INVESTIGATION OF THE TRAVELLING WAVE EFFECTS IN ANALYSIS OF SYNCHRONOUS MACHINE TRANSIENT BEHAVIOUR

II . THE SYNCHRONOUS MACHINE PERFORMANCE

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دراسة لتقيم أثر الموجات المسافرة في تحليل السلوك العابر للآلات (المكائن) المتزامنة – (٢) سلوك المكائن المتزامنة

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خلاصة المقالة

يمثل البحث المقدم الثاني من سلسلة من الابحاث الخاصة والتي تهتم بالسلوك الكلي للنظم الكهربائية المتكاملة والتي تضم مكائن متزامنة مرتبطة بشبكة كهربائية كبرى من خلال خط كهربائي طويل . وقد اهتم هذا البحث بوضع المعادلات الخاصة بالمكائن المتزامنة في صورة جديدة آخذا في الاعتبار الشكل الجديد والذي اقتراحه المؤلف في البحث الاول من هذه السلسلة .

بذلك امكن تصميم نموذج رقمي جديد لدراسة النظام الكهربائي متكاملا اثناء السلوك العابر للفترات الموجية والكهرومغناطيسية وذلك بعد استبعاد أثر منظم الجهد للمكائن المتزامنة .

ABSTRACT:

This is the second of a series of papers interested mainly in the overall transient behaviour of an integrated power system constituting a synchronous machine connected to a large network through a long line. After deriving an expression for the long line as a synchronous machine's terminal constraint in the first paper, this paper investigates the machine's electromagnetic transients involving line's short lived travelling wave effects. A digital model is derived for the study of the transient behaviour of the system involving the long line effects but excluding, the action of machine's automatic excitation regulator.

NOMENCLATURE

[ ], A	Matrix
p	d /dt.
d, q, O	Direct, quadrature and zero components respectively.
D, Q	Direct and quadrature damper respectively.
a, b, c	Phase values.
i, v,	Instantaneous current, voltage and flux linkage respectively.
a, f, D, Q	Stator, field, direct axis and quadrature axis damper circuits respectively.
R,L,C,G,Z	Resistance, Inductance, capacitance, conductance and impedance respectively.
w, w_b	Rotor angular velocity in per unit and radians respectively.

1. INTRODUCTION

Analysis of an integrated power system's behaviour makes it desirable to recognize three distant periods; namely the surge period, the dynamic period and the steady state period. This classification depends mainly upon the fact that always exists a wide span between the system's time constants. Such classification of course greatly facilitates the solution of the problem whose nature necessitates the applications of more than one transformation to the involved equations. However, such a classification; with the use of modern derivative type excitation system regulator's becomes inadequate. The use of regulators whose action is dependent upon changes of first, second and /or higher erivatives of the controlled variables; makes it necessary to allow for wave propagation in a long line simultaneously with the dynamic transient phenomena [6. Hence, with the long line represented as in the first paper, the whole system electromagnetic transient allowing for line's is investigated throughout the subject paper.

2. SYSTEM EQUATIONS

From the accompanying paper, a long line can be represented by the matrix equation:

$$G\,V + P\,C\,V = N'\,I + P\,M\,I \qquad (1)$$

where for completely transposed system we have:-

$$M = \begin{vmatrix} M & m & m \\ m & M & m \\ m & m & M \end{vmatrix}$$

$$M = \pm \frac{\sqrt{L(C + 3c)}}{2}$$

$$m = - \sqrt{L(C - c} + M$$

$$N' = \begin{vmatrix} N' & n' & n' \\ n' & N' & n' \\ n' & n' & N' \end{vmatrix}$$

$$N' = \frac{\sqrt{r'(G + 3g')}}{2}$$

$$n' = - \sqrt{r'(G - g')} + N'$$

A capital letter designates diagonal element and a small letter designates a off-diagonal element of a matrix.

Applying the modified park's transformation [3] (power invariant) to both sides of equation (1), we obtain:

$$k\,[G\,V + P\,C\,V] = K\,[N'\,I + P\,M\,I] \qquad (2)$$

Substituting for K from equation (22), bearing in mind, for balanced operation,

$$\sum_{i=a}^{c} F_i = 0$$

where F may be i, v or Ψ, we get:-

For direct axis:-

$$(G - g).\, V_d + (C - c)\,(pV_d - wV_q)$$

$$= (N' - n')I_d + (M - m)\,(pI_d - wI_q) \qquad (3)$$

For quadrature axis:

$$(G - g)\,.\, V_g + (C - c)\,(wV_d + pV_q)$$

$$= (N' - n')\, I_q + (M - m)\,(wI_d + pI_q) \qquad (4)$$

Re-writting equations (3) and (4) in matrix form, and after collecting voltage terms in one side, and current terms in the other sids, we get:-

$$\begin{vmatrix} (G-g)+p(C-c) & -w(C-c) \\ w(C-c) & (C-g)+P(C-c) \end{vmatrix} \begin{vmatrix} V_d \\ V_q \end{vmatrix} = \begin{vmatrix} (N-n')+p(M-m) & -w(M-m) \\ w(M-m) & (N'-n')+p(M-m) \end{vmatrix} \begin{vmatrix} i_d \\ i_q \end{vmatrix}$$

From the last equation, we get:

$$\begin{vmatrix} V_d \\ V_q \end{vmatrix} = \begin{vmatrix} (G-g)+p(C-c) & -w(C-c) \\ w(C-c) & (G-g)+p(C-c) \end{vmatrix}^{-1} \begin{vmatrix} (N'-n')+p(M-m) & -w(M-m) \\ w(M-m) & (N'-n')+p(M-m) \end{vmatrix} \begin{vmatrix} i_d \\ i_q \end{vmatrix}$$

$$= \frac{1}{(A+pB)^2 + K^2} \begin{vmatrix} (A+pB)(e+pf) + Kh & K(e+pf) - h(A+pB) \\ h(A+pB) - K(e+pf) & (A+pB)(e+pf)+Kh \end{vmatrix} \begin{vmatrix} i_d \\ i_q \end{vmatrix}$$

where:

$A = (G - g)$, $\qquad B = (C - c)$

$e = (N' - n'),$ $\quad f = (M - m)$

$K = w\,(C - c),$ $\quad h = w\,(M - m)$

Re-writting from the appendiix equation (30):

$$\begin{vmatrix} V_d \\ \\ V_q \end{vmatrix} = \begin{vmatrix} r_a & 0 \\ \\ 0 & r_a \end{vmatrix} \begin{vmatrix} i_d \\ \\ i_q \end{vmatrix} - w \begin{vmatrix} 0 & 1 \\ \\ -1 & 0 \end{vmatrix} \begin{vmatrix} \Psi_d \\ \\ \Psi_q \end{vmatrix} - \frac{p}{w_d} \begin{vmatrix} \Psi_d \\ \\ \Psi_q \end{vmatrix}$$

Substituting for the voltage matrix in this last equation from (5), and collecting the flux terms in one side, we obtain:-

$$-w \begin{vmatrix} 0 & 1 \\ \\ -1 & 0 \end{vmatrix} \begin{vmatrix} \Psi_d \\ \\ \Psi_q \end{vmatrix} - \frac{p}{w_b} \begin{vmatrix} \Psi_d \\ \\ \Psi_q \end{vmatrix}$$

$$= \frac{1}{(A+pB)^2 + K^2} \begin{vmatrix} (A+pB)(r_aA+e) + p(r_aB+f) + K(r_aK+h) & (Ke - Ah) + p(Kf - Bh) \\ \\ (Ah - Ke) + p(Bh - Kf) & (A+pB)(r_aA+e) + p(r_aB+f) + K(r_aK+h) \end{vmatrix} \begin{vmatrix} i_d \\ \\ i_q \end{vmatrix} \tag{6}$$

Re-writting equation (29) from appendix, then:

$$\begin{vmatrix} i_d \\ \\ i_q \end{vmatrix} = \begin{vmatrix} 1/x_d' & 0 \\ \\ 0 & 1/x_q'' \end{vmatrix} \begin{vmatrix} \Psi_d \\ \\ \Psi_q \end{vmatrix} - \begin{vmatrix} -(x_{aD} - \dfrac{x_{af}\,x_{fD}}{x_f}).i_D \\ + E_q \\ E_d'' \end{vmatrix}$$

Substituting for the current matrix in equation (6) from this last equation, and multiplying both sides by $\{(A + pB)^2 + K^2\}$, we get:-

$$\{(A + pB)^2 + K^2\}. \begin{vmatrix} w \begin{vmatrix} 0 & -1 \\ \\ 1 & 0 \end{vmatrix} - pw_b \end{vmatrix} \begin{vmatrix} \Psi_d \\ \\ \Psi_q \end{vmatrix}$$

$$
= \begin{vmatrix} K(r_a+h)+(a+pB).(r_aA+e) +p(r_aB+f) & (Ke-Ah)+p(Kf-Bh) \\ (Ah-Ke)+p(Bh-Kf) & K(r_aK+h)+(A+pB).(r_aA+e)+p(r_aB+f) \end{vmatrix}
$$

$$
\times \begin{vmatrix} 1/x'_d & 0 \\ 0 & 1/x''_q \end{vmatrix} \begin{vmatrix} \Psi_d \\ \Psi_q \end{vmatrix} - \begin{vmatrix} E'_q-(x_{aD}-\dfrac{x_{af}x_{fD}}{x_f}).i_D \\ E''_d \end{vmatrix}
$$

Collecting homogeneous terms, and rearranging, we get:-

$$
\begin{vmatrix} (\dfrac{m}{x_d}+pw_bU) & (\dfrac{n}{x''_q}+wu) \\ (wu-\dfrac{n}{x'_d}) & (\dfrac{m}{x''_q}+pw_bu) \end{vmatrix} \begin{vmatrix} \Psi_d \\ \Psi_q \end{vmatrix} = \begin{vmatrix} \dfrac{-m}{x'_d} & \dfrac{n}{x''_q} \\ \dfrac{-n}{x'_d} & \dfrac{-m}{x''_q} \end{vmatrix} \begin{vmatrix} E'_q-(x_{aD}-\dfrac{x_{af}.x_{fD}}{x_f})i_d \\ E''_d \end{vmatrix} \qquad (7a)
$$

where

$$M = K(r_aK+h) + (A+pB)\ .\ (r_aA+e) + p(r_aB+f)$$

$$= P^2B(r_aB+f) + p\ \ B(r_aA+e) + A(r_aB+f) + A(r_aA+e) + K(r_aK+h)$$

$$n = h\ (A+pB) - K(e+pf) = p(Bh-Kf) + Ah-Ke)$$

$$u = (A+pB)^2 + K^2 = P^2B^2 + P(2AB) + A^2+K^2) \qquad (7b)$$

Substituting from appendix equations (25b), (24) and (27b) for E'_q, i_D and D''_d into equation (7a), we get:

$$
\begin{vmatrix} (\dfrac{m}{x'_d}+pw_bu) & (wu-\dfrac{n}{x''_q}) \\ (wu-\dfrac{n}{x'_d}) & \dfrac{m}{x''_q}+pw_bu) \end{vmatrix} \begin{vmatrix} \Psi_d \\ \Psi_q \end{vmatrix} =
$$

$$
\begin{vmatrix} \dfrac{-m}{x'_d} & \dfrac{n}{x''_q} \\[2ex] \dfrac{-n}{x'_d} & \dfrac{m}{x''_q} \end{vmatrix}
\begin{vmatrix} \dfrac{x_{af}\,\Psi_f}{x_f} + \left(\dfrac{x_{af}\,x_{aD}}{x_f} - x_{aD}\right)\dfrac{p\,\Psi_D}{w_b r_D} \\[2ex] \dfrac{x_{aQ}\cdot\Psi_Q}{x_Q} \end{vmatrix}
$$

Re-aranging this last equation, we get:

$$
\begin{vmatrix} \left(\dfrac{m}{x'_d} + p w_b u\right) & \left(wu - \dfrac{n}{x''_q}\right) & \dfrac{pmb}{w_b r_D x'_d} & \dfrac{mnx_a Q}{x''_q \cdot x_Q} & \dfrac{mx_{af}}{x_f x'_d} \\[2ex] \left(mu - \dfrac{n}{x'_d}\right) & \left(\dfrac{m}{x''_q} + p w_b u\right) & \dfrac{pnb}{w_b r_D x'_d} & \dfrac{m\,x_a q}{x''_q \cdot x_Q} & \dfrac{nx_{af}}{x_f x'_d} \end{vmatrix}
\times \begin{vmatrix} \Psi_d \\ \Psi_q \\ \Psi_D \\ \Psi_Q \\ \Psi_f \end{vmatrix} = \begin{vmatrix} 0 \\ \\ 0 \end{vmatrix} \qquad (8)
$$

where $b = (x_{af}\,x_{fD}/x_f) - x_{aD}$

Now, let us accomplish the coefficient matrix by adding 3 rows, and hence it becomes a square matrix, whose inverse, generally (if nonosignular) exists. These rows, as the author proposes, can be derived by differentiating twice the appendix equations (36), (37) and (38). Thus equation (8) now can e changed into:

$\left(\frac{m}{x_d'} + p\,w_b u\right)$	$\left(wu - \frac{n}{x_q''}\right)$	$\frac{mp}{w_b \cdot r_D x_d'}\left(\frac{x_{af} x_{fD}}{x_f} - x_{aD}\right)$	$\frac{-n x_{aQ}}{x_q'' \cdot x_Q}$	$\frac{m x_{af}}{x_f x_d'}$	Ψ_d
$\left(wu - \frac{n}{x_d'}\right)$	$\left(\frac{m}{x_q'} + p w_b u\right)$	$\frac{nP}{w_b \cdot r_D \cdot x_d'}\left(\frac{x_{af} x_{fD}}{x_f} - x_{aD}\right)$	$\frac{n\,x_{af}}{x_f x_d}$	Ψ_q	
$\frac{p^2 . r_D}{a} \times (x_{fD} x_{af} - x_{aD} x_f)$	0	$\frac{p^3}{w_b} + \frac{p^2 . r_D (x_f x_d - x_{af}^2)}{a}$	0	$\frac{p^2 . r_D}{a} \times (x_{aD} x_{af} - x_d x_{fD})$	Ψ_Q
0	$\frac{p^2 r_Q x_{aQ}}{(x_{aQ}^2 - x_q x_Q)}$	0	$\frac{p^3}{w_b} + \frac{p^2 . r_Q . x_q}{(x_q x_Q - x_{aQ}^2)}$		Ψ_f
$\frac{p^2 x_{af} . x_{aD}}{(x_{aD} x_{af} - x_d x_{fD})}$	0	$\frac{p3(x_{aD}^2 - x_d x_D)}{w_b . r_D} - p^2 x_d \times \frac{x_{af}}{(x_{aD} x_{af} - x_d x_{fD})}$		$\frac{p^3 . x_{af} T_{do}'}{x_f}$	Ψ_D

$$= [\ 0 \quad 0 \quad 0 \quad 0 \quad p^2 E_{fd}\ ]^T \qquad (9)$$

where $a = x_d x_f x_D + x_{aD}^2 x_f - x_{aF}^2 X_D - x_{fD}^2 x_d$

equation (9) can be put in the matrix concise form:

$$\{P^3 W_3 + P^2 W_2 + P W_1 + W_0\} \quad Y = F \tag{10}$$

where:

$$W_3 = \begin{bmatrix} w_b . B^2 & 0 & \dfrac{B(r_a B + f)}{w_b . r_D . x_d'} . \left(\dfrac{x_{af} x_{fD}}{x_f} - x_{aD} \right) & 0 & 0 \\ 0 & w_b . B^2 & 0 & 0 & 0 \\ 0 & 0 & \dfrac{1}{w_b} & 0 & 0 \\ 0 & 0 & 0 & \dfrac{1}{w_b} & 0 \\ 0 & 0 & \dfrac{x_{fa}(x_{aD}^2 - x_d x_D)}{w_b . r_D (x_{aD} x_{af} - x_d x_{fD})} & 0 & \dfrac{x_{af} T_{do}'}{x_f} \end{bmatrix} \tag{11}$$

$$W_2 = \begin{bmatrix} \dfrac{B(r_aB+f)}{x_d'} & wB^2 & \left(\dfrac{x_{af}x_{fD}}{x_f} - x_{aD}\right)\cdot\dfrac{1}{w_b \cdot r_D \cdot x_d'} \times \{B(r_aA+e)+A(r_aB+f) & 0 & \dfrac{x_{af}B(r_aB+f)}{x_f \cdot x_d'} \\ wB^2 & \dfrac{B(r_aB+f)}{x_q''} + 2AB\ w_b & \left(\dfrac{x_{af}x_{fD}}{x_f} - x_{aD}\right)\cdot\dfrac{(Bh-Kf)}{w_b \cdot r_D \cdot x_d'} & \dfrac{x_{aQ}}{x_Q \cdot x_q''}\{B(r_aA+e)+A(r_aB+f) & 0 \\ \dfrac{r_D(x_{fD}x_{af}-x_{aD}x_f)}{a} & 0 & \dfrac{r_D(x_fx_d-x_{af}^2)}{a} & 0 & \dfrac{r_D(x_{aD}x_{af}-x_dx_{fD})}{a} \\ 0 & \dfrac{r_Q \cdot x_{aQ}}{(x_{aQ}^2-x_qx_Q)} & 0 & \dfrac{r_Q \cdot x_q}{(x_qx_Q-x_{aQ}^2)} & 0 \\ \dfrac{x_{af}\cdot x_{aQ}}{(x_{aD}x_{af}-x_dx_{fd})} & 0 & \dfrac{x_d\,x_{af}}{(x_dx_{fD}-x_{aD}x_{af})} & 0 & 0 \end{bmatrix} \qquad (12)$$

$W_1 =$

$w_b(A^2+K^2) + \frac{B(r_aA+e)}{x_d'} + \frac{A(r_aB+f)}{x_d'}$	$\frac{(Kf-Bh)}{x_q'} + 2ABw$	$(\frac{x_{af}x_{fD}}{x_f} - x_{aD}) \times \frac{\{A(r_aA+e)+K(r_aK+h)\}}{w_b \cdot r_D \cdot x_d'}$	$\frac{x_{aQ}(Kf-Bh)}{x_q' \cdot x_Q}$	$x_{af}B(r_aA) + A(r_aB+f) /(x_f x_d')$
$2ABw + \frac{(Kf-Bh)}{x_d'}$	$w_b(A^2+K^2) + \frac{B(r_aA+e)}{x_q''} + \frac{A(r_aB+f)}{x_q''}$	$(x_{aD} - \frac{x_{af}x_{fD}}{x_f}). \frac{(Ke-Ah)}{w_b r_D x_d'}$	$\frac{x_{aQ}}{x_q'' x_Q} - B (r_aA+e) + A(r_aB+f)$	$\frac{x_{af}(Bh-Kf)}{x_f x_d'}$
		0		

(13)

$W_o =$

$\frac{1}{x_d'}\{A(r_a+e)+K(r_aK+h)$	$w(A^2+K^2)+\frac{(Ke-Ah)}{x_q''}$	0	$\frac{x_{aQ}}{x_Q \cdot x_q''}(Ke-Ah)$	$—G—\{A(r_aA+e)+K(r_aK+h)\}$
$w(A^2+K^2)+\frac{(Ke-Ah)}{x_d'}$	$\frac{1}{x_q''}(A(r_aA+z)+k(r_aK+h)$	0	$\frac{A(r_aA+e)+(K(r_aK+h)}{x_q \cdot x_Q}$	$\frac{x_{af}(Ah-Ke)}{x_f \cdot x_d'}$
		0		

(14a)

$$Y = \begin{vmatrix} \Psi_d & \Psi_q & \Psi_D & \Psi_Q & \Psi_f \end{vmatrix}^T \quad (14b)$$

$$F = \begin{vmatrix} 0 & 0 & 0 & 0 & p^2E_{fd} \end{vmatrix}^T \quad (14c)$$

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In solving the investigated system equation (9), throughout this paper, we shall discuss its solution with the discard of the effect of the excitation regulator, i.e P^2E_{fd} will be replaced by zero. However, inclusion of excitation regulator's effect will be discussed in a third paper.

Thus equation (10) becomes:-

$$(p^3W_3 + p^2 W_1 + pW_1 + W_0)\, Y = 0 \qquad (15)$$

To solve this last equation, let us proceed as follows:-

Let $Y_1 = Y$ (16a)

$Y_2 = Y_1 = p\,Y$ (16b)

$Y_3 = Y_2 = p^2Y$ (16c)

and from equation (15)

$Y4 = Y_3 = p^3\, Y$

$$= -W_3^{-1}(W_2\, Y_3 + W_1Y_2 + W_0\, Y_1) \qquad (16d)$$

The set of equations (16a) up to (16d) can be rearranged into the form:

$$p \begin{vmatrix} Y_1 \\ Y_2 \\ Y_3 \end{vmatrix} = \begin{vmatrix} 0 & I & 0 \\ 0 & 0 & I_1 \\ -W_3^{-1}W_0 & -W_3^{-1}W_1 & -W_3^{-1}W_2 \end{vmatrix} \begin{vmatrix} Y_1 \\ Y_2 \\ Y_3 \end{vmatrix} \qquad (17)$$

where

Y_1, Y_2 and Y_3 designate 5×1 matrices.

0, I designate zero and identity matrices of 5^{th} order respectively.

Equation (17) can be re-written in the concise form:

$$PX = A\,X \qquad (18)$$

where

$$X = \begin{vmatrix} Y_1 \\ Y_2 \\ Y_3 \end{vmatrix}$$ i.e. a 15^{th} order column matrix.

$$A = \begin{vmatrix} 0 & I & 0 \\ 0 & 0 & I \\ -W_3^{-1}W_0 & -W_3^{-1}W_1 & -W_3^{-1}W_2 \end{vmatrix}$$

Owing to the fact that thediagonal elements involve zeros, hence a convergent solution by Rung-Kutta method is practivally impossible.

Solution of equation is readily Known (4) and is:-

$$X = e^{tA}.\, X_0 \qquad (19)$$

where X_0 is the matrix X at $t = 0$

Substituting from the appendix equation (42) for the transition matrix e^{tA}, we finally get the solution:

$$X = -\Big[\frac{e^{-\lambda_1 t}}{(\lambda_1-\lambda_2)..(\lambda_1\lambda_{15})}\{A+\lambda_2 I\}\ \ldots\ \Omega+\lambda_{15}I$$

$$+\frac{e^{-\lambda_2 t}}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)..(\lambda_2-\lambda_{15})}\{A+\lambda_1 I\}\{A+\lambda_3 I\}\ldots$$

$$\{A+\lambda_{15}I\} + \ldots$$

$$+\frac{e^{-\lambda_{15}t}}{(\lambda_{15}-\lambda_1)..(\lambda_{15}-\lambda_{14})}\{A+\lambda_1 I\}..\{A+\lambda_{14}I\}\Big]X_0 \qquad (20)$$

where $\lambda_1, \lambda_2 \ldots \lambda_{15}$ are the eigen values of $-A$.

Once finding out the time variation of the flux linkage (and its first and second derivative) matrix X, then: back substitution from equation (20) into the appendix equation (29a): i_d and i_q are calculated. Again, substituting into into equation (30) V_d and V_q are calculated, and consequently the machine terminal voltage is calculated by aid of equation (24i).
ltage and current any point along the line are calculated by aid of the formula (from the accomanying paper):-

$$\begin{vmatrix} V \\ I \end{vmatrix}_x = e^{-xA}\begin{vmatrix} V \\ I \end{vmatrix}_0$$

where 0 designates sending end.

NUMERICAL EXAMPLE

Using the above mentioned algorithm, a computer run is made with the data given in Table I, referred to 500 kV and 1236 MVA base.

Table I

Gen. stator values p.v.					Total reacts. of rotor wdgs.		
X_d	X_q	X'_d	X''_q	r_a	X_f	X_D	X_Q
1	0.67	0.33	0.23	0.003	1.11	0.995	0.61

Mutual reacts. of rotor wdgs				Rotor circuit resistances		
X_{af}	X_{fD}	X_{aD}	X_{aQ}	r_f	r_D	r_Q
0.87	0.87	0.87	0.52	0.0004	0.018	0.0063

Time const			(Line Parameters/Km) x 10^{-6}						
T'_{do}	T''_{qo}	w_b	r	L	G	g	C	c	
8.54	0.31	314	60	3	0.5	0.2	3	1	800

Figure (1) shows the computed values for the per unit generator terminal voltage (v_b), receiving end voltage (v_R), line sending end current variations for the case of sudden switching off of line remote end breadder when the system initially with the following conditions:

Receiving end load	= 1 + j0.3 P.U
Receiving end voltage	= 0.955 P.U
Sending end voltage	= 1.035 P.U
Generator terminal voltage	= 1.045 P.U.
Shunt reactor reactive power compenstion	= 0.5 P.U.

CONCLUSION:

Throughout this paper formulae are derived to link the fast transient line switching domain with the synchronous machine electromagnetic transient time domain. Although the excitation regulator effect is not investigated in the subject paper, yet the derived equations are the natural entrance for this target. however, the influence of excitation regulator upon the overall line switching and synchronous generator's electromagentic (stator) transient behaviour is reserved for a future paper.

APPENDICES

I. SYNCHRONOUS MACHINE EQUATIONS

According to Park equations [1, 2 and after proper selection of transformation matrices such that power should be invariant anywhere in the system [we have:

$$\begin{vmatrix} F_d \\ F_q \\ F_o \end{vmatrix} = K \begin{vmatrix} F_a \\ F_b \\ F_c \end{vmatrix}$$

$$= \sqrt{\tfrac{2}{3}} \begin{vmatrix} \cos\theta & \cos(\theta - \tfrac{2}{3}\pi) & \cos(\theta + \tfrac{2}{3}\pi) \\ \sin\theta & \sin(\theta - \tfrac{2}{3}\pi) & \sin(\theta + \tfrac{2}{3}\pi) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{vmatrix} \begin{vmatrix} F_a \\ F_b \\ F_c \end{vmatrix} \qquad (22)$$

where F designates v, i, and Ψ

d,q,O, designate direct, quadrature and zero components respectively.

a,b,c designate phase values.

Conversely

$$\begin{vmatrix} F_a \\ F_b \\ F_c \end{vmatrix} = K^{-1} \begin{vmatrix} F_d \\ F_q \\ F_c \end{vmatrix} = K^{t} \begin{vmatrix} F_d \\ F_q \\ F_0 \end{vmatrix} \qquad (23)$$

Applying transformation matrices (22) and (23) it had been proved elsewhere [7] , that the following relationships are derived:

$$I_d = X_d i_d + x_{af} i_f + x_{aD} i_D \qquad (24a)$$

$$I_q = x_q i_q + x_{aQ} i_Q \qquad (24b)$$

$$I_o = x_o i_o \qquad (24c)$$

$$I_f = x_{af} i_d + x_f i_f + x_{fD} i_D \qquad (24d)$$

$$I_D = x_{aD} i_d + x_{fD} i_f + x_D i_D \qquad (24e)$$

$$I_Q = x_{aQ} i_q + x_Q i_Q \qquad (24f)$$

$$V_d = -r_a i_d - (1/w_b)\, p\Psi_d - w\Psi_q \qquad (24g)$$

$$V_d = -r_a i_q - (1/w_b)\, p\Psi_q + w\Psi_d \qquad (24h)$$

$$V_t = \sqrt{V_d^2 + V_q^2} \qquad (24i)$$

$$V_o = -r_o i_o - (1/w_b)\, p\Psi_o \qquad (24j)$$

$$V_f = r_f i_f + (1/w_b)\, p\,\Psi_f \qquad (24k)$$

$$0 = r_D i_D + (1/w_b)\, p\,\Psi_D \qquad (24L)$$

$$0 = r_Q i_Q + (1/w_b)\, p\,\Psi_Q \qquad (24m)$$

Multiplying out both sides of equation (24d) by (x_{af}/x_f) then subtracting from equation (24a), we get:

$$\Psi_d - (\frac{x_{af}}{x_f})\Psi_f = (x_d - \frac{x_{af}^2}{x_f})id + (x_{aD} - \frac{x_{af} \cdot x_{fD}}{x_f}) \cdot i_D \tag{25}$$

But $$x_d - \frac{x_{af}^2}{x_f} = x_d' \tag{25a}$$

= Direct axis transient reactance and

$$\frac{x_{af}}{x_f} \cdot \Psi_f = E_q' \tag{25b}$$

e.m.f. behind transient reactance.

Thus, equation (25) becomes, after rearrangement:-

$$x_d' i_d = (\Psi_d - E_q') + (x_{aD} - \frac{x_{af} \cdot x_{fD}}{x_f}) \cdot i_D \tag{26}$$

Again, multiplying out both sides of equation (24f) by (x_{aQ}/x_Q); then subtacting from equation (24b), we get:

$$\Psi_q - (\frac{x_{aQ}}{x_Q}) \cdot \Psi_Q = (x_q - \frac{x_{aQ}}{x_Q})i_q \tag{27}$$

and $$x_q'' = x_q - \frac{x_{aQ}^2}{x_Q} \tag{27a}$$

= Quadrature axis subtransient reactance.
and let us define:

$$E_d'' = (\frac{x_{aQ}}{x_Q}) \cdot \Psi_Q \tag{27b}$$

Thus, equation (27) becomes:

$$\Psi - E_d'' = x_q'' i_q \tag{28}$$

From equations (26) and (28), we can weite down:-

$$\begin{vmatrix} \\ i_q \end{vmatrix} = \begin{vmatrix} i/x & 0 \\ 0 & 1/x''_q \end{vmatrix} .\{ \begin{vmatrix} \Psi_d \\ \Psi_q \end{vmatrix} - \begin{vmatrix} E'_q - (\frac{x_{af} \cdot x_{fD}}{x_{aD} \cdot x_f}) i_D \\ E''_d \end{vmatrix} \} \quad (29)$$

Substituting into equation (29) for i_D from equation (24L), and for E''_d from equation (27b) and for E'_q from equation (25b) we get:

$$\begin{vmatrix} i_d \\ i_q \end{vmatrix} = \begin{vmatrix} 1/x'_d & 0 \\ 0 & i/x''_q \end{vmatrix} .\{ \begin{vmatrix} \Psi_d \\ \Psi_q \end{vmatrix} - \begin{vmatrix} (x_{af}\Psi_f/x_f) + (x_{aD} - (x_{aD} - \frac{x_{af}x_{fD}}{x_f}) p\Psi_D/w_b r_D \\ (x_{aQ}/x_Q)\Psi_Q \end{vmatrix} \} \quad (29a)$$

Also, rewritting equations (24g), (24h) in matrix form, then:

$$\begin{vmatrix} V_d \\ V_q \end{vmatrix} = A_L \begin{vmatrix} i_d \\ i_q \end{vmatrix} + A_M \begin{vmatrix} \Psi_d \\ \Psi_q \end{vmatrix} + A_R\, P \begin{vmatrix} \Psi_d \\ \Psi_q \end{vmatrix} \quad (30)$$

where

$$A_L = -r_a \cdot I_{2\times2} \quad (30a)$$

$$A_M = -w \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \quad (30b)$$

$$A_R = -1/w_b \cdot I_{2\times2} \quad (30c)$$

To find out explicit expressions for machin's current, then, from the set of appendix equations (24a) up (24f) excluding (24c), we have:

$$\begin{vmatrix} i_d \\ i_q \\ i_D \\ i_Q \\ i_f \end{vmatrix} = \begin{vmatrix} x_d & 0 & x_{aD} & 0 & x_{af} \\ 0 & x_q & 0 & x_{aQ} & 0 \\ x_{aD} & 0 & x_D & 0 & x_{fD} \\ 0 & x_{aQ} & 0 & x_Q & 0 \\ x_{af} & 0 & x_{fD} & 0 & x_f \end{vmatrix}^{-1} \begin{vmatrix} \Psi_d \\ \Psi_q \\ \Psi_D \\ \Psi_Q \\ \Psi_f \end{vmatrix}$$

Or

$$\begin{vmatrix} i_d \\ i_q \\ i_D \\ i_Q \\ i_f \end{vmatrix} = \begin{vmatrix} a_{11} & \cdot & \cdot & \cdot & a_{15} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{51} & \cdot & \cdot & \cdot & a_{55} \end{vmatrix} \begin{vmatrix} \Psi_d \\ \Psi_q \\ \Psi_D \\ \Psi_Q \\ \Psi_f \end{vmatrix} \qquad (31)$$

Since relations between i_d, i_q and machine fluxes have be implicity obtained when investigating the machine's terminal constraint, then we shall the i_D, i_Q, and i_f – fluxes relationships.

Thus, after executing (31a), we get:

$$i_D = \frac{(x_f x_d - x_{af}^2)\,\Psi_D + (x_{fD} x_{af} - x_{aD} x_f)\,\Psi_d + (x_{aD} - x_d x_{fd})\,\Psi_f}{(x_d x_f x_D + x_{aD}^2 x_f - x_{af}^2 x_D - x_{af}^2 x_D - x_{fD}^2 x_d)} \qquad (32)$$

To avoid complication, i_D in the expression of i_f will be substituted from (24L) and i_q in the expression of i_Q will be substituted from (24m). Thus, after elimininating i_d from equations (24a) and (24d), we get:

$$i_f = \frac{1}{(x_{aD} x_{af} - x_d x_{fD})} \left\{ x_{aD}\,\Psi_d - x_d\,\Psi_D + \frac{(x_{aD}^2 - x_d x_D)}{w_b \cdot r_D} \cdot p\,\Psi_D \right\} \qquad (33)$$

Substituting for i_Q from equation (24b), we get:-

$$i_q = \frac{1}{x_q}(\Psi_q - x_{aQ}\, i_Q)$$

Substituting from this last equation into (24 L), we get:

$$\Psi_Q = \frac{x_{aQ}}{x_q}(\Psi_q - x_{aQ} i_Q) + x_Q i_Q$$

$$= \frac{x_{aQ}}{x_q}\{\Psi_q + i_Q (x_Q - \frac{x_{aQ}^2}{x_q})\}$$

$$\text{Thus } i_Q = \frac{x_q\{\Psi_Q - (x_{aQ}/x_q)\Psi_q\}}{(x_Q . x_q - x_{aQ}^2)}$$

$$= \frac{(x_q \Psi_Q - x_{aQ}\Psi_q)}{(x_Q x_q - x_{aQ}^2)} \qquad (34)$$

Multiplying out both sides of equation (24K) by x_{af}/r_r' we get:
Lastly substituting for i_Q from equation (34) into (24m) we get the equality:

$$\left| \frac{r_Q}{x_Q x_q - x_{aQ}^2} \right| \left| -x_{aQ}\Psi_q + x_q + \frac{p}{w_b . r_Q}(x_Q x_q - x_{aQ}^2)\Psi_Q \right| = 0 \qquad (38)$$

II. SOLUTION OF A MATRIX DIFFERENTIAL EQUATION:

To evaluate the solution of the equation:

$$p\,X = A\,X$$

The solution is generally given by:

$$X = e^{tA} . X_o = e^{-t(-A)} . X_o = e^{-tA'} . X_o \qquad (39)$$

where X_o is the vector X at t = 0, and $A' = -A$.

To calculate the transition matrix $e^{-t\,A'}$, the eigen values of the state matrix A' are calculated. Let these be $\lambda_1, \lambda_2, \ldots, \lambda_{15}$. This, with the aid of sylvester: formula, the expansion of the exponential function is given by:

$$e^{-A't} = \sum_{k=1}^{n} P(\lambda_K) Z_K(A') \qquad (40)$$

where

$$Z_K(A') = \frac{\prod_{K \neq r} (A' - \lambda_r I)}{\prod_{K \neq r} (\lambda_K - \lambda_r)}$$

$$P(\lambda_K) = e^{-\lambda_K t} \tag{41b}$$

where λ_K' λ_r are the eigen values of A'. K. r = 1, 2, ... 15 and I is the unit matrix of order 15.

Interpreting equation (40) for the 15^{th} order state matrix A', we get:-

$$e^{-tA'} = \frac{e^{-\lambda_1 \tau}}{(\lambda_1 - \lambda_2)..(\lambda_1 - \lambda_{15})} \{A' - \lambda_2 I\} .. \{A' - \lambda_{15} I\}$$

$$+ \frac{e^{-\lambda_2 t}}{(\lambda_2 - \lambda_1)(\lambda_2 - \lambda_3)..(\lambda_2 - \lambda_{15})} \{A' - \lambda_1 I\}.$$

$$.\{A' - \lambda_3 I\} \ldots \{A' - \lambda_{15} I\} + \ldots$$

$$+ \frac{e^{-\lambda_{15} t}}{(y_{15} - \lambda_1)..(\lambda_{15} - \lambda_{14})} \{A' - \lambda_1 I\}.. \{A' - \lambda_{14} I\} \tag{42}$$

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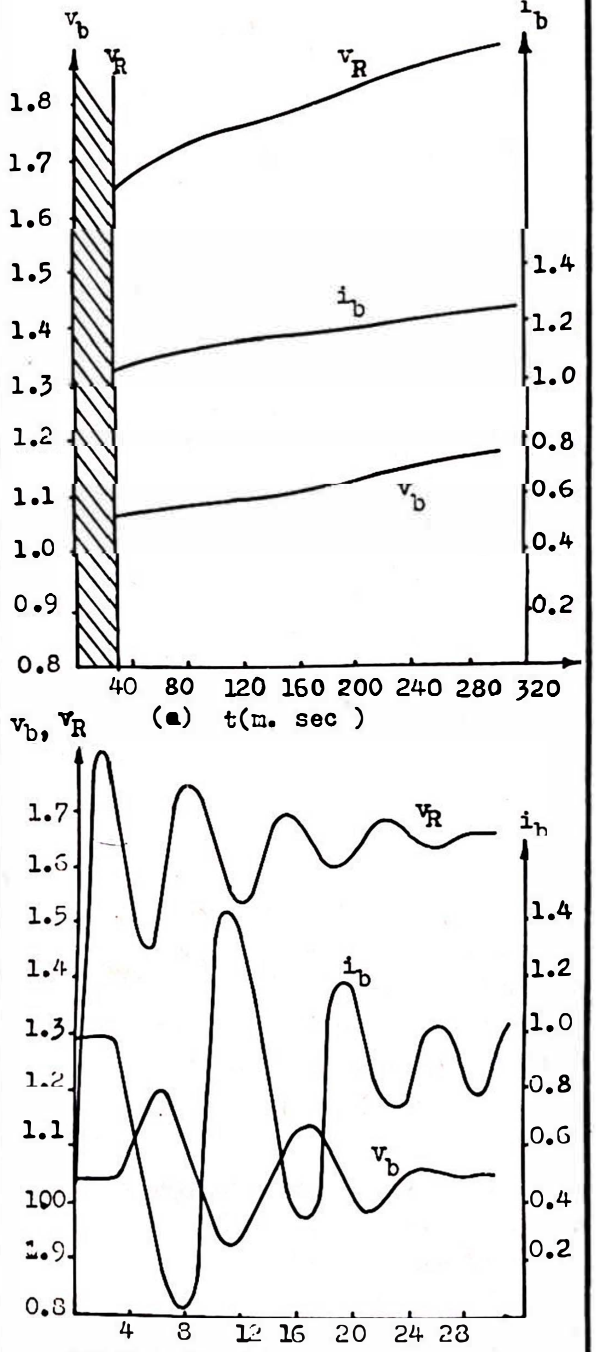

(b) Detail of Hatched zone in(a)

Figure (1) : Time response of Gen. terminal voltage v_b, Gen cunent I_b and receiving volt v_R .