A Simple Model of Capacitor Discharge Through a Spark Gap

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Abstract
A simple computational model is established to simulate a capacitor discharge process through a spark gap. The model constitutes of three intervals, the first one is concerned with charging the capacitor by a D.C. voltage source, where the voltage across the capacitor raises to a certain critical value regarded as the breakdown voltage of the spark gap. The second interval describes the gap breakdown where the resistance of the ionized gas in the gap decreases very sharply as a result of heating the plasma by the electrical current. This interval is denoted as the resistive phase of the discharge. The third interval describes the discharge through the previously heated plasma in the gap; for this interval the plasma resistance is assumed to have a constant value which is considered as the minimum value obtained at the end of the previous interval (the resistive phase interval). The temporal evolution curves obtained from the model exhibit reasonable trends that conform to the physical situation under study. Also, the comparison made with published data shows an acceptable agreement. The model is employed to perform a parametric comparison to examine the rule of the gap parameters on the voltage and current evolution curves.

نموذج مبسط لتفريغ متسعة خلال فوجة الشرارة

تم إنشاء نموذج حاسوبي لمحاكاة عملية التفريغ الكهربائي لتسعة خلال فوجة شراة. يتضمن النموذج ثلاث فترات زمنية، الفترة الأولى خاصة بشحن التسعة بواسطة مصدر فولتية مستمر، حيث ترتفع قيمة فولتية على طرف التسعة لحين بلوغ قيمة حرجة تمثل قيمة فولتية الأنهيار الكهربائي لفوجة شراة. بينما يتم خلال الفترة الثانية توصيف عملية الأنهيار الكهربائي لفوجة شراة، حيث تضمن المقاومة الكهربائية لغاز المشتاء في الفوجة بشكل حاد نتيجة للتسخين البلازما بالتيار الكهربائي، ويطلق على هذه الفترة بالطور المقاوم للفوجة الكهربائي. أما الفترة الثالثة فتمثل عملية استمرار التفريغ الكهربائي خلال بلازما الفوجة بعد تسخينها خلال الفترة السابقة، حيث يتم هذا افتراض أن مقاومة البلازما ثابتة خلال هذه الفترة، وهي مساوية للقيمة الدنيا لمقاومة النهاية الفجوة السابقة (فترة الطور المقاوم).

忍受ت المنحنى البيانية المتغيرة من هذا النموذج، وال المتعلقة بالتغير الزمني للتيار، والفولتية، منحنى منطقي، يتوافق مع الواقع الفيزيائي تحت دراسة كما بنيت المقارنة التي تم اجراؤها مع بيانات مشتاء توافقا مفولا.
تم استخدام النموذج لأجراء مقارنة يبيان تأثير معاملات الشراة على منحنين التغيرات الزمنية لكل من الفولتية والتيار.

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INTRODUCTION

Electric discharge in a gaseous medium may be established simply by exerting an electric potential difference across a gas contained between two electrodes. The potential difference must be sufficient to cause ionization, and eventually, breakdown of the gaseous medium.

However it is stated that the existence of initiatory free electrons in the gas is necessary for launching an ionization process. These initiatory electrons may be generated by any external energy source (background radioactivity or cosmic ray for example) [1].

Three successive states may be distinguished, the first is ionization, during which the population of charged particles in the gas increases steadily, the second is breakdown, which is a transient state through which the gas is converted from an isolating to a conducting medium, and the third is the discharge state where the ionized gas conducts electric currents [1].

At initial stages of ionization process, the initial electrons are accelerated by the electric field applied between the electrodes. After gaining sufficient energy, these accelerated electrons may ionize other molecules and atoms (neutral particles) in the gas by collisions, hence generating more free electrons and positive ions (electrons and positive ions constitute the main charged particles), as well as negative ions [2].

In addition to collision effect, free electrons may also be generated by secondary emission of electrons from the cathode as it is being bombarded by accelerated positive ions [3]. In fact, secondary emissions may also occur by other processes like field emission, photo emission, and thermionic emission [1,2].

Accordingly, the population of the charged particles increases during ionization, where the generated charged particles may produce additional charged particles and this leads to a steady increment of the electrical conductivity of the gaseous medium.

When the population of the charged particles in the gas reaches sufficient levels (depending on the gas nature and experimental conditions), the formerly isolating gas starts conducting electric currents, and this transient state is the electric breakdown of the gas. The instant of occurrence of this transition is known as the breakdown point, after which the gas undergoes electric discharge (conducts electric currents) provided the existence of an applied electric field through the gas (power source)[1].

Providing sufficient rates of charged particles generation, the discharge process may sustain itself; hence no external agent requirement is necessary any longer to provide free electrons to the gaseous medium [2].

Regarding discharge, several discharge regimes may be classified depending on gas pressure, electrodes spacing and configuration. These discharge modes varies from low current (like glow or corona discharges) to high current modes (like arc discharges) [4].

However, there is a transient mode of discharge known as “spark”, which can be thought of as a transition state toward an arc discharge [2]. Spark discharge is not a steady process like other modes of discharge, where it has a limited life time after which it develops into an arc if the potential difference persists across the gas, otherwise it may last for only a fraction of a second before it extinguishes [5].

In this study, the spark discharge is described through a simple simulation. In a spark, the current rises rapidly due to the fast decrease in the resistance of the discharged gas (plasma) as a result of plasma heating [2,5].
Determining the conditions of a spark breakdown is essential in designing high-voltage apparatus, where many experimental efforts concerning practical problems related to dielectric behavior of external insulators have been established [6]. Practically, sparks are generated through devices known as “spark gaps”, which are composed essentially of electrodes in certain configurations, contained inside a tube filled with a gas. One of the mostly wide applications of spark gaps is to be employed as fast switches devices (FED) in many electrical circuits that require high voltage and high current switching, where spark gaps are considered as heavy-duty closing switches [7].

Theoretical studies and modeling, in addition to computer simulation are strongly required in spark gaps investigation since many difficulties and requirements may arise in experimental work [6]. Moreover, theoretical work and computer simulation may provide insight about different processes that occur in the plasma during spark formation, as well as determining the conditions of a spark breakdown which is a vital issue in designing high voltage apparatus [6].

In the present work, a simple simulation is achieved for the discharge of a capacitor through a spark gap, a situation frequently encountered in electro discharge machining (EDM) [8]. However, air is considered as the dielectric between the electrodes in this work.

The essential issue here is to include the variation of plasma resistance during the spark discharge process period [9], where it is stated that the knowledge of the temporal behavior of the spark gap resistance is sometimes required in the design of heavy-duty switches circuits as well as in improving switches performance [7].

Some Theoretical Aspects

It is instructive to give a brief presentation of some aspects concerning electric discharge development in general. In the ionization state, the multiplication of free electrons due to collisions is referred to as “avalanche” [4]. Townsend is one of the pioneers in studying ionization and breakdown in gases throughout avalanche process. According to Townsend formulation, the increase in electronic current ($i_e$) as the electrons move toward the anode is given as [2]:

$$\frac{di_e}{dx} = \alpha i_e$$  \hspace{1cm} .... (1)

where ($x$) is the distance travelled by the electrons, and ($\alpha$) is known as the Townsend’s first ionization coefficient. If the distance between the electrodes is ($d$), then eq.(1) gives the current at the anode as:

$$i_e = i_o e^{\alpha d}$$  \hspace{1cm} .... (2)

where ($i_o$) is the current resulted by the initial free electrons at the cathode [2].

Townsend expressed the coefficient ($\alpha$) in terms of parameters ($A$) and ($B$) which are obtained experimentally as [1]:

$$\alpha = A p e^{(B p/E)}$$  \hspace{1cm} .... (3)
In eq.(3), the parameters (A) and (B) vary for different gases, while (p) and (E) are respectively the gas pressure and the electric field intensity across the gas. When including secondary emission of electrons from the cathode surface, the rate of current growth will be faster than that obtained by eq.(1). To include the secondary emission into the avalanche process, a coefficient (γ) is introduced, known as the effective secondary emission coefficient (third Townsend coefficient) [4]. By introducing both coefficients (α) and (γ) into the avalanche process, the following expression of the electric current at the anode may be deduced [2]:

\[
    i = \frac{i_o e^{\alpha d}}{1 - \gamma (e^{\alpha d} - 1)} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots (4)
\]

The above equation is an expression of the Townsend mechanism for current growth in the discharged medium [2].

As long as free electrons sources exist, the discharge process may proceed. This is referred to as a “non-self sustained discharge” in which the discharge process ceases when the source of free electrons generation is turned off. However, the discharge may be self-sustained when the number of electrons generated by avalanche processes is sufficient to cause more electrons multiplication and hence sustaining the discharge. The condition for a discharge to be self-sustained is met when the denominator in eq.(4) becomes zero [1], i.e.:

\[
    \gamma (e^{\alpha d} - 1) = 1
\]

or,

\[
    \alpha d = \ln(1 + \frac{1}{\gamma}) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots (5)
\]

The above condition is known as the Townsend criterion for breakdown as it represents the condition where the discharge transforms to a self sustained state. From Esq. (3) and (5) we can get:

\[
    A p d \exp \left( \frac{-B p}{E} \right) = \ln \left( 1 + \frac{1}{\gamma} \right) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots (6)
\]

Considering the electric field intensity (E) as the ratio of the potential difference (V) across the gas and the spacing (d) between the electrodes (i.e. E=V/d), and taking specifically the breakdown potential (V_B) since eq.(5) is a criterion for breakdown, then the above equation may be written in the form [1,2]:

\[
    V_B = \frac{B p d}{\ln(p d) + C} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots (7)
\]

where \[ C = \ln \left[ \frac{A}{\ln \left( 1 + \frac{1}{\gamma} \right)} \right] \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots (8) \]
Equation (7) is known as the Paschen’s law for breakdown, and it can be thought of as an extension of the Townsend’s avalanche mechanism [2].

**Spark Discharge**

It is important to note that Paschen’s law has its limitations [2] which arise due to space charges throughout the discharge zone. Space charges result in electric field distortion among the discharged gas between the electrodes. This circumstance is met for large gaps (i.e. large electrode spacing), electrode irregularity and surface roughness, and also for relatively high pressures [2].

In spark discharges, space charges are likely to be formed in the gaseous medium, causing electric field deformation. Distortions in electric field lead to enhancements of the field intensity at distortion positions. As a result, streamers may be formed, which are thin discharge channels that propagate rapidly between the distortion locations and the electrodes so as to establish electric connections between the electrodes [6]. Streamers cause the breakdown to occur at voltages lower than those predicted by Paschen’s law as well as to initiate at earlier times [2].

After the ignition of breakdown, the ionized gas (plasma) is heated by the electric current. Heating the plasma column (or channel) causes the plasma resistance to decrease, and this is associated with a current increase and voltage drop during the plasma heating period. This period (or phase) is referred to as the “closure phase”, where the spark gap converts from a non-conducting to a conducting state, it is also known as the “resistive phase” since it is the period during which the discharge resistance changes (decreases) [2]. This phase has a significance importance in fast switches design and improvement since it specifies the time-dependant of the spark gap during closure, and hence affecting the current rise time [7].

As a mathematical mean of determining the resistive phase period ($\tau_R$), is the empirical equation that was stated by Sorensen and Ristic for gaseous nitrogen, given as [2]:

$$\tau_R = \frac{44 p^{\frac{3}{2}}}{E Z^3} \quad \ldots \ (9)$$

Where

$(Z)$ is the circuit impedance. Also Sorensen and Ristic established an empirically-based formulation for the spark gap resistance as a function of time in the form [2]:

$$r(t) = 0.23 \left(\frac{\tau_R}{t}\right)^3 Z \quad \ldots \ (10)$$

Being empirical in nature, equation (10) is expected to have its limitations in application. Since the gap resistance varies with time during the resistive phase, it is referred to as the “dynamic resistance” [2].

The Present Calculation Procedure

The present calculations are established to simulate the function of the simple circuit shown in fig. (1) taking into account the dynamic resistance of the plasma generated in the spark gap.
The calculation procedure is composed of three temporal intervals:

1- Capacitor charging.
2- Electric breakdown of the gap; where the capacitor begins to discharge through the gap. In this interval, the gap is considered to be in the resistive phase, where the plasma resistance decreases with time (dynamic resistance).
3- The discharge interval, where the discharge is assumed to proceed with a steady gap resistance (i.e. the plasma resistance is in its minimum).

Throughout the calculations of the present work, the initiating time instant of each of the above intervals is taken as zero; this is possible since each interval is described by its own relations; however, the time factor during the whole process is determined as an accumulative quantity in presenting the final results of the model.

The first interval begins when the switch of the circuit in fig.(1) is closed and the power supply of potential (V_o) starts charging the capacitor. The voltage across the capacitor (v) increases with time according to the known relation [10]:

\[ v(t) = V_o \left( 1 - e^{-\frac{t}{RC}} \right) \]  \hspace{1cm} \text{... (11)}

The charging process continues until the voltage across the capacitor reaches the breakdown value (V_B) of the gap; this happens at a time (t_B) which is regarded as the breakdown instant. As an approximation adopted in this work, equation (7) is used to determine the breakdown voltage (V_B).

The associated current during this interval is found by Ohm’s law with the voltage as determined by eq.(11) and the resistance as the circuit resistance [shown in fig.(1)].

As soon as the capacitor voltage reaches the breakdown value (V_B), the second interval begins.

Throughout the second interval (the gas breakdown), the capacitor is considered to discharge through the spark gap where the ionized gas resistance decreases rapidly. The Ristic and Sorencen’s equation [eq.(10)] is employed in this work to describe the plasma dynamic resistance [r(t)]. The scheme in figure (2) helps to illustrate the situation.
For a capacitor discharge we have [10]:

$$i(t) = -C \frac{dv(t)}{dt} \quad \ldots (12)$$

To formulate the voltage across the gap as a function of time, both of the dynamic resistance [eq.(10)] and the current as expressed in eq.(13) are plugged in Ohm’s law yielding:

$$v(t) = -C \frac{dv(t)}{dt} \times 0.23 \left( \frac{R}{t} \right)^3 Z \quad \ldots (13)$$

Arranging and performing integration, one may obtain an expression for the gap voltage as a time dependent quantity as follows:

$$\int_{v_B}^{v(t)} \frac{dv(t)}{v(t)} = -\frac{1}{0.23 \frac{C Z \tau R^3}{t}} \int_0^t t^3 dt \quad \ldots (14)$$

Here the voltage at the beginning of this interval (at t=0) is considered as the breakdown voltage ($V_B$). Perfroming the integration of eq.(14) leads to:

$$\ln \left( \frac{v(t)}{V_B} \right) = -\frac{1}{0.92 \frac{C Z \tau R^3}{t}} t^4$$

Putting $g = \frac{1}{0.92 \frac{C Z \tau R^3}{t}}$, the above relation gives:

$$v(t) = V_B e^{-g t^4} \quad \ldots (15)$$
This equation indicates that the voltage decreases with time through this interval. The corresponding current is obtained from Ohm’s law with the gap resistance and voltage as given by equations (10) and (15) respectively.

The capacitor continues discharging through the gap in its resistive phase until the period ($\tau_R$) of this interval is elapsed. At the end of this interval (at $t=\tau_R$), the voltage across the gap ($V_R$) and the gaps resistance ($R_g$) may be determined by the substitution $t=\tau_R$ in equations (10) and (15) respectively, hence obtaining:

$$V_R = V_B e^{-g \tau_R^4} \quad \ldots \text{ (16)}$$

And

$$R_g = 0.23 Z \quad \ldots \text{ (17)}$$

Just beyond the instant $\tau_R$, the third interval begins where the capacitor will be considered here to continue discharging through the gap of a constant plasma resistance ($R_g$). In this situation, the voltage across the gap is expected to continue to decrease with time starting from the value ($V_R$) at the beginning of this interval (at $t=0$). Hence, by the traditional equation for the capacitor discharge process [10], and putting the proper parameters, one may write the equation for the voltage across the gap for the third interval as:

$$v(t) = V_R e^{-t / c R_g} \quad \ldots \text{ (18)}$$

and the Ohm’s law is employed again to determine the corresponding current with the voltage as determined by eq.(18) and the resistance as ($R_g$).

In this interval, the capacitor is expected to continue discharging until the plasma state in the gap disappears where the plasma heating is no longer sufficient to overcome the recombination of the plasma components due to the diminishing voltage and current across the gap as time elapses.

As a simple and crude approximation, the time duration for this interval is estimated conventionally as ($T_g \approx 5 R_g C$) which is a practical-purpose approximation to estimate the discharge period of a charged capacitor [10]. However, more accurate values of this timing may be obtained experimentally or via appropriate mathematical modeling of plasma decay.

A computer program based on the calculation procedure described above is constructed and used to obtain a simulation of the spark gap circuit operation. Also, the program is used to achieve a simple parametric investigation regarding the spark gap parameters, namely, the gas pressure and electrode spacing in order to examine their effects on the voltage and current temporal variations.

**Results and Discussion**

The proposed model is applied examine the time evolution curves of the voltage and current for a simple spark gap circuit [fig.(1)]. In order to specify a certain case to be
regarded as a “reference” for comparison purposes, the following parameters were chosen as: \( V_0=1000 \text{v} \), \( C=30 \text{pF} \), \( R=100 \ \Omega \), and \( z=50 \ \Omega \). The spark gap parameters were chosen as: \( P=1 \ \text{atm.} \) and \( d=0.1 \ \text{mm} \).

The time evolution curves for voltage and current are shown in figures (3) and (4) respectively. Figure (3) shows the increase of the voltage across the gap during the charging process until the breakdown time \( t_B \approx 6 \text{ns} \) is reached, where the voltage at this instant reaches the breakdown value \( V_B \approx 865 \ \text{v} \) as determined. Beyond the instant \( t_B \), the voltage decreases with time due to the rapid diminishing of the plasma dynamic resistance throughout the resistive phase of a period \( \tau_R \approx 1.4 \ \text{ns} \). After the resistive phase is elapsed, the voltage continues to decrease with the plasma resistive considered in its minimum value (as assumed in this work as an approximation).

Figure (4) shows the current evolution curve. It is noticed that the initial current value is \( 10 \ \text{A} \) which is compatible with the values chosen for the charging voltage \( V_0=1000 \ \text{v} \) and the resistance \( R=100 \ \Omega \) [fig(1)]. In figure (4), the portion of the curve that extends from \( t=0 \) to \( t=t_B \approx 6 \ \text{ns} \) represents the charging current that flows from the source to the capacitor, while the portion that extends beyond the time \( t_B \) represents the current flowing from the capacitor through the spark gap (a discharge current).

Figure (3) Temporal evolution of voltage across the gap during the whole process (i.e. charging and discharging of the capacitor).
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Figure (4) Current temporal evolution during the capacitor charging and discharging intervals.

It is noticed from fig.(4) that the gap current rises immediately after the instant ($t_B$) ($\approx 6$ ns) despite the associated decrease in the potential difference across the gap as fig.(3) indicates. This is due to the rapid decrease in the plasma dynamic resistance during the resistive phase [Sorenen and Ristic equation [2]]. After the elapse of the resistive phase (of a duration $\tau_R \approx 1.4$ ns), the current decreases with time due to the continuous diminishing of the voltage across the gap as time goes on. This explains the formation of the crest in the current curve during the discharge process.

The effect of the spark gap on the circuit is examined by considering the gas pressure and electrode spacing. The effect of the gas pressure on the gap voltage is illustrated in fig.(5), where two cases are considered: case (1) which is the same reference case tested above, and case (2) for which the pressure in the first case is increased from 1 atm. to 1.2 atm. with the rest of the parameters kept unchanged.

It is noticed from fig.(5) that increasing the gas pressure causes the breakdown time ($t_B$), and consequently the breakdown voltage ($V_B$) to increase. As it is expected, the two curves coincide during the capacitor charging process where no effect of the spark is taking place yet. This trend of the voltage curves is reflected on the current curves as shown in fig.(6), where the two curves coincide through the charging phase and the deviation between them starts when breakdown starts. It is apparent that the higher ($t_B$) associated with the higher pressure causes the current pulse to occur at a later time compared with the lower pressure condition. The higher peak current for the higher pressure case is due to the higher pressure voltage ($V_B$) as it is indicated in fig.(5).

The effect of the gas pressure on the shape of the voltage and current pulses during discharge (i.e. beyond the breakdown instant) is viewed through figures (7) and (8) respectively. Here the breakdown instant is considered as the starting time ($t=0$) for both cases for the sake of the pulse shape comparison.

In fig.(7) it is noticed that the voltage curve for the higher condition decays more steeply compared with the lower pressure case. The current curves associated with this comparison are shown in fig.(8). It may be noticed that the current pulse associated with the higher pressure condition is characterized by steeper rising and decaying, this is considered as an advantageous property regarding high speed switches [2].

A similar comparison is made to test the effect of the electrode spacing on the voltage and current pulses during discharge as shown in figures (9) and (10) respectively. The trends of the voltage and current curves are similar to those observed in figures (7) and (8).
Figure (5) The effect of gas pressure in the gap on the voltage time evolution curves throughout the whole process (charging and discharging).

Figure (6) The effect of gas pressure in the gap on the current time evolution curves throughout the whole process (charging and discharging).
Figure(7) Voltage across the gap during discharge process for two values of gas pressure.

Figure(8) Current passing through the gap during discharge process for two values of gas pressure.
Figure (9) Voltage across the gap during discharge process for two values of electrodes spacing.

Figure (10) Current passing through the gap during discharge process for two values of electrodes spacing.
However, by examining figures (7) - (10) it may be noticed that steeper curves are obtained by increasing the pressure rather than the electrode spacing. It is believed that this is due to the unequal effect of pressure and electrode spacing on the duration of the resistive phase ($\tau_R$).

This may be explained easily by expressing equation (10) in the form: 

$$\tau_R = \frac{44 d^{1/3}}{V P^{1/2}}$$

(where the definition $E=V/d$ is used). The above expression indicates that increasing ($P$) and/or ($d$) leads to an increase in ($\tau_R$), however, it also leads to an increase in the voltage between the electrodes {according to Paschen’s law [eq.(7)]} which tends to diminish ($\tau_R$). Now, since the power of ($P$) is half then the rule of the pressure in increasing ($\tau_R$) would not overcome the rule of the voltage in the denominator, while it can be verified that the effect of ($d$) would predominate over that of ($V$) by substituting the expression of the voltage from eq.(7) into the above form of ($\tau_R$). Hence it is concluded that increasing the pressure leads to shorter ($\tau_R$) while increasing the electrodes spacing leads to longer ($\tau_R$).

Table (1) lists some important parameters as determined through the present model for the three cases considered above: the reference case (for which the pressure is 1 atm. and the electrodes spacing is 1 mm), case (1) for which the pressure is increased to (1.2 atm.) relative to the reference case, and case (2) for which the electrodes spacing is increased to (1.2 mm) relative to the reference case.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Reference Case [P=1 atm., d=0.1 mm]</th>
<th>Case (1) [P=1.2 atm., d=0.1 mm]</th>
<th>Case (2) [P=1 atm., d=1.2 mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_B$</td>
<td>865 v</td>
<td>981 v</td>
<td>981 v</td>
</tr>
<tr>
<td>$I_B$</td>
<td>8.6 A</td>
<td>9.816 A</td>
<td>9.816 A</td>
</tr>
<tr>
<td>$t_B$</td>
<td>6 ns</td>
<td>12.02 ns</td>
<td>12.02 ns</td>
</tr>
<tr>
<td>$\tau_R$</td>
<td>1.4 ns</td>
<td>1.33 ns</td>
<td>1.46 ns</td>
</tr>
<tr>
<td>Resistive Phase Termination Time $t_r$ ($t_r=t_{th}+\tau_R$)</td>
<td>7.3 ns</td>
<td>13.3 ns</td>
<td>13.4 ns</td>
</tr>
<tr>
<td>$V(t_r)$</td>
<td>394 v</td>
<td>409.66 v</td>
<td>401.434 v</td>
</tr>
<tr>
<td>$I(t_r)$</td>
<td>29 A</td>
<td>33.04 A</td>
<td>30.769 A</td>
</tr>
<tr>
<td>$R_g$</td>
<td>13.8 $\Omega$</td>
<td>12.39 $\Omega$</td>
<td>13.04 $\Omega$</td>
</tr>
</tbody>
</table>

It is noted from this table that the shortest interval ($\tau_R$) corresponds to case (1) where the pressure is raised relative to the reference case, and causes the steeper the steeper voltage and current curves relative to the other two cases.

It is worth mentioning that two main approximations have been made in this simple modeling. The first one is employing Paschen’s law in determining the breakdown voltage, where this law has its limitations related with the electric field uniformity, electrode shape and surface roughness, in addition, it does not include the streamers mechanism in breakdown phenomena which represents the most probable mechanism in
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spark gap breakdown [1,2]. However, the validity of the Paschen’s law [eq.(7)] regarding the proper Pd- value [2] is taken into consideration and dealt with in the program. The second approximation is the assumption of a steady gap resistance beyond the resistive phase interval. However, this assumption may be compensated by choosing a small period of time after the end of the resistive phase, so that the calculations may not proceed so far beyond the resistive phase. Fortunately, it is not necessary (or even important) to advance in time beyond the resistive phase since this phase by itself represents the transition of the gap into a closed circuit (or closed switch) which is the significant factor in fast switches devices.

For a comparison purpose, the present model was applied to some measurement cases presented in reference (11) in which different electrode materials were used, with air pressure in the gap ranges approximately from 0.03 to 1 atm. The maximum currents obtained via the present calculations [i.e. I(tr)] were compared with the corresponding measured maximums presented in Ref. (11).

Table (2) lists the values of the peak currents obtained from Ref.(11) (Measured) and those obtained from the present model (Determined) at the end of the resistive phase [I(t2)].

Table (2) reveals an acceptable agreement between theoretical and the available measurement results for maximum currents.

Table (2) Measured and determined electric currents at the end of the resistive phase. The measured data are obtained from Ref. (11). Electrodes materials correspond to the measured data are indicated in the first column, and the air pressures values adopted in the present model determination are listed in the last column.

<table>
<thead>
<tr>
<th>Electrode spacing (µm) \ Electrode material</th>
<th>Measured current (Peak) [Ref.(11)] (A)</th>
<th>Determined current [I(tr)] (Present model) (A)</th>
<th>Pressure (atm.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 \ (Brass)</td>
<td>9</td>
<td>12.4</td>
<td>0.25</td>
</tr>
<tr>
<td>60 \ (Steel)</td>
<td>23.6</td>
<td>23.03</td>
<td>1</td>
</tr>
<tr>
<td>60 \ (Aluminum)</td>
<td>22.7</td>
<td>21.4</td>
<td>0.9</td>
</tr>
<tr>
<td>90 \ (Brass)</td>
<td>16.4</td>
<td>16.146</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Conclusions

It is obviously concluded that experimental work is strongly needed in such studies to confirm and aid the theoretical effort, in addition of the benefit of using some measured quantities as inputs for the simulation software. However, the present model provides a simple and fast tool for simulating the function of circuits that involve capacitors and spark gaps.

The reasonable temporal behavior of the voltage and current curves obtained from this modeling, as well as the comparison made with some measured current data, support the model in principle.

The parametric comparison made had led to a conclusion regarding the usage of spark gaps as fast switches devices (FSDs), where it is concluded that raising the gas pressure is
preferable than increasing the electrode spacing when high operating voltages and currents are involved, where steeper voltage and current curves are gained by increasing the pressure rather than the electrode spacing.

Of course, the model can be developed and modified with respect to many aspects, especially those concerning the plasma neutralization due to recombination processes, which have crucial role in determining the termination time of the discharge phase.

Reference