INVESTIGATION OF THE TRAVELLING WAVE EFFECTS IN ANALYSIS OF SYNCHRONOUS MACHINE TRANSIENT BEHAVIOUR

I. A NEW APPROACH FOR LONG LINE REPRESENTATION

by

Dr . M . Serry taha .
College of Engineering
Sulaimaniya University
Sulaimaniya Iraq .

Scientific Journal ,
The University of Technology ,
Baghdad .

Vol . 1
No . 2
December 1977

https://doi.org/10.30684/etj.1977.2.3
2412-0758/University of Technology-Iraq, Baghdad, Iraq

دراسة لتقيم اثر الموجات المسافرة في تحليل السلوك العابر للآلات (المكائن) المتزامنة –

(١) اتجاه جديد لتمثيل الخطوط الطويلة

الدكتور محمود سري طه

جامعة السليمانية – كلية الهندسة

خلاصة المقاله

يمثل البحث المقدم الاول من سلسلة من الابحاث الخاصة التي تهتم بالسلوك الكلي للنظم الكهربائية المتكاملة ويهتم هذا البحث اساسا باستنباط تعبير (او صورة رياضية) جديدة للخطوط الكهربائية الطويلة يمكن استخدامه في حل المعادلات الخاصة بالنظم الكهربائية المتكاملة من وجهة نظر خاصة باعتبار ان الخط الطويل هو نفسه نهاية طرفية للماكينة المتزامنة.

وقد اقترح المؤلف خمسة بدائل ممكنة ويمكن لمن يشاء من الباحثين تناول كل منها بتفصيل اكثر ولقد اشار المؤلف الى اختياره لواحد من هذه البدائل مع ذكر وجهة نظره الخاصة لهذا الاختيار.

هذا وقد تناول البحث كذلك الطرق المعروفة لحل معادلات الخطوط الكهربائية بدلالة القيم التي يتم حسابها عند طرف الماكينة المتزامنة باعتباره طرف ارسال للخط.

وبذلك يمكن معرفة قيم الجهد والتيار عند اي نقطة في هذا النظام المتكامل اثناء اي لحظة مما يجنب الباحث كثيرا من التعقيدات الناتجة عن اختلاف النظريات الرياضية لتناول الماكينة على حدة مثل استخدام The d-q-o transformation لحل معادلات الماكينة ثم استخدام The eigenvalve techniques لحل معادلات الخط ثم حلها كلها آنيا.

ABSTRACT:

This is one of a series of papers interested mainly in the overall transient behaviour of integrated power systems. With a target of introducing the whole system in a synchronous machine-wise frame of reference: the subject paper is devoted for derivation of an expression for the long line as a machine-terminal constraint. Five versions for long line representation are proposed in an operational impedance matrix form. These can be left for further investigation for selection of the optimum, although the author recommends the fifth one. Modification of line parameters due to inclusion of other distributed and lumped parameter series or shunt elements are discussed. Besides, available methods in literature for calculations of voltage and current vectors at any point along the line are given.

LIST OF SYMBOLS

[ ], Z	Matrix, and impedance matrix.
i, v	Instantanous current and voltage at any point along the line.
I, V	Fourier transforms of current and voltage.

x	distance along the line.
Z_c, Z_0	Caracteristic and surge impedance respectively.
R,L,C,G	Line constant matrices per conductor per unit length.
Z, Y	Longitudnal impedance and shunt admittance per conductor per unit length.
λ_i, λ_e	eigen values determining propagation current and voltage modes.
F_{pq}, G_{pq}, F_{pq}, G_{pq}	Four terminal network parameter matrices.

1. INTRODUCTION

The general behaviour of a power system is defind during the transient, dynamic and steady state periods. Analysis of an integrated power system's behaviour makes it desirable to recognize three distinct periods:

i) A surge pericd in which travelling wave effects predominate and in which system elements are represented by their surge impedances.
ii) A dynamic period which is transitional between the surge period and the steady state and is characterized by variable's variations in an enveloppe which varies aperiodically with time.
iii) And lastly, the steady state period.

It is justifying to note, however, that such classification may become inadequate now. The use of exciters with automatic control whose action is dependent of first, second and /or higher derivatives of the controlled variables may make it necessary to allow for wave propagation in a long transmission line simultaneously with the dynamic transient phenomena.

One aspect to realize such a step, is to introduce a 3-phase long line with its series and shunt elements in a convenient impedance-or admittance-matrix form. Solution of this terminal's impedance matrix with the generator's differential equations can give the required solution of the problem.

2. REVIEW OF AVALIABLE METHODS

For the purpose of involving wave propagation phenomena, the terminal elements are divided into two types namely; those whose parameters are essentially lumped, such as transformers, reactors and capacitors; and distributed; like transmission lines and cables. Idealy the method of transient calculations should be capable of representing both lumped and distributed parameters, in addition to the effect of nonlinearities which may be involved equally well. However and up to author's knowledge, there methods of calaulations (2) are avaliable in literature.

(a) The lumped parameter method in which lines and cables are represented by artificial lines made up of lumped elements in a series of "T" or "Π" sections. One of the solutions is to write down the differential equations of the individual elements, then these are digitally solved by Runge-Kutta or any similar routine. However, although this method is accurate when applied to elements having lumped constants, but error is introduced by the representation used for lines and cables.The artificial line used behaves in more or less exactly the same way as the actual line for a particular frequency only but not convenient for generalization.

(b) The Fourier transform method which requires the calculation of response of the connected system over a range of frequencies and the use of the inverse Fourier transform to calculate the system's time response.In this method. a three phase line's equation must first be transformed into another set of components which have no mutual coupling between them.The propagation coefficients for each of the components or modes may then be calculated as for the single phase line.To perform such transformation. again it is necessary to calculate the eigenvalues of the transmission coefficient matrix $\gamma(w) = Z(w) \cdot Y(w)$.However.due to the much involved operations. this method is not always considered the most favourite.

(c) The travelling wave methods which are principally based up on the solution of transmission line partial differential equations.Two of which in wide use are:solution of the line voltage and current equations by laplace transform method. and the second by the well known lattice diagram method.In this latter method which is widely used for digital computation of switching transients: lines and cables are specified by their surge travel times and surge impedance matrices.Reflected and refracted voltage and current waves at junctions of terminations are calculated by the use of reflection and refraction coefficients.In this method. lumped elements are represented by transmission line stubs i.e. short lines which are shorted at remote ends. or open circuited. or infinitely long lines.However. due to the presence of the synchronous generator. this method will not be typically applied.

3. ADOPTED METHOD

Due to complex function of the synchronous machine involved with its associated regulators. hence a thorough analysis probibits its representation by a lumped reactance (or impedance) system.So for a synchronous generator-wise solution it is assumed that the generator will look to the line as if it were a three phase element represented by its characteristic impedance (surge impedance: for perfect earth) in matrix operational form.This is right so long as voltage and current at any point along the line: including its sending end: are related always by its surge impedance.The effect of intermediate lumped series and shunt elements will be involved by modifying the line constants: and consequently its surge impedance matrix.Once the surge impedance matrix is obtained in operational form: it can be solved simultaneously with the generator's differential equations in any selected machine's frame of reference. as will be seen in the accompanying paper.

4.TRANSMISSION LINE EQUATIONS

The involved transmission line is assumed to be three phase and competely transposed.The magnitude of the distributed parameters associated with each conductor is considered to be the same for all conductors considering the case balanced components of potentials and currents, then the differential equations describing the electromagnetic waves on the transmission line will be:

$$-\frac{\partial V}{\partial x} = L.\frac{\partial i}{\partial t} + R.I \tag{1}$$

and
$$-\frac{\partial i}{\partial x} = C.\frac{\partial V}{\partial t} + G.V \tag{2}$$

Where
i and V are column matrices of line currents and voltages respectively.

$$R = R\,U, L = L\,U = (L_s - M)\,.\,U$$

Where L_s is line self inductance per unit length
M is line mutual inductance per unit length
U is a unit 3×3 matrix.
C is a square symmetrical matrix whose main diagonal elements are equal to C, i.e. line capacitance to ground per unit length, and the sub-diagonal elements are "c" i.e. line capacitance between any pair of conductors per unit length.
G is a square symmetrical matrix whose main diagonal elements are equal to G i.e. line conductance to ground per unit length and the sub-diagonal elements are "g" i.e. line conductance between any pair of conductors per unit length.

Line surge impedance matrix is given by:

$$Z_o = Y^{-1}\,.\,(Y\,Z)^{1/2} \tag{3}$$

where
$$Y = G + PC \tag{4a}$$
and
$$Z = R + PL \tag{4b}$$

P designates $\frac{d}{dt}$.

Obtaining Z_o rigorously and explicity in operational form is very difficult if not impossible.What already published in this concern was given by reference (1) for a single frequency.

5. DERIVATION OF AN APPROXIMATE EXPRESION FOR $(Y\,Z)^{1/2}$

Let us assume; approximately that:

$$(Y\ Z)^{1/2} = P\ M + N \tag{5}$$

Squaring both sides of this last equation and substituting from (4a) and (4b), we get:

$$(G + p\ C)\ .\ (R + PL) = (P\ M + N)^2$$

Equating corresponding terms on each side, we get the following identities, which should be consistent:

a) $M^2 = C\ L = L\ C$ (6a)
b) $M\ N + N\ M = C\ R + G\ L = R\ C + L\ G$ (6b)
c) $N^2 = G\ R = R\ G$ (6c)

It is clear that, we have three equations and two unknown matrices, namely M and N, and hence solution is indeterminate.

However, some assumptions can be made, which result in a solvable set of equations.These may be:-

1) Assume $R = 0$, and from equations (6c) and (6b), this will lead necessarily to the identity $G = 0$.Thus, we have only one equation (6a) in one unkown matrix M.
   Solving this latter will yield:-

$$M = \begin{matrix} M & m & m \\ m & M & m \\ m & m & M \end{matrix}$$

$$\text{where } M = \pm \frac{\sqrt{L(C + 3c)}}{2} \tag{7a}$$

$$\text{and } m = -\sqrt{L(C - c)} \pm \frac{\sqrt{L(C + 3c)}}{2} \tag{7b}$$

   However, this assumption, will lead to imbigiousity while calculating currents and voltages at other points along the line, due to discard of R and G, hence attenuation effect.
2) Assumption (1) can be refined by taking the resistance and leakance matrices outside the line, as a lumped series impedance and shunt admittance matrices respectively.
   Although expected results will be better than first assumption, yet it will lead to zero attenuation results for the artificial line.
3) Assume a value for G matrix, matched with the R matrix such that equation set (6) reduces to only two.In other words, let us proceed as follows:

   From equation (6a) we get:

$$M = L^{1/2} C^{1/2}$$

From equation (6c) we get:

$$N = R^{1/2} G_a^{1/2}$$

Where G_a is assumed value matched with R to obtain only two equation set.
Substituting for M and N into (6b), we get:

$$\text{L.H.S} = (RL)^{1/2} \{ C^{1/2} G_a^{1/2} + G_a^{1/2} . C^{1/2}$$

$$\text{R.H.S} = R\ C + L\ G_a .$$

Thus equality of both sides of equation (6b) necessitates the assumption that both the C and G_a matrices should be diagonal ones.Such an assumption may lead to serious errors as the nondiagonal elements of above mentioned matrices may be quite appreciable with respect to diagonal elements.

Hence this assumption will be rejected.

4) Assume the following hybrid method:

ii — Assume a part of resistance R, let, it be denoted by r', as a distributed element.The remainder: for all the line i.e. l(R − r') as a lumped element.Value of r' will be determind sooner.

ii — Assume that the conductance matrix is imparted into a distributed parameter one given by:-

$$G_a = \begin{vmatrix} G & g_a & g_a \\ g_a & G & g_a \\ g_a & g_a & G \end{vmatrix}$$

and a lumped parameter matrix given by:

$$G_b = \begin{vmatrix} 0 & g_b & g_b \\ g_b & 0 & g_b \\ g_b & g_b & 0 \end{vmatrix}$$

Where

g_a is mutual conductance per phase per unit length and will be calculated sooner.

$g_b = g - g_a$

From equations (6a), and (6c) respectively we get:

$$M = \begin{vmatrix} M & m & m \\ m & M & m \\ m & m & M \end{vmatrix} , \text{ and } \dot{N} = \begin{vmatrix} N & n & n \\ n & N & n \\ n & n & N \end{vmatrix} \qquad (8a)$$

where:

$$M = \pm \frac{\sqrt{L(C + 3c)}}{2} \qquad (8b)$$

$$m = -\sqrt{L(C - c)} \pm \frac{\sqrt{L(C + 3c)}}{2} \qquad (8c)$$

$$N = \pm \frac{\sqrt{r'(G + 3ga)}}{2} \qquad (8d)$$

$$n = -\sqrt{r'(G - g_a)} \pm \frac{\sqrt{r'(G + 3ga)}}{2} \qquad (8e)$$

From equation (6b), we have

$$M\,N + N\,M = LG_a + r'\,C \qquad (9)$$

From this last equation, we get the following versions Either:-

$$M\,N = LG_a \qquad (10a)$$

and $$N\,M = r'\,C \qquad (10b)$$

Or $$M\,N = r'\,C \qquad (10c)$$

and $$N\,M = L\,G_a \qquad (10d)$$

Substituting from equation (6a) and (6c) into (10a), we get:

$$MN = \sqrt{L}.C^{1/2}.\sqrt{r'}G_a^{\ 1/2}$$

$$= \sqrt{r L}\,.C^{1/2}G_a^{1/2} \qquad (11)$$

But, from equation (10a), we have

$$MN = LG_a$$

Thus $$LG_a = \sqrt{r'L}\,.C^{1/2}G_a^{1/2}$$

But squaring both sides of this last equation, we get:—

$$G_a = (\frac{r'}{L}).C$$

It can be easily proved that equation (12) will be arrived at by trying any of versions (10b), (10c) or (10d).

From equation (12), we can write down:-

$$\begin{vmatrix} G & g_a & g_a \\ g_a & G & g_a \\ g_a & g_a & G \end{vmatrix} = \left(\frac{r'}{L}\right) \begin{vmatrix} C & c & c \\ c & C & c \\ c & c & C \end{vmatrix}$$

i.e. $G = \left(\frac{r'}{L}\right).C$

From which r' can be calculated

$$r' = \left(\frac{L}{C}\right).G \tag{13}$$

and

$$g_a = \left(\frac{r'}{L}\right).c = \left(\frac{G}{G}\right).c \tag{14}$$

Summary of this method is given as:-

The actual line simulated hybridly by:-

i - Artificial distributed parameter line with inductance and capacitance parameters as the actual line, but with resistance diagonal matrix with elements r' given by equation (13). The conductance matrix, having the actual line diagonal elements, but with subdiagonal elements g_a given by equation (14).

ii - A lumped series diagonal 3 ×3 matrix given by l r' U where U is 3 ×3 indentity matrix.

iii - A lumped shunt admittance 3 ×3 matrix whose diagonal elements are zero, and subdiagonal elements or $l(g - g_a)$.

5) Another, hybrid method similar to method (4) but with the exception that the subdiagonal elements of the conductance matrix of the artificial line are assumed to be the same as the actual line. The main diagonal elements of artificial line will be assumed to be G_a.

Thus, repeating procedure mentioned in method (4) we get:-

For the distributed parameter artificial line

$$G_a = \begin{vmatrix} G_a & g & g \\ g & G_a & g \\ g & g & G_a \end{vmatrix}$$

and for the lumped parameter conductance matrix:--

$$G_b = \ell. \begin{vmatrix} G_b & & \\ & G_b & \\ & & G_b \end{vmatrix} = \ell.G_b.U$$

Where: $G_b = G - G_a$

From equations (6a), and (6c), respectively we get:

$$N = \pm \frac{\sqrt{r'(G_a + 3g)}}{2} \tag{15a}$$

$$n = - \sqrt{r'(G_a - g)} \pm \frac{\sqrt{r'(G_a + 3g)}}{2} \tag{15b}$$

M, and m are the same given by (8b) and (8c) respectively. Proceeding as above, we finally get:

$$r' = L.(\frac{g}{c}) \tag{16}$$

and $$G_a = (\frac{r'}{L}) .C = C.(\frac{g}{c}) \tag{17}$$

Summary of this method can be given as:-

The actual line can be hybridly simulated by:-

i) Artificial distributed-parameter line the inductance and capacitance parameters of which are as the actual line, but with resistance diagonal 3×3 matrix with element r' given by equation (6). The conductance matrix, having diagonal elements G_a given by equation (17), but with the actual line subdiagonal elements.

ii) A lumped series per phase resistance given by $l(r - r')$.

iii) A lumped shunt conductance of admittance per phase given by $l\ (G - G_a)$.

Although the five methods, mentioned above, need more investigation, yet it seems to the author that the fifth method is the most favourite, as it imparts the line into an artificial distributed-parameter line and a lumped diagonal matrix element, which allows to be easily combined into perphase values of the other lumped elements. Besides, it generaily deos not involve matrices with zero diagonal elements (like metohd (4)), which may make time of computation more lengthly.

6. CALCULATION OF VOLTAGES AND CURRENTS ALONG THE LINE

After decomposition of a line a lumped element and an artificial distributed parameter line; the equations of system as a whole, are solved simultaneously with the generator<s equations to find out the generator<s terminal voltage and line current. The line sending end voltage and current are consequently found out at any time. Voltage and current at any point along the line; at a definite time can be found after the equations (1) and (2) are transformed by Fourier transformer into:-

$$\begin{vmatrix} \frac{dV_x}{dx} & = & Z(w) \, . \, I_x \\ \frac{dI_x}{dx} & = & Y(w) \, . \, V_x \end{vmatrix}$$ (18a) (18b)

and

where

$$V_x = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}_x \qquad I_x = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}_x$$

and

$$Z(w) = R(w) + jw\,L(w)$$

$$Y(w) = G(w) + jw\,C(w)$$

$$Z(w) = R(w) + jw\ L(w)$$
$$Y(w) = G(w) + jw\ C(w)$$

It is worthmentioning here. that an approximation is made here whrn assuming that voltages and currents are purely sinusoidal with a single frequency.
Equations (18a) and (18b) can be combined into the matrix form:-

$$\frac{d}{dx} \begin{vmatrix} V_x \\ I_x \end{vmatrix} = \begin{vmatrix} O & Z \\ X & O \end{vmatrix} \begin{vmatrix} V_x \\ I_x \end{vmatrix} \qquad (19a)$$

which is the state transition equation:

$$\frac{d}{dx} X = A\,X \qquad (19b)$$

where

$$X = \begin{vmatrix} V_x \\ V_x \end{vmatrix} \qquad A = \begin{vmatrix} O & Z \\ Y & O \end{vmatrix} \qquad (19c)$$

Equation (19b) has the solution:

$$X = e^{-x.A} . X_o \qquad (20)$$

where X_o is the vector centaining the voltage and current at line sending end.
The calculation of the state transition matrix $e^{x.A}$ can be carried out in several ways (6). Up to author's knowledge three possible ways can be applied: the direct expansion, the use of eigen values and eigenvectors, and the use of sylvester formula. The direct expansion is not recommended due to slow conversion of exponential series (Appendix I).
The Sylvester formula yields an efficient method for calculating the transition matrix $e^{x.A}$ (Appendix II).
The eigen values and eigen vectors of matrix A (order 6) can be related to the eigen values and eigen vectors of the transmission coefficient matrix Z Y (order 3). However, when this technique is applied, the eigen values, eigen vector method becomes more efficient than the Sylvester method.
Using this technique, yields the solution (Appendix III).

$$e^{x A} = \begin{vmatrix} H(x) & K(x) \\ D(x) & E(x) \end{vmatrix} \qquad (21)$$

where
G(x), K(x), D(x) and E(x) are given in equations (48b) up to (48e).
Thus, one can summarize the calculational process of a synchronous machine with connected long line at its terminal as:

i - The line is considered as imparted into a lumped element and an artificial line whose characteristic (surge) impedance Z_o can be easily calculated in the operational form. For this purpose, equation sets (5) and (15a) up to (17) are proposed for the artificial line.

ii - The synchronous machine's equations can be solved simultaneously, with the line: represented as mentioned in (i); as a terminal constraint to give the line sending and receiving end variables with travelling wave transients involved. Detail of this step will be given in the accompanying paper.

iii - Voltage and current vectors at any point along the line are calculated by equations (20) and (21).

7. INCLUSION OF OTHER CONNECTED ELEMENTS

In usual cases, other series or shunt elements are connected with the long line, and hence the above mentioned equations should be modified to take into account the effects these connected elements. These are calssified here into three types; naely; distributed parameter elements (lines or cables); series lumped elements (such as series connected reactors, transformers and capacitor banks);and shunt lumped elements (such as loads, shunt reactors, shunt capacitors, voltage transformers and generators).

7.1 Distributed Parameter Elements:

For an element (line or cable) of length pq connected between points p and q, the current-voltage relation at both ends of the line is given by (see Appendix IV).

$$\begin{vmatrix} I_{pq} \\ I_{qp} \end{vmatrix} = \begin{vmatrix} F_{pq} & G_{pq} \\ G_{qp} & F_{qp} \end{vmatrix} \begin{vmatrix} V_p \\ V_q \end{vmatrix} \qquad (22)$$

where:

I_{pq} is vector containing the set of line currents flowing at end p towards q.

V_p is vector containing the set of phase to ground voltages at bus bars p.

F_{pq}, G_{pq}, G_{pq} and F_{qp} are 3×3 matrices given by equations set (54a) and (54b).

Thus, for any number of distributed parameter elements connected to bus bar p, the admittance matrix T_{pp} is given by

$$Y_{pp} = \sum_q F_{pq} \qquad (23)$$

and

$$Y_{pq} = G_{pq} \qquad (24)$$

The summation in equation (23) is carried out over all the lines connected to bus bar p. Y_{pa} is equal to O if there is no connection between bus bars p and bus bars q.

7.2 Series Elements:

An element connected between bus bars p and q can be represented in a similar form as in equation (22). The parameter matrices F_{pq}, F_{qp} and G_{qp} can be claculated from the equivalent circuit or they can be measured. In case that the series element is, or can be replaced by an impedance Z_{pq}, these parameters matrices take the values

$$F_{pq} = F_{qp} = Z_{pq}^{-1} \quad \text{and} \quad G_{pq} = G_{qp} = -Z_{pq}^{-1} \qquad (25)$$

These parameter matrices can now be incorporated in the Y matrices of equations (23) and (24).

If an element connected between bars p and q; has the parameter matrices F_{pq}, F_{pq}, G_{pq} and G_{pq}; is connected in series with an element qn, having the parameter matrices F_{qn}, F_{nq}, G_{qn} and G_{nq}, then we shall be have the current-voltage relation:

$$\begin{vmatrix} I_p \\ I_q \\ I_n \end{vmatrix} = \begin{vmatrix} F_{pq} & G_{pq} & O \\ G_{qp} & F_{qp} + F_{qn} & G_{qn} \\ O & G_{nq} & F_{nq} \end{vmatrix} \begin{vmatrix} V_p \\ V_q \\ V_n \end{vmatrix} \qquad (26)$$

Eliminating V_q, and remembering that $I_q = 0$, finally get:

$$\begin{vmatrix} I_{pn} \\ \\ I_{np} \end{vmatrix} = \begin{vmatrix} F'_{pn} & G'_{pn} \\ \\ G'_{np} & F'_{np} \end{vmatrix} \begin{vmatrix} V_p \\ \\ V_n \end{vmatrix} \tag{27a}$$

where:

$$F'_{pn} = F_{pq} - F_{pq}(F_{qp} + F_{qn})^{-1} G_{pq} \tag{27b}$$

$$F'_{nnp} = F_{nq} - G_{nq}(F_{qp} + F_{qn})^{-1} G_{qn} \tag{27c}$$

$$G'_{pn} = (Gp_q (F_{qp} + F_{qn})^{-1} G_{qn} \tag{27d}$$

$$G'_{np} = G_{nq}(F_{qp} + Fqn)^{-1} G_{qp} \tag{27e}$$

The combined element can be incorporated in the Y matrix as mentioned before.

7.3 Shunt Elements:

A shunt element connected to the bus bare p and of admittance Y_{pp} equal to the inverse of its impedance matrix can be included in the Y matrix by adding the submatrix Y_{pp} to the revelant diagonal matrix.Also, if a network can be divided into two independent parts, the equivalent impedance of one part can be treated as a shunt element to the other.

CONCLUSION

1) Five methods are proposed here to take into account travelling wave effects of a long line while investigation of transient behaviour of an integrated power system contituting a synchronous machine.The one which is recommended by the author imparts the long line into an artificial distributed parameter line plus a lumped series resistance plus shunt conductance element.
2) The paper gives a general survey of available methods for calculation of voltages and currents along the line in terms of its sending end values, and recommends the eigen values eigen vectors method for such type of problems.
3) It discusses cases of inclusion of more than one long line connected to the system: of connected series and shunt elements upon the changes of system's parameters.

However, how to calculate the line's sending end values is the problem which is solved in the accompanying paper.

APPENDIX I

EVALUTION OF I AND V BY DIRECT EXPANSION (2)

By transforming equations (1) and (2) into Fourier domain. we get:

$$\frac{dV}{dx} = Z(w) . I \tag{28a}$$

and

$$\frac{dI}{dx} = Y(w) . V \tag{28b}$$

where

$$Z(w) = R(w) + jw\,L(w)$$

$$Y(w) = G(w) + jw\,C(w)$$

Sumultaneous solution of equations (28a). (28b) gives:-

$$\frac{d^2V}{dx^2} = Z(w) . Y(w) . V \tag{29a}$$

and

$$\frac{d^2I}{dx^2} = Y(w) . Z(w) . I \tag{29b}$$

Equations (29a) and (29b) are solved by assuming a solution of the form:-

$$V = e^{-\lambda_e x} . T \tag{30a}$$

and

$$I = e^{-\lambda_i x} . W \tag{30b}$$

where λ_e and λ_i are square matrices. refered to as propagation constants. T. and W are column matrices of voltages and currents at line's sending end.

The exponentional function can be expanded by Cayley-Hamilton theorm (7):

$$e^{-\lambda_e x} = I - \lambda_e \cdot \frac{x}{11} + \lambda_e^2 \frac{x^2}{21}$$

$$- \lambda_e^3 \frac{x^3}{31} + \ldots\ldots\ldots \qquad (31)$$

Taking the x derivative, we get:

$$\frac{d e^{-\lambda_e x}}{dx} = - \lambda_e \{ I - \lambda_e \frac{x}{11}$$

$$+ \lambda_e^2 \frac{x^2}{21} - \lambda_e^3 \frac{x^3}{31} + \ldots\ldots$$

$$= - \lambda_e \cdot e^{-\lambda_e x}$$

and the second derivative of V and I yield:—

$$\frac{d^2 V}{dx^2} = \lambda_e^2 \cdot V \qquad (32)$$

and

$$\frac{d^2 I}{dx^2} = \lambda_i^2 \cdot I \qquad (33)$$

Substituting from equation (32) into (29a), we get:

$$\lambda_e^2 \, V = Z(w) \,.\, Y(w) \,.\, V$$

Again substituting from equation (33) into (29b) we obtain:-

$$\lambda_i^2 \, I = Y(w) \,.\, Z(w) \,.\, I$$

Hence, λ_e and λ_i are now calculated to be:

$$\lambda_e = (Z(w) \,.\, Y(w))^{1/2}$$

which can be substituted into equation (31) to get the required state transition matrix.Simularly.

$\lambda_i = (Y(w) . Z(w))^{1/2}$

APPENDIX II

EVALUTION OF I AND V BY SYLVESTER FORMULA (2.7)

The expansion of the exponential function is given by:

$$e^{\lambda_f} = \sum_{k=1}^{n} P(\lambda_K) . Z_K(\lambda_f) \qquad (34)$$

$$Z_K(\lambda_f) = \frac{\prod_{K \neq r} (\lambda_f - \lambda_r U)}{\prod_{K \neq r} (\lambda_K - \lambda_r)} \qquad (35)$$

where

$$P(\lambda_K) = e^{-\lambda_K x}$$

f designatis i.e

and λ_K . λ_r are the eigen values of λ_f.

U is 3×3 identity matrix.
For three conductor line. three distinct characteristic roots (eigen values) will exist for each of λ_e . and λ_i.
Let these for λ_f be $\lambda_1$and $\lambda_2$and λ_3. then:-

$$e^{\lambda_f x} = \frac{e^{-\lambda_1 x}}{(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)} \{\lambda_f - \lambda U\} \{\lambda_f - \lambda_3 U\}$$

$$+ \frac{e^{-\lambda_2 x}}{(\lambda_2 - \lambda_1)(\lambda_2 - \lambda_3)} \{\lambda_f - \lambda_1 U\} \{\lambda_f - \lambda_3 U\}$$

$$+ \frac{e^{-\lambda_3 x}}{(\lambda_3 - \lambda_1)(\lambda_3 - \lambda_2)} \{\lambda_f - \lambda_1 U\} \{\lambda_f - \lambda_2 U\}$$

Thus. substituting into equations (30a) and (30b) . we get the voltage and current vectors at any point along the line.

APPENDIX III

EVALUATION OF TRANSITION MATRIX e^{xA} BY THE EIGEN VALUES METHOD (1,4,5,6)

To calculate the transition matrix ,the eigen values and eigen vectors of the state matrix A are calculated.However, it has been proved that the eigen values of it has been proved that the eigen values of the A matrix are the positive and negative square roots of the eigen values of the transmission coefficient matrix Z .Y (7).This reduces the computation time considerably since the transmission coefficient matrix is of the order 3, while that of the state matrix is 6.To find the relation between the eigen vectors of the state matrix A and the eigen vectors of the transmission coefficient matrix Z.Y, let us proceed as follows:

Let us denote the eigen values of the matrix Z Y by λ_1^2, λ_2, λ_3^2, and let us storethem in the diagonal matrix Ω_2.The corresponding eigen vectors stored in the matrix Q columnwise.Q and Ω_2 satisfy the relation:

$$Z\ Y\ Q = Q\ \Omega^2 \tag{36}$$

The eigen vectors matrix to the matrix A is denoted by P and is divided into four 3×3 square matrices P_1 , P_2, P_3 and P_4.

Thus ,the eigen values equation of matrix A will be:

$$\begin{vmatrix} 0 & Z \\ Y & 0 \end{vmatrix} \begin{vmatrix} P_1 & P_2 \\ F_3 & P_4 \end{vmatrix} = \begin{vmatrix} P_1 & P_2 \\ P_3 & P_4 \end{vmatrix} \begin{vmatrix} \Omega & 0 \\ 0 & \Omega \end{vmatrix} . \tag{37}$$

where Ω is a diagonal matrix containing λ_1, λ_2 and λ_3.

Equation (37) gives the equation set:-

$$Z\ P_3 = P_1\ \Omega \tag{38a}$$

$$Z\ P_4 = -P_2\ \Omega \tag{38b}$$

$$Y\ P_1 = P_3\ \Omega \tag{38c}$$

and $$Y\ P_2 = -P_4\ \Omega \tag{38d}$$

From equation (38c) we get:-

$$P_3 = Y\ P_1\ \Omega^{-1} \tag{39}$$

Substituting P_3 from equation (39) into (38a) and postmultiplying by Ω yields:

$$Z\,Y\,P_1 = P_1\,\Omega^2 \tag{40}$$

Comparison between equations (40) and (36) shows that P_1 is the eigen vector matrix Z Y.Calculation of P_2 , P_3 and P_4 from equations (38b) , (38d) and (39) gives the eigen vector matrix P of the state matrix A as:

$$P = \begin{vmatrix} Q & Q \\ Z^{-1}Q\,\Omega & \times Z^{-1}Q\,\Omega \end{vmatrix} \tag{41}$$

To calculate P^{-1}, then P is divided into four 3×3 submatrices S_1 , S_2 , S_3 and S_4 , which satisfy the relation:

$$\begin{vmatrix} Q & Q \\ (Z^{-1}Q\,\Omega) & (-Z^{-1}Q\,\Omega) \end{vmatrix} \begin{vmatrix} S_1 & S_2 \\ S_3 & S_4 \end{vmatrix} = \begin{vmatrix} U & O \\ O & U \end{vmatrix} \tag{42}$$

where U and O are 3×3 unit and zero matrices prespectively.
Solving this last equation for S_i , i =1,2,3,4 yields:

$$P^{-1} = \tfrac{1}{2} \begin{vmatrix} Q^{-1} & \Omega^{-1}Q^{-1}Z \\ Q^{-1} & -\,\Omega^{-1}Q^{-1}Z \end{vmatrix} \tag{43}$$

But ,as P: which is denoted by the modal matrix: is containing the eigen vectors of matrix A as mentioned above:then it should satisfy the relations:-

$$A\,P = P\,\Omega \tag{44}$$

and

$$e^{xA} = P\,e^{x\Omega}.P^{-1} \tag{45}$$

Substituting from equations (41) and (43) into (45) , we get:-

$$e^{xA} = \begin{vmatrix} Q\cosh(x\,\Omega)\,Q^{-1} & Q\,\Omega^{-1}\sinh(x\,\Omega)\,Q^{-1}Z \\ Z^{-1}Q\sinh(x\,\Omega)\,Q^{-1} & Z^{-1}Q\cosh(x\,\Omega)\,Q^{-1}Z \end{vmatrix} \tag{46}$$

But ,it has been shown in reference (2), that the surge impedance $Z_o = Z_c$ (for perfect earth), and is given by:

$$Z_o = Q\,\Omega^{-1}\,Q^{-1}\,Z \tag{47}$$

Substituting from equation (47) into (46) we finally:

$$e^{xA} = \begin{vmatrix} H(x) & K(x) \\ D(x) & E(x) \end{vmatrix} \tag{48a}$$

where

$$H(x) = \cos h\ x\ (Z\ Y)^{1/2} \tag{48b}$$

$$K(x) = \sin\ x\ (Z\ Y)^{1/2}.Z_o \tag{48c}$$

$$D(x) = Z_o^{-1}\ \sin h\ x\ (Z\ Y)^{1/2} \tag{48d}$$

$$E(x) = Z_o^{-1}\ \cos h\ x\ (Z\ Y)^{1/2}\ Z_o \tag{48e}$$

APPENDIX IV

DERIVATION OF CURRENT-VOLTAGE RELATIONS ALONG A LINE (4,5,6)

Re-writing equation (20):

$$X = e^{xA}.\ X_R \tag{20}$$

and for line connecting bus bars p and q, we have:-

$$\begin{vmatrix} V_p \\ I_{pq} \end{vmatrix} = e^{xA}. \begin{vmatrix} V_q \\ I_{qp} \end{vmatrix} \tag{49}$$

substituting for $e^{xA}$from equation (21) into (49) we get:-

$$\begin{vmatrix} V_p \\ I_{pq} \end{vmatrix} = \begin{vmatrix} H(x) & K(x) \\ D(x) & E(x) \end{vmatrix} \begin{vmatrix} V_q \\ I_{qp} \end{vmatrix}$$

$\therefore\ I_{pq} = D(x).\ V_q + E(x).\ I_{qp}$

But $V_p = H(x).\ V_q + K(x).\ I_{qp}$

or $I_{qp} = K^{-1}(x).\ \{V_p - H(x).\ V_q$ (50b

Substituting from (50b) into (50a), we get:-

$$I_{pq} = D(x).\ V_q + E(x)\ K^{-1}(x)\ V_p - E(x)\ K^{-1}(x)\ H(x)\ V_q$$

$$= \{D(x) - E(x)\ K^{-1}(x).\ H(x)\} V_q + E(x)\ K^{-1}(x)\ V_p \tag{51}$$

Combining the current vectors into one matrix we obtain:-

$$\begin{vmatrix} I_{pq} \\ I_{pq} \end{vmatrix} = \begin{vmatrix} E(x)K^{-1}(x) & D(x)-E(x)K^{-1}(x)H(x) \\ K^{-1}(x) & -K^{-1}(x)\ H(x) \end{vmatrix} \begin{vmatrix} V_p \\ V_q \end{vmatrix} \tag{52}$$

Substituting from the set of equations (48b) up to (48e) into (52) we get:

$$\begin{vmatrix} I_{pq} \\ I_{qp} \end{vmatrix} = \begin{vmatrix} F_{pq} & G_{pq} \\ G_{qp} & F_{qp} \end{vmatrix} \begin{vmatrix} V_p \\ V_q \end{vmatrix} \tag{53}$$

where:

$$F_{pq} = F_{qp} = Z_{pq}^{-1}\ Q_{pq}\ \Omega_{pq}\ \coth(\Omega_{pq} l_{pq})\ (Q_{pq}^{-1} \tag{54a}$$

and

$$G_{pq} = G_{qp} = Z_{pq}^{-1}\ Q_{pq}\ \Omega_{pq}\ \mathrm{cosech}(\Omega_{pq} l_{pq})\ Q_{pq}^{-1} \tag{54b}$$

REFERENCES

1. L.M. WEDEPHOL: "Application of matrix methods to the solution of travelling wave phenomena in polyphase systems". Proc. IEE. Vol. 110. No. 12. 1963. pp. 2200-2212.
2. D. E. HEDMAN: "Propagation of Overhead Transmission lines-Theory of Modal Analysis". LEEE Transactions on PAS. March. 1965. pp. 200-205.
3. J.P. BICKFORD. P.S. DOEPEL: "Calculation of Switching Transients with Particular Reference to LIne Energization" Proc. of IEE. Vol. 114. April 1967. pp. 465 - 477.
4. T. CHOLEWICKI: "Matrix methods applied to the analysis of transients in a long transmission line" Bulletin De l'Academie Polonaise des Sciences. Serie des Sciences Techniques. Vol. XVI. No.7. 1968.pp. 611 - 616.
5. T. CHOLEWICKI: "Analysis of transients in a uniform unsymmetric three phase long transmission line" Ibid. Vol. XVI. No. 1. pp. 97 - 104.
6. L.M. WEDEPHOL. R.C. WASLEY: "Solution of travelling wave problems in multiconductor lines using eigen value techniques". Proc. PSCC. Rome 1969. paper S.3.
7. F.B. HILDEDRAND: "Methods of Applied Mathematics". Prentice-Hall. Inc.. 1965.