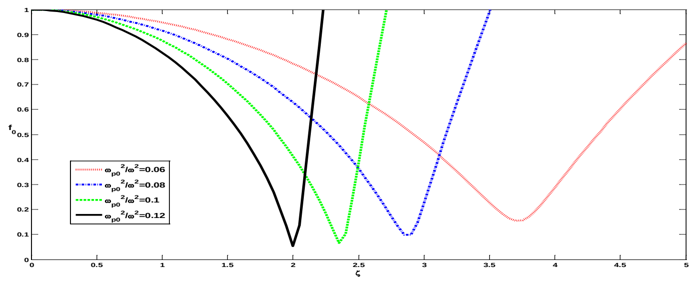

Figure (4) Variation of laser beam width parameter f_0 with normalized distance propagation $\zeta = (z/R_d)$ for different plasma density $(\omega_P^2/\omega_0^2 = 0.06, 0.08, 0.1, 0.12)$ at a pump laser intensity $\alpha E_0^2 = 0.3$,(ω_c/ω_0=0.6), and $k_0^2 r_0^2 = 400$.

Figure (5) describes the variation of the dimensionless laser beam width parameter (f_0) with the normalized distance of propagation (ζ), for the following parameters: $\omega_p^2/\omega_0^2 = 0.06$ and $k_0^2 r_0^2 = 400$,at different values of laser pump intensities (αE_0^2). For all values of laser intensities, the laser beam gets self-focused at minimum values and then diverges due to the natural diffraction by the plasma medium. When the pump laser intensity is increased from $0.4 - 0.7$ the laser beam gets self-focused at relatively less propagation distance and attains lowest minimum beam width value.

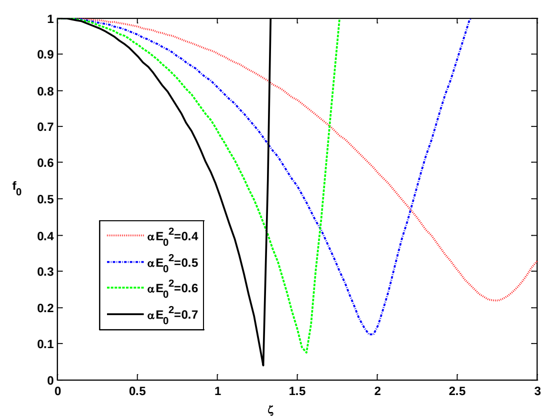

Figure (5) Variation of laser beam width parameter f_0 with normalized distance propagation $\zeta = (z/R_d)$ for different pump laser intensities $(\alpha E_0^2 = 0.4, 0.5, 0.6, 0.7)$,at a plasma density $(\omega_P^2/\omega_0^2 = 0.06)$,$(\omega_c/\omega_0 = 0.45)$ and $k_0^2 r_0^2 = 400$.

Figure (6) describes also the variation of the dimensionless laser beam width parameter (f_0) with the normalized distance of propagation (ζ), for the following parameters: $\alpha E_0^2 = 0.3$ and $\omega_p^2/\omega_0^2 = 0.06$, but for different values for the pump laser beam widths $(k_0^2 r_0^2)$. For all values the pump laser beam widths, the laser beam gets self-focused at minimum values and then diverges due to the natural diffraction by the plasma medium. When the pump laser beam widths is increased from 300 -700 the laser beam gets self-focused at relatively less propagation distance and attains lowest minimum beam width value and observed the effect of magnetic field when the beam width needs less propagation to focused than without magnetic field.

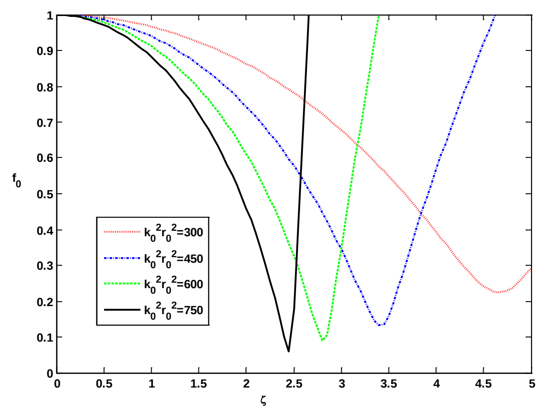

Figure (6) Variation of laser beam width parameter f_0 with normalized distance propagation $\zeta = (z/R_d)$ for different laser beam widths $(k_0^2 r_0^2 = 300{,}450{,}600{,}750)$ at pump laser intensity $(\alpha E_0^2 = 0.5)$, $(\omega_c / \omega_0 = 0.6)$, and plasma density $(\omega_P^2/\omega_0^2 = 0.06)$.

Figure (7) describe the variation of the dimensionless laser beam width parameter (f_0) with the normalized distance of propagation (ζ),under the effect of dc magnetic field for the following parameters: $\alpha E_0^2 = 0.5$, $k_0^2 r_0^2 = 400$, $\omega_p^2/\omega_0^2 = 0.06$ .The beam width parameter initially decreases for all our values of ω_c^2/ω_0^2, attains a minimum beam width due to relativistic self-focusing and then diverges due to diffraction effect, the minimum beam width is the lowest for higher value of ω_c^2/ω_0^2 .

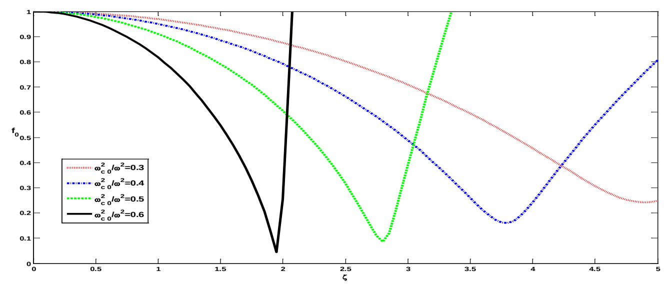

Figure (7) Variation of laser beam width parameter f_0 with normalized distance propagation $\zeta = (z/R_d)$ for different electron cyclotron frequencies $(\omega_c^2/\omega_0^2 = 0.3, 0.4, 0.5, 0.6)$ at plasma density $(\omega_P^2/\omega_0^2 = 0.06)$, pump laser intensity $\alpha E_0^2 = 0.5$, and $k_0^2 r_0^2 = 400$.

Figure (8) and 9 indicate the variation of $f_{\min}$ as a function of ω_{co}/ω for different value of $\omega_{po}{}^2/\omega^2$ and αE_0^2 Both the minimum spot size and the self focusing length decrease as the magnetic field increases for the selected values of $\omega_{po}{}^2/\omega^2$ and αE_0^2.

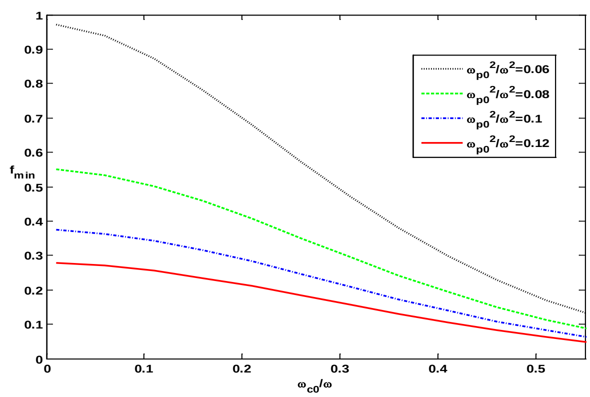

Figure (8) Variation of minimum beam width parameter $f_{\min}$ with normalized electron Cyclotron frequency ω_{c0}/ω for different plasma density $(\omega_P^2/\omega_0^2 = 0.06, 0.08, 0.1, 0.12)$ at a pump laser intensity $\alpha E_0^2 = 0.3$,and $k_0^2 r_0^2 = 400$.

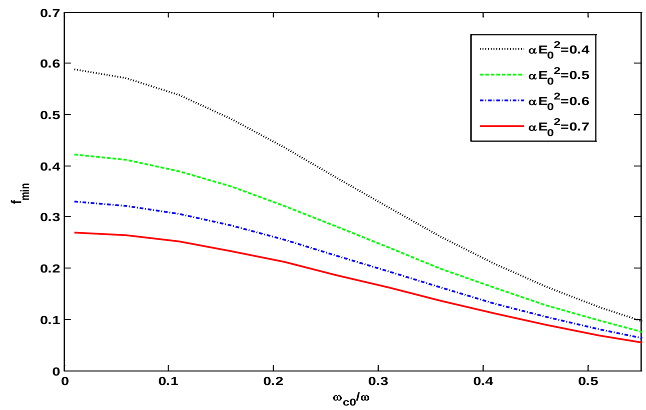

Figure (9) Variation of minimum beam width parameter $f_{\min}$ with normalized electron cyclotron frequency ω_{c0}/ω for different pump laser intensities $(\alpha E_0^2 = 0.4,0.5,0.6,0.7)$,at a plasma density $(\omega_P^2/\omega_0^2 = 0.06)$ (ω_c/ω_0=0.45) and $k_0^2 r_0^2 = 400$.

DISCUSSION

The relativistic self-focusing of intense laser beam $(\sim 10^{19}\, Wcm^{-2})$ in underdense magnetized plasma has been studied in the present chapter. Analytical formulation and numerical solution for the relativistic self-focusing process have been carried out for different laser powers (intensities), plasma frequencies (densities) and static magnetic field intensity values.

The physical conditions for the analysis of the self-focusing are for linearly polarized Gaussian laser beam propagating in collision less under dense magnetized plasma. The analysis has been done by assuming cylindrical symmetry and that the initial laser beam width having a Gaussian intensity distribution in the plane transverse to the direction of propagation (i.e. viz, z-direction) and the static magnetic field is assumed parallel to the direction of propagation of the laser beam. The analysis is valid within the paraxial approximation $(r \langle\langle\, r_0 f_0)$.

Intense Gaussian laser beam propagates through plasma produces greater electron quiver motion on axis than off axis, hence results change in the refractive index of the plasma which depends on the relativistic electron mass and then relativistic self-focusing of the laser beam take place in the nonlinear plasma medium.

The presence of static magnetic field on the plasma can significantly influence the self-focusing process via the effect of Lorentz force on the plasma electrons.

It was found that the static magnetic field enhanced the efficiency of the self-focusing process due to electron cyclotron resonance and when $(\omega_c \sim \omega_0)$ the electron quiver shows resonant enhancement, hence the nonlinear effect. As the value of the magnetic field increased the laser beam become self-focused at a faster rate and smaller minimum beam width value.

It was found also that the relativistic self-focusing process depends or related directly on the initial laser intensity, plasma density and initial laser beam width (radius). As these parameters increased in their values, the laser beam become self-focused at a faster rate and smaller minimum beam width value, i.e., the distance (where the laser beam attains minimum beam radius) when the relativistic nonlinear term and the natural diffraction term balance each other will be shorter.

As a result, when laser beam or pulse undergoes self-focusing/guiding process, it can propagates over a long distance through the plasma while maintaining high interaction intensity, drastically increasing the efficiency of various applications such as high harmonic generation, parametric instabilities, electron accelerator, x-ray lasers, advanced fusion energy and others.

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