Design and Development Dynamic Functional Models of Radio Systems by Using the MATLAB System

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ABSTRACT
In this study, we have presented brief theoretical information regarding the design of dynamic functional models of radio systems and practical recommendations for their development and investigation in the MATLAB system. The paper mainly concentrates on three points, as follows: 1. Development of a dynamic functional model filters shaping to obtain the Gaussian random processes with given correlation properties using the MATLAB environment. 2. A development study of a dynamic model rectangular pulse detector against the self-noise. Keywords: Dynamic Functional Models, Filters Shaping, Gaussian Random Processes, Detector, Self - Noise.

INTRODUCTION
Function radio object - this process purposeful transformation of the input signal (effects). The purpose can be used to change the conversion waveform, separation it's from the mixture signal even in case of interference, and, finally, measuring the signal parameters. Description of function object in any language called functional model. The aim of the work is to develop dynamic functional models and study its characteristics of radio systems. Convenient tool for the development and study dynamic functional models is the MATLAB system. To perform the above points, firstly, we have to obtain the following basic information about the MATLAB system: 1. the basics of working with data in the MATLAB with help command Line. 2. Writing programs in MATLAB, consisting of M-script and M-functions. 3. Creation of dynamic models in a graphical environment visual simulation SIMULINK. 4. Using the special features of the MATLAB system - creating your own library of blocks, masked subsystems and others. These details can be obtained from [1,2,3,4,5,6], more detailed - sources [7,8]. Note that the study of the fundamentals of working with MATLAB system is not a goal in itself laboratory work. MATLAB System is used only as a convenient tool for creating dynamic...
functional models. In preparation for the laboratory work is necessary to study or to repeat the theoretical foundations of radio systems, explaining the principle of the simulated device. The necessary theoretical knowledge fully contained in the sources [9,10,11,12].

Simulation of Gaussian Random Processes

In this section, the purpose is a developing dynamical functional models shaping filters to obtain Gaussian random processes with given correlation properties in the environment MATLAB. This is by considering the principles of modeling implementations of narrowband Gaussian random processes by complex envelope. Narrowband is the process of satisfying the condition:

\[ \Delta f << f_0 \]  

Where \( f_0 \) – center frequency spectrum of the process, \( \Delta f \) – bandwidth. To describe models narrowband processes (both deterministic and random) is convenient to use the mathematical apparatus of the complex envelope. Complex envelope \( \bar{U}(t) \) narrowband process can be regarded as its low-frequency model that contains all the information about the amplitude and phase of the process and can be written as

\[ \bar{U}(t) = A(t) \exp[j\Phi(t)] \]  

Where \( A(t) \) and \( \Phi(t) \) – the instantaneous amplitude and phase of the narrowband process which represent the slowly varying (with respect to the center frequency) function of time. Complex envelope can be written and otherwise, through its quadrature components \( A_c(t) \) and \( A_s(t) \):

\[ \bar{U}(t) = A_c(t) + jA_s(t) \]  

If the spectrum of narrowband Gaussian random processes is symmetric about the center frequency \( f_0 \), this process is called quasi-harmonic Gaussian random processes. It can be shown that the quadrature components of the complex envelope of the quasi-harmonic Gaussian random processes are two independent Gaussian random processes. Therefore, the discrete model of the complex envelope of the quasi-harmonic Gaussian random processes shaped in the form of implementation of discrete complex random process, which is a real and imaginary part are two independent real Gaussian random processes with a given correlation function. Correlation function \( R(k) \) and the power spectral density of random processes \( W(w) \) in accordance with the theorem of Wiener - Khinchin connected pair of Fourier transforms:

\[ W(w) = \sum_{k=-\infty}^{\infty} R(k) e^{-jwk} \]  

\[ R(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} W(w) e^{jwk} dw \]
Power spectral density process at the output $W_{out}(\omega)$ linear system associated with power spectral density process at the entrance $W_{in}(\omega)$ the following relationship:

$$W_{out}(\omega) = W_{in}(\omega)K^{2}(\omega) \tag{5}$$

Where $K^{2}(\omega) –$ square modulus of the frequency response systems.

In turn, square modulus of the frequency response systems is associated to the Fourier transform with the autocorrelation function of the impulse response of the system. Hence, taking into account (4), (5), it follows the correlation function process at the systems output when exposed at its input a white noise is equal to the autocorrelation function of the impulse response. Hence, in order to form implementation quasi-harmonic complex envelope of a Gaussian random processes, it is necessary convert realization white Gaussian noise using the shaping filter, the autocorrelation function of the impulse response which is equal to the desired correlation function of the simulated quasi-harmonic process. In the simulation of random processes with given correlation properties are commonly used discrete filters shaping with finite impulse response FIR filters. The method of formation of a process with a given correlation functions by using shaping FIR filter is called moving summation method [12]. There are known methods for calculation of the coefficients forming a FIR filter for a given correlation function, or power spectral density of the simulated process [12]. Here, we look at some examples of design filters shaping, to calculate the coefficients are not required to perform complex calculations.

The procedure for modeling complex envelope quasi-harmonic Gaussian random processes following:

1. Formed vector counts the complex envelope of the discrete white Gaussian noise required dimension with the correlation function

$$R(k) = \begin{cases} 1, & K = 0 \\ 0, & K \neq 0 \end{cases} \tag{6}$$

2. Located the autocorrelation function of the impulse response of the filter shaping.

3. Calculate the amplitude-frequency characteristics of the filter.

4. Found the amplitude-frequency characteristics designed discrete filter shaping.

5. Implementation of the discrete white Gaussian noise is converted using the shaping filter designed.

Consider the example of the design of the shaping filter. Let it be required to build a model of a Gaussian random process with the correlation function

$$R(\tau) = \exp[-\alpha \tau] \tag{7}$$

Where $\alpha –$ constant factor.

Calculating the Fourier transform of the function (7), we obtain an expression for the power spectral density of the simulated random processes:

$$W(\omega) = \frac{2\alpha}{\alpha^2 + \omega^2} \tag{8}$$

In accordance with the above, the amplitude-frequency characteristic of the filter shaping is equal to
\[ K(\omega) = \frac{2\alpha}{\sqrt{\alpha^2 + \omega^2}} \] (9)

It is known that the amplitude-frequency characteristic of the form (9) corresponds to the \( RC \) – filter of the first order with the impulse response \( h(t) = \alpha \exp[-\alpha t] \). Since in this example the known form of the impulse response of the analog filter, then applying window design method of discrete FIR filter, we obtain an expression for the impulse response of a discrete filter shaping:

\[ h(t) = \begin{cases} \alpha \exp[-\alpha n], & 0 \leq n < N \\ 0, & n > N \end{cases} \] (10)

Where \( N \) – filter order.

There are also methods for calculating the coefficients of discrete shaping filters infinite impulse response (IIR filters). In this example, the discrete model of forming IIR filter is described by the difference equation of the first order:

\[ y[n] = a_0 x[n] + b_1 y[n-1] \] (11)

Where \( x[n] \) and \( y[n] \) – samples the input and output process in time \( n \), \( a_0 \) and \( b_1 \) – the filter coefficients, \( b_1 = e^{-\alpha}, a_0 = K_0(1-b_1), K_0 \) – gain.

Discrete process at the output of the filter shaping has a discrete correlation function \( R(k) \), representing is an approximation given the correlation function of a continuous process \( R(\tau) \). The accuracy of approximation is determined by the selected method of designing filter shaping and its parameters. For example, in the special case when the modeling process with exponential correlation function (7) is used IIR filter described by the difference equation (11), samples discrete correlation function \( R(k) \) will be exactly equal to the values of a continuous function \( R(\tau) \) at the appropriate times. At the same time last statement is not true for a filter with impulse response (10). In the latter case, approximation accuracy is determined by the size of the time window, limiting the length of the impulse response of the FIR filter.

**Conclusion**

Design shaping filter for modeling complex envelope quasi-harmonic Gaussian random processes with the correlation function of the following form:

\[ R(\tau) = \begin{cases} 1 - \frac{\tau}{T}, & |\tau| \leq T \\ 0, & |\tau| > T \end{cases} \] (12)

Where \( T \) – a predetermined time interval.

1. Working from the command line to get the implementation of the complex envelope of random processes with given correlation properties.
2. Writing a MATLAB program that allow of measuring the pulse and frequency characteristics of the digital filter, and exploring the transformation of random processes that filter. The program must contain \( M \) – script, \( M \) – function, calculates the coefficients of the FIR filter for a given frequency response, and \( M \) – function implements the filter.
3. Using the developed program, design a filter shaping for modeling the complex envelope of random processes with power spectral density of the form:

\[ W(w) = \exp(-aw^2) \]  

(13)

To answer the question: what forms will the impulse response of the shaping filter? necessary to bring the results calculating both filters shaping, timing diagrams and processes spectrogram at the inlet and outlet of each filter, impulse response graphs and the amplitude-frequency characteristic of the second filter.

**Development Dynamic Functional Models of Radio Elements on the Basis of the Complex Envelope Apparatus.**

The purpose of this section of the work is development and study dynamic model rectangular pulse detector against the background self-noise. Consider the model matched filter for a simple radio pulse with an arbitrary shape of the envelope. Analog matched filter for such a pulse is realized in a form of serial connection highly selective radio-frequency filter, the envelope of the impulse response which corresponds to the envelope of the RF pulse, and the amplitude detector providing invariance of the matched filter to the initial phase of the signal. In practice, generally it is used quasi-optimal filters in which as the radio-frequency filter applies resonant circuit. The complex frequency response of the resonant circuit \( K(jw) \) has the form

\[ K(jw) = \frac{K_0}{1 + j(w - w_R)\tau_k} \]  

(14)

Where \( w_R \) – resonant frequency of the circuit; \( \tau_k \) – time constant; \( K_0 \) – transfer coefficient at the resonance frequency.

Since this filter is a linear frequency-selective device, for drawing up its mathematical model using the apparatus of the complex envelope. Note that the equivalent low-frequency selective RF filter is a low pass filter (LPF) impulse response which corresponds to the complex envelope of the impulse response of the radio frequency filter. Block diagram of the model optimal detector radio pulse signal (it is part of a matched filter) is shown in Fig. (1).

![Block diagram of the model optimal detector radio pulse signal](image)

Figure (1): Block diagram of the model optimal detector radio pulse signal

Low-frequency resonant circuit is equivalent \( RC \) – circuit. Its frequency \( K_{eqv}(j\Omega) \) and pulse \( \bar{G}(t) \) characteristics written in the form
\[ K_{eq}(j\Omega) = \frac{K_0}{1 + j\Omega \tau_k} \]  

(15)

\[ \frac{1}{2} \tilde{G}(t) = G_0 e^{-\alpha t}, \alpha = \frac{1}{\tau_k} \]  

(16)

Where \( \Omega = w - w_k \).

In accordance to method of the overlap integral the complex envelope of the process at the filter output \( \overline{U}_z(t) \) is calculated as

\[ \overline{U}_z(t) \approx \frac{1}{2} \int_{-\infty}^{\infty} \overline{U}_1(t) \tilde{G}(t-x) dx \]  

(17)

Where \( \overline{U}_1(t) \) – the complex envelope of the process at the input filter.

In case of using a cascade connection of multiple resonant circuits, the resulting frequency (pulse and envelope) characteristics shape close to the Gaussian. It is understood that the appropriate model for the complex envelope in this case would be a low-pass filter of \( n - th \) order, where \( n \) – number of cascaded loops (low pass filter in the model for the complex envelope). Since the initial phase of the unknown signal, to detect it necessary to use two quadrate channels, i.e. the impulse response of the low-frequency equivalent principle is complex. In each square channel model for the complex envelope using a valid low-pass filter shape of the impulse response is determined the shape of the envelope of the radio pulse. A model for the complex envelope amplitude detector representing as a function block that calculates the square root of the sum of squares of samples of the complex envelope at the output of the quadrate filter Fig. (1).

If we considered a model of the analog quasi optimal matched filter then the parameters of the impulse response of the quadrate low-pass filter should be selected in such a way that their bandwidth has been agreed with the width of the signal spectrum in the sense of providing a maximum signal to noise ratio. For example, if the quadrate low-pass filter uses a simple RC-filter, it can be shown that the maximum signal to noise ratio will be observed at the output when the bandwidth \( RC \) – filter on the level of 0.707 is approximately \( 0.2/\tau_s \), where \( \tau_s \) – duration rectangular pulse.
$$\frac{(f-0.5f_s)}{f_s}$$

**Figure (2): The amplitude frequency characteristic**

In Fig. (2) shows the amplitude spectrum of a rectangular video pulse (graph. 1), amplitude frequency characteristic quasi-optimal $RC$-filter (graph. 2), amplitude frequency characteristic of an ideal low-pass filter whose pass band is equal to the equivalent noise bandwidth of quasi-optimal filter (graph. 3), amplitude frequency characteristic of an ideal low-pass filter whose pass band is equal to bandwidth passing the quasi-optimal filter on the level of 0.707 (graph. 4). The horizontal axis of the graphs in Fig.(2) is plotted normalized frequency $(f-0.5f_s)/f_s$, where $f_s$ – the sampling frequency.

In case of an input quasi-optimal filter mixture signal and noise at the filter output, we can explore that the process has a pronounced maximum at the time of expiration of the input signal. If the bandwidth is more required, the filter will pass on the noise output more power, which will lead to a deterioration of the signal to noise ratio. In this case, termination of signal, we will see at the output filter a maximum peak. If the bandwidth less than required (too much time constant) the filter cannot for the duration of the signal to save its energy, which also leads to a decrease in SNR. Fig. (3) shows the timing diagram corresponding to the three situations described.

Fig. (3 a) shows the implementation of $S_1[n]$ input process as a mixture of rectangular video pulse with noise; In Fig. (3 b) - process $S_2[n]$ at the output of the quasi-optimal filter; c and d - timing diagrams of processes $S_3[n]$ and $S_4[n]$ at the outputs of filters whose bandwidth is greater than 3 times and 3 times less bandwidth quasi-optimal filter, respectively. The dashed line in timing diagram shows the adaptive threshold. It is evident that only at the output quasi-optimal filter, bandwidth is matched with spectral width of the signal, received SNR, providing excess threshold.

**Figure (3): Block diagram of the model optimal detector radio pulse signal**

As noted above, when using a cascade connection of several $RC$ – filter frequencies response resulting shape closes to the Gaussian. Such a filter will therefore be optimal for Gaussian pulse shape.
If the matched filter is implemented in digital form, is a block diagram for
represents a block diagram of a system digital signal processing, and not only its functional
case, the operation of digital filters described by a discrete convolution:

\[ \bar{U}_2[n] = \sum_{k=0}^{n} \bar{U}_i[k] \bar{G}[n-k] \]  \hspace{1cm} (18)

Where \( \bar{U}_{1,2}[n] \) – samples of the complex envelope of the processes at the inlet and outlet of the
filter, \( \bar{G}[n] \) – Discrete samples of the impulse response.

Clearly, that the research works of both digital and analog systems on electronic - computer
used their discrete model (18). When designing a digital system sampling interval \( T_s \) is selected
in accordance with the Nyquist theorem:

\[ T_s \leq 1/2F_{\text{max}} \]  \hspace{1cm} (19)

Where \( F_{\text{max}} \) – maximum frequency in the signal spectrum.

In practice, used signals of limited duration, having unlimited wide spectrum. With the value of
frequencies \( F_{\text{max}} \) provisionally selected according to some predetermined level, and the
condition (19) performed with a margin.

To select the sampling frequency in the model of a continuous system may use different
approaches, which, along with the condition (19), provide performance or that criterion of
adequacy of model a real system, taking into account the correlation properties of the simulated
process. In practice generally used simple approach in which the sampling interval is chosen
much smaller than the time constant of the simulated device.

Note that if the noise samples at the inlet of the filter are modeled by independent discrete white
Gaussian noise, this corresponds to impact inlet study systems noise with power spectral
density, uniform in the frequency range \([0,1/T_s]\). If required to generate noise model with
limited stripe and a predetermined shape of the spectrum, it is necessary first convert samples of
discrete white Gaussian noise using the filter shaping.

Simulation package SIMULINK allows you to explore the functional models both analog and
digital systems.

On Fig. (1) after the matched filter and amplitude detector included a threshold device (TD),
task which is decision-making about availability signal in the input process according to the
Neyman-Pearson. Recall that the decision rule, the best according to the Neyman-Pearson,
maximizes the probability of correct detection of a signal at a fixed level of probability of false
alarm.

Since the noise power is unknown a priori, in the model included the evaluation unit standard
deviation self-noise. As an estimate of the standard deviation of noise, this unit calculates the
sample standard deviation:

\[ \hat{\sigma}_n = \sqrt{\frac{1}{N} \sum_{k=0}^{N-1} [\overline{U}[n]]^2} \]  \hspace{1cm} (20)

Where \( N \) – averaging window size. threshold value \( U_0 \) formed as follows:

\[ U_0 = k \hat{\sigma}_n \]  \hspace{1cm} (21)
Where the constant $k$ is determined based on the acceptable level of false alarm probability. Assuming that the intrinsic noise of the receive path in each quadrate channel is an independent Gaussian random processes; noise at the output of the amplitude detector in the absence of the desired signal is distributed in law Rayleigh. Then the probability of false alarm $F$ is given by:

$$F = \exp \left( -\frac{U_n^2}{\sigma_n^2} \right)$$

(22)

Where $\sigma_n^2$ – the noise variance at the matched filter output. Given that $\frac{U_\omega}{\sigma_n} = k$, from formula (22) can easily find the value of $k$.

**CONCLUSION**

1. Design a mathematical model functional analog detector rectangular impulse radio signal consisting of a cascade connection of three resonant circuits, the amplitude detector, Adaptive Threshold forming apparatus and a threshold device. When modeling used mathematical apparatus of the complex envelope. Ensure that the probability of false alarm is not more than $10^{-6}$. To explore the process of passing a mixture of the complex envelope of white Gaussian noise and the complex envelope of the rectangular pulse radio with random initial phase through the model of the detector.

2. Design a functional mathematical model of the digital detector complex envelope of the radio pulse with a Gaussian envelope form with random initial phase and amplitude. Explore the passage of a mixture of discrete white Gaussian noise and Gaussian pulse through developed a functional model. Calculate the probability of correct detection of a signal at a given signal to noise ratio at the input of the quasi-filter and a spectral width of the input process.

Developed functional models implemented in the package SIMULINK. Upon completion of the work to make a report that lead block diagrams and mathematical models of the studied detectors, timing diagrams and amplitude spectra of processes at all points of the circuit.

**Investigation Characteristics of Digital FIR and IIR Filters Using Dynamic Functional Models in SIMULINK.**

The purpose is study some special features of the SIMULINK system, An example of the measuring system of amplitude - frequency response of the filter. In practice, often have to solve the problem of measuring the amplitude - frequency response of some analog device using sweep frequency generator (SFG). Block diagram of the measurement is shown in Fig. (4).
Figure (4): Block diagram of the measuring system of amplitude-frequency response of the filter

If the rate of frequencies change sweep frequency generator $K_f$ such that the condition

$$K_f \tau < 1$$

(23)

Where $\tau$ – the expected duration of the impulse response of the investigate device, the signal at the output of the amplitude detector (AD) is proportional to the level of amplitude-frequency response at the current frequency. Then, if on the inputs horizontal and vertical apply voltage proportional to the instantaneous frequency of the oscillator, respectively rocking frequencies and the voltage at the output of the amplitude detector, the screen image will get the amplitude-frequency response of the device.

We have constructed mathematical models of functional devices in the circuit in Fig. (4) using the apparatus complex envelope. Consider the model of the signal at the output of the sweep frequency generator. To control the frequency of the generator is used periodic sawtooth signal, but for the simulation of the circuit in the SIMULINK system it suffices to consider one period, in a limits which the signal at the output of the sweep frequency generator is a signal with linear frequency modulation [13,14].

If the duration of the period is $T$, law changes frequencies on the period has the form:

$$f(t) = f_0 - \frac{\Delta f}{2} + \Delta f \cdot \frac{t}{T}$$

(24)

Where $f_0$ – center frequency, $\Delta f$ – frequency deviation. The phase of this signal:

$$\Phi(t) = 2\pi \left( f_0 t - \frac{\Delta f}{2} t + \frac{\Delta f}{2T} \right)$$

(25)

Varies according to the quadratic law. Complex envelope formed linear frequency modulation-signal $U_{LFM}(t)$ has the form:

$$U_{LFM}(t) = U_0 \exp \left[ j2\pi \left( -\frac{\Delta f}{2} t + \frac{\Delta f}{2T} \right) \right]$$

(26)

Where $U_0$ – constant amplitude.

In a SIMULINK system there are several ways of forming complex envelope linear frequency modulation-signal. The first and easiest-direct use of formula (26).
The second way is connected using block Chirp signal, which is a model of the sweep frequency generator. The output signal this block is valid linear frequency modulation-signal, whose frequency at a given time interval varies from $f_{min}$ to $f_{max}$. Quadrate components of the complex envelope of a linear frequency modulated-signal can be obtained using digital down frequency conversion circuit (DLPF) shown in Fig. (5), where $\Delta t$ - sampling interval.

If for modeling the complex envelope of the linear frequency modulation-signal using the first approach, the minimum frequency sampling digital mode $f_s$ determined by the width of the spectrum linear frequency modulation signal and corresponds approximately to the frequency deviation $\Delta f$. If you use the second approach, the minimum sampling frequency in the simulation of a real linear frequency modulation signal should be $2\Delta f$ avoid imposing spectral copies of the region of positive and negative frequencies. In practice, the sampling frequency is selected in accordance with the following expression:

$$ f_s = \frac{4f_0}{(4k + 1)} $$

(27)

Where $k = \left[ \frac{f_0}{f_{s_{min}}} \right]$, $f_{s_{min}} = 2\Delta f + 2\Delta f_{prot}, \Delta f_{prot} = $ protective interval between spectral copies of a digital signal, defines the operation of taking the nearest integer.

Functional mathematical model investigated devices Fig. (4) is its low frequency equivalent. The use of low-frequency equivalent to simulate electoral systems, as well as a functional model of the amplitude detector discussed in the section on the previous lab.

Consider the possibility of using the system SIMULINK block Matlab Function for modeling arbitrary functional transformation of input samples. The parameters of this block are the name of the function, written in the language of MATLAB, which converts the input sequence samples. Below is an example modeling of the matched filter for the complex envelope pulse linear frequency modulation signal, implemented as a digital FIR filter.

```matlab
% The function simulates the operation of the FIR filter
function [output]=LFM_Filter(input,N)
    global Delay Coefs; % global arrays
    for i=N:-1:2
        Delay(1,i)=Delay(1,i-1); % shift delay line
    end
    Delay(1,1)=input; % recording a new reference
    output=0;
    for(i=0:1:N-1)
        output=output+Delay(1,i+1)*Coefs(1,i+1); % calculation of output
    end
end
```
end % reference

% The initialization script “filter_init”
script
global Delay Coefs;
L=61;
Delay=zeros(1,L);
Coefs=ones(1,L);
delta_f=750000;
fs=6000000;
tau=0.00001;
a=delta_f/fs;
b=delta_f/(tau*fs^2);
for n=0:1:L
Coefs(1,n+1)=cos(pi*a*n-pi*b*n*n)+j*sin(pi*a*n-pi*b*n*n);
end

Parameters in block Matlab Function must specify the name of the _LFM _filter (u, N), and before running the model SIMULINK in the MATLAB command prompt to run the script _filter _init.

CONCLUSION
In conclusion, the formation of the complex envelope linear frequency modulation signal has been explored and developed using two methods as described above. In another case, the implementation of the models for the digital low-pass filter of shown previously in figure (5) has been developed and built using a block MATLAB functions. Further, the investigation has been carried out to show a block diagram, mathematical models devices and the overall pattern block diagram of meter amplitude - frequency response and timing diagrams spectrogram processes in all points of the circuit.

REFERENCES