# An Improve Differential Box Counting Methodto Estimate Fractal dimension

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## Abstract

The mathematical concept that used to measure the complexity of the fractal sets is known as fractal dimension. It is considered as a global feature for fractal images. Many methods have been proposed to estimate this value according to object nature. Differential Box Counting method is one of the most important methods used to estimate the dimension of gray scale image. In this paper, we proposed a new method based on the aggregate gray level values in the image. The results of the experiments show an improvement over the traditional DBC method.

**Keywords:** Iterated Function System (IFS), fractal dimension, box-counting method, Differential Box Counting DBC, grayscale images.

## طريقة عد الصناديق المحسنه لتخمين البعد الكسوري

**الخلاصه**

يطلق على المصطلح الرياضي الذي يستخدم لقياس درجة التعقيد بالبعد الكسوري. ويعتبر عامل مهم في الصور الكسورية.الكثير من الطرق التي اقترحت لتخمين قيمته اعتماداً على طبيعة الشكل.طريقة الصندوق المشتقه تعتبر من اهم الطرق المستخدمة لايجاد البعد الكسوري للصور الرمادية.في بحثنا هذا. سنقوم بأقتراح طريقة جديده تعتمد على معدل تاثير القيم اللونية الرمادية للصور. النتائج العملية ستوضح بأن الطريقة المقترحه تعتبر طريقه محسنه لطريقة الصندوق المشتق التقليدية.

## INTRODUCTION

To describe an irregular phenomena and self-similar objects a new branch of nonlinear science known as fractal has been emerged and serves as a good tool for this purpose. It was first introduced by the IBM mathematician Benoit Mandelbrot [1] in 1975 coined from the Latin word "Fractus", which means the broken to describe realistic objects. Since then, this concept has been proven to be suitable in many applications. Practically, in image processing and data vision [2,3]. Beside this important applications, it was been used recently to develop the security

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of cryptography and steganography system [4-6].This discovery made our way of thinking enter to a nonlinear stage and provide us a new view to the world.

Fractals can be generated by an interpolation method called Iterated Function system (IFS) introduced by Barnsley [7] based on the self-similar objects. Then fractal is an image represents the attractor generated by this method. He assume that, the use of the IFS transformation led to generate self-similar objects that can be used to describe some phenomenon which cannot been explained with Euclidean geometry.

Fractal dimension is a mathematical concept used to measure the geometrical complexity of fractal set. It is defined for fractal geometric images, and considered as global features for them. Due to its importance for image analysis and classification, many methods have been emerged and developed to estimate the fractal dimension of images.

Mandelbrot [1] was the first who described an approach to find the fractal dimension while he tried to find the length of the coastlines. In 1984, S. Peleg [8] worked on the 2D images and in this case the image can be represented as a hilly terrain surface. Pentland in [2] noted that Fourier power spectrum of fractal Brownian function (*f*) is proportional to the following expression: (*f-2h-1*), and $h = 2 - D$, where *D* is the Fractal Dimension (*FD*). It can be calculated using the least square linear fit between *P( f )* and *f*, where *P(f)* is the Fourier power spectrum. J.Gangepain and R.Carmes in [9] propose a method named the reticular cell counting approach. In their approach they consider a *3D* space *(x,y)* coordinates to represent *2D* position and the third *z* coordinate to represent the image gray level, then for a given scale *L*, they partitioned the *3D* space into boxes of size $L \times L \times L'$ and by calculating the number $N_L$ of the subcubes that at least one sample from the intensity surface, they concluded the following relation:

$$N_L = \frac{1}{r^D} = \left[\frac{L_{max}}{L}\right]^D \qquad \dots(1)$$

Where

*D* represents the fractal dimension

After that, Nirupam Sarkar and B. B. Chaudhuri [10] designed one of the popular methods in box counting theorem named as Differential Box Counting method (*DBC*), they used it to estimate the dimension for gray level images. S.Bczkowski et al. [11] proposed a new box counting method named as Modified Differential Box Counting MDBC. They tried to decrease the errors occurring from traditional differential box counting method through working on two principles; the border effects, and the non-integer value of $\varepsilon$. They proved that their method is more powerful and very simple to use. N. Theera in [12] proposes a new method to estimate the fractal dimension. His method is an extension of differential box counting method by incorporating order statistically. The new technique yields a good performance, and provided better discrimination than the differential box counting. S.Tao Liu in [13] discusses the problem of differential box counting method, which is the range of linear scales and the precise number of boxes that cover the image plane; he also proposes a new method to find the number of boxes. The results show that his method is not just more effective but also takes less computational time. J. Mishra and S.P. Pradhan in [14] propose a new effective box counting method to estimate fractal dimension, and from the experimental on gray scale images, they showed that their method not only has more precise estimated value of fractal dimension, but also takes less computational time compared with the traditional differential box counting method. R. Abiyev and K. Ihsan in [15] proposed a new method to estimate fractal dimension. Their method differs from the differential box counting method in computing the number of boxes that cover the image. The new method takes the aggregate effects of the gray levels not just for the maximum and the minimum gray levels as in differential box counting. An improved *DBC* method is presented in this paper by modifying the basic ideas for partitioning of the image plan. It was used to estimate the fractal dimension for gray scale images, the experimental results show that the proposed method produced more precise value of the fractal dimension compared to Sarkar *DBC* method. The reminder of this work is structured as follows: Section 2 is summarized the basic definition of the fractal dimension approach and the *DBC* basic concepts, and drawbacks of the Sarkar *DBC* method. Section 3 introduced our proposed method with the procedure for the fractal dimension estimation. Section 4 analyzed the experimental results, whereas, Section 5 exposes the conclusions.

## Basic Concepts for DBC Approach

In this section, some basic concepts are given to fill in some background for the reader. A more detailed review of the topics in this section can be found in [10, 16,17]

## Basic Fractal Dimension

The original approach to estimate the fractal dimension was suggested by Mandelbrot [1] when he tried to find the length of the coastlines. In his approach he considers all point that has $\varepsilon$ distance from the coastlines. These points are formed together a $2\varepsilon$ strip width and suggested the length of the coastlines equal to $L(\varepsilon)$ which is equal to the width of the strip divided by $2\varepsilon$. Hence, when $\varepsilon$ decreased, $L(\varepsilon)$ is increased. Mandelbrot applied this thought for many coastlines and put the following form:

$$L(\varepsilon) = F\varepsilon^{1-D} \qquad \dots (2)$$

Where

$F$ and $D$ are constants for specific coastline. Then $D$ is Fractal Dimension (*FD*) of the line. $D$ can be derived from least square linear fit of log $L(\varepsilon)$ and *log* ($\varepsilon$).

This great definition of fractal dimension was considered as the key that the others depended on in their researches. The developments of computers play an important role in increase the knowledge about fractal dimension. It is used to describe the complexity of the objects. The main advantage of finding the fractal dimension is to get the information about the geometric structures of the fractals, and it refers to the measurement of roughness in the geometrical objects. Thus, there are a lot of attempts to know this dimension based on self-similarity property as criteria of describing fractals. The general definition of the fractal dimension is as follows:

Given a bounded self-similar set $A \subset R^n$ which is a union of $N_r$ different non-overlapping set each of which is similar to $A$, scaled down this set by a ratio $r$, then the fractal dimension $D$ is calculated by [7]:

$D = \lim_{r \to 0} \frac{\log(N_r)}{\log(1/r)}$ …(3)

The *D* value in (3) is a statistical quantity and it clearly shows the complexity of given fractal. Hence, due to this importance, many researchers are tried to give perfect form to estimate it, not only for full self-similar shapes but also for different images have a type of self-similarity. In this work, the comparisons of the results have been done with the Hausdorff dimension [7], where the Hausdorff dimension is considered as the precious value of the dimension for the fractals objects. It is can be defined as follows:

Let *(X, d)* be a metric space in *s*-dimensional set. If $k \subset X$, $d \in [0, \infty)$, then the Hausdorff measure of *k* is defined by:

$H_\varepsilon^s = inf\{\sum_{i=1}^{\infty} r_i^s : \text{where } r_i \text{ is } \varepsilon - cover \text{ for } k\}$ …(4)

Where

$\epsilon > 0$ is a non-negative number.

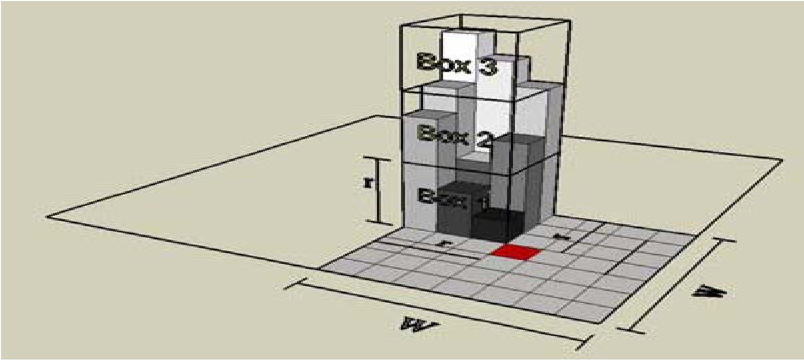
Here, $r^s(k)$ is the *s*-dimensional Hausdorff measure of *k*. For example, if we have a set in a smooth surface of infinite area situated in three dimensional Cartesian space, then $H_\varepsilon^s$ is just the area of *A*, while for *s*<2, $H_\varepsilon^s$=∞, and $H_\varepsilon^s = 0$ for *s*>0. This type of dimension can be applied for both fractal and non-fractal sets.

**The Differential Box Counting Method**

This method is used to estimate the fractal dimension of gray scale images. The main idea of this method is to divide the image surface into grids, cover each grid with boxes, a box is counted if it contains a gray level value. According to DBC method, the 3*D* image of size *M*×*M* pixels with (*x*, *y*) denoting 2*D* position and the 3rd coordinate z denotes the gray level, the (*x*,*y*) space is partitioned into grids of size *s*×*s* by the scale *r=s/M*, where *M*/2≥ *s* >1 and *s* is an integer. On each grid there is a column of boxes of size $s \times s \times \acute{s}$, where $G/_{\acute{s}} = M/_s$ and *G* is the total number of gray level as shown in Figure 1. Assign numbers 1, 2, … to these boxes, so the minimum and the maximum gray level of the image in the $(i,j)^{th}$ grid are fall in box number *k* and *I* respectively. They proposed that,

$n_r(i,j) = I - k + 1$ …(5)

Then the Fractal Dimension *FD* can be estimated from the least square linear fit of log ($N_r = \sum_{r=1}^{M} \sum n_r$) / log (1/r) [10].




**Figure(1). The representation of the gray levels in differential box counting method**

## Drawbacks of the Differential Box Counting Method

Sarkar compared his method with other algorithms. His method has lower computational complexity, best in terms of efficiency and dynamic range of *FD*s, but has lower accuracy.

However, some disadvantages of *DBC* have been discussed in other literatures [18,19]. They pointed that the *DBC* method has two major problems.

- The overcounting of the total number of boxes that is cover the image intensity surface through the *x* and *y* directions or also for *z* direction
- The undercounting in total number of boxes, this problem occurs when the image contains sharp gray edges.

The two previous problems give an inaccurate value of the fractal dimension, or give the same *FD* of two images with the same roughness and different gray levels. For simplicity, if we generate a new image by increase the gray levels with fixed value, then the fractal dimension of the new image should remains the same as the original image because they have the same roughness, but when the *DBC* is uses this two *FD* values will be distinct.

However, according to the analysis of these two shortcomings, a new method has been proposed to prove that the accuracy of the *DBC* is limited. The improvement in the *DBC* method is concealed in counting the total number of boxes that cover the image surface.

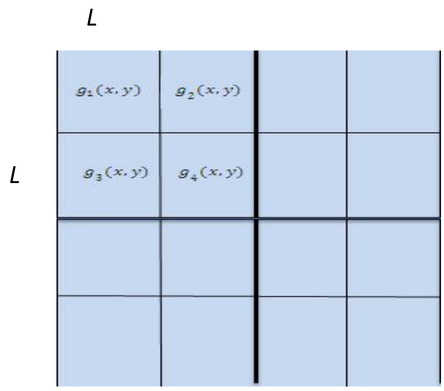
## The Proposed Method

From the authority of DBC structure, it is clear that the exactness of Sarkar *DBC* method is limited, thus, some modification has been proposed to increase its accuracy, so the purpose of this modification is to solve the exactness property and as follows:

For an image of size $P \times P$ pixels in 3*D* space, the $(x, y)$ space is partitioned into grids of size $t \times t$ as in the traditional differential box counting method. For the scale $r = t/P$, where $t$ is an integer and $P/2 \geq t > 1$, the total number of boxes $N_r$ is counted in the proposed method as follows:

Let $g_1(x,y), g_2(x,y) \dots g_{r\times r}(x,y)$ be the gray level values in each box as shown in Figure 2. For each box of size $r \times r$, the number of boxes can be calculated by the following relation

$$n_r = \frac{\sum_{i=1}^{r} \sum g_i(x,y)}{r^2} + 1 \qquad \dots(6)$$




**Figure(2). The structure of the proposed method**

Where

the total number of boxes $N_r$ that cover the image intensity surface is calculated as follows:

$$N_r = \sum_{r=1}^{p} n_r \qquad \ldots(7)$$

Therefore, the *FD* can be estimated from the least squares linear fit of $log\ (N_r)$ over *log (1/r)*. The most important property for the new algorithm is to estimate the dimension of the *3D* images based on the aggregate effect of the gray level at each box of size *r*, i.e. the number of boxes $n_r$ at each grid is based on the total number of gray levels in that grid. In classical differential box counting algorithm, only two pixel values namely the difference between the minimum and the maximum gray levels are included in the calculation of the dimension, which causes a non-precise dimension. The algorithm of the proposed *DBC* method to calculate the fractal dimension for gray scale images is as follows.

**The algorithm of the proposed *DBC* method**

**Algorithm 1:** (this algorithm is used to find the dimension of the gray scale images by the proposed method).

Input: image of size $M \times M$.

Output: fractal dimension FD.

i. On each grid there is a column of boxes of size$s \times s \times s$, where ${}^{G}/_{s} = {}^{M}/_{s}$ and *G* is the total number of gray level.

ii. Assign numbers 1, 2, … to the boxes as shown in Figure 3.4. Let the minimum and the maximum gray level of the image in the $(i,j)^{th}$ grid fall in box number *K* and *I* respectively. By the proposed method

)$n_r(i,j) = (\sum_{r=1}^{M} {}^{g_i(x,y)}/_{r^2}) + 1$

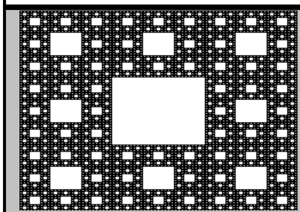
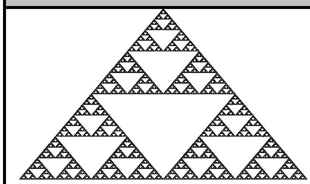
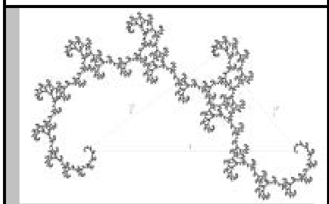
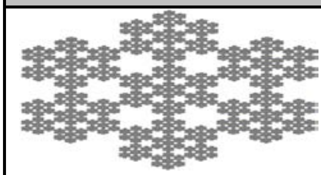
iii. The fractal dimension (FD) can be estimated from the least square linear fit of log ($N_r = \sum n_r$) / log (1/r).

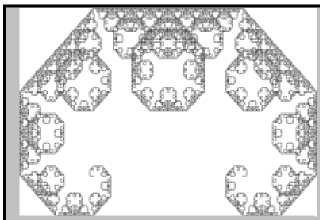
iv. **End**

**The experimental results**

In this section, our proposed method to estimate the fractal dimension have been compared with Sarkar DBC method which have been applied on set of five gray scale images as shown in Table 1. These five images have the same size and the same resolution with distinct gray levels. The gray scale images that have well known Hausdorff dimension is tested.

**Table (1). Comparison between Sarkar DBC, the proposed DBC, and the Hausdorff dimension**

| Fractal name | Hausdorff dimension | Sarkar DBC | The proposed DBC |
|---|---|---|---|
|  S1 | **1.8928** | **1.4779** | **1.7464** |
|  S2 | **1.5850** | **1.8760** | **1.8114** |
|  S3 | **1.61803** | **1.7292** | **1.6661** |
|  S4 | **1.7712** | **1.5556** | **1.8710** |

| | | | |
|---|---|---|---|
|  S5 | **1.9340** | **1.3086** | **1.8951** |

The reason for choosing images of well-known Hausdorff dimension in order to show the improvement in *DBC* value over Sarkar method, where the Hausdorff dimensions for the fractals that are used in the experiments are manual calculated, the Hausdorff dimension always gives the precious value for the dimension, so it has been used in this experiment as a tester of the efficiency.

Table1 shows that, the estimated dimension by the proposed method is closer to the Hausdorff dimension than the traditional *DBC* dimension, and since the Hausdorff has been used as a tester then it can be obviously notice that the proposed DBC is more accurate (always closer to the Hausdorff dimension) than the traditional DBC . Figure3shows the accuracy of the proposed *DBC* fitted curve over Sarkar *DBC*. The proposed DBC at each point in x-axis which represent the fractal dimension for the tested images always closer together with the Hausdorff dimension rather than DBC. It is clear from Figure3 that the proposed *DBC* dimension is closer to the Hausdorff dimension than Sarkar dimensions.

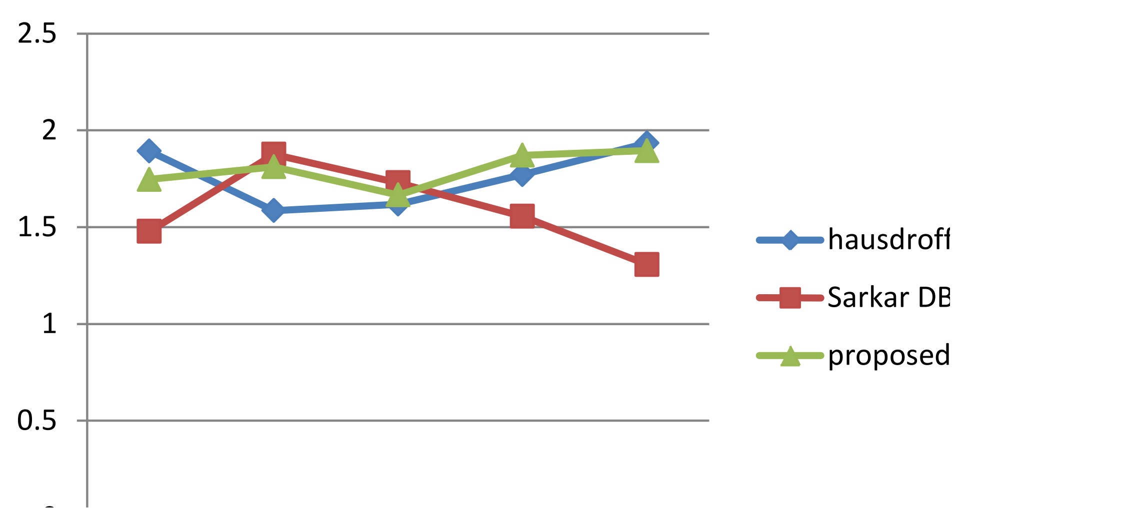



**Figure (3). Comparison between Sarkar DBC and the proposed method with the Hausdorff method**

## Conclusions

For the Sarkar DBC method, there is an accuracy problem, so the fractal dimension estimated by this method sometimes is not precise. In this paper, Sarkar method is modified through proposing a new approach based on the aggregate gray level values in the image to give better estimation for the fractal dimension value. Since the traditional DBC have the draw backs of over counting or under counting the number of boxes that cover the image surface, this problem actually happened since the

Sarkar DBC based mainly on just the maximum and the minimum values to estimate the dimension, this problem has of the gray levels in the image surface.

The experiments is done on a set of five images, the results showed that the proposed method is an improvement over the Sarkar DBC.

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