INVESTIGATION OF THE TRAVELLING WAVE EFFECTS IN ANALYSIS OF SYNCHRONOUS MACHINE TRANSIENT BEHAVIOUR

11. THE SYNCHRONOUS MACHINE PERFORMANCE

by

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ABSTRACT:
This is the second of a series of papers interested mainly in the overall transient behaviour of an integrated power system constituting a synchronous machine connected to a large network through a long line. After deriving an expression for the long line as a synchronous machine's terminal constraint in the first paper, this paper investigates the machine's electromagnetic transients involving line's short lived travelling wave effects. A digital model is derived for the study of the transient behaviour of the system involving the long line effects but excluding, the action of machine's automatic excitation regulator.

NOMENCLATURE
[ ] A Matrix
d, q, O Direct, quadrature and zero components respectively.
D, Q Direct and quadrature damper respectively.
a, b, c Phase values.
i, v Instantaneous current, voltage and flux linkage respectively.
a, f, D, Q Stator, field, direct axis and quadrature axis damper circuits respectively.
w, w_b Rotor angular velocity in per unit and radians respectively.

1. INTRODUCTION
Analysis of an integrated power system's behaviour makes it desirable to recognize three distant periods; namely the surge period, the dynamic period and the steady state period. This classification
depends mainly upon the fact that always exists a wide span between the system's time constants. Such classification of course greatly facilitates the solution of the problem whose nature necessitates the applications of more than one transformation to the involved equations. However, such a classification: with the use of modern derivative type excitation system regulator's becomes inadequate. The use of regulators whose action is dependent upon changes of first, second and/or higher derivatives of the controlled variables: makes it necessary to allow for wave propagation in a long line simultaneously with the dynamic transient phenomena [6. Hence, with the long line represented as in the first paper, the whole system electromagnetic transient allowing for line's is investigated throughout the subject paper.

2. SYSTEM EQUATIONS

From the accompanying paper, a long line can be represented by the matrix equation:

\[ G V + P C V = N' I + P M I \]  

(1)

where for completely transposed system we have:

\[ M = \begin{bmatrix} M & m & m \\ m & M & m \\ m & m & M \end{bmatrix} \]

\[ M = \pm \frac{\sqrt{L(C + 3c)}}{2} \]

\[ m = -\frac{\sqrt{L(C - c + M)}}{2} \]

\[ N' = \begin{bmatrix} N' & n' & n' \\ n' & N' & n' \\ n' & n' & N' \end{bmatrix} \]

\[ N' = \frac{\sqrt{r'(G + 3g)}}{2} \]

\[ n' = -\frac{\sqrt{r'(G - g)}}{2} + N' \]

A capital letter designates diagonal element and a small letter designates a off-diagonal element of a matrix.

Applying the modified park's transformation [3] (power invariant) to both sides of equation (1), we obtain:

\[ k \ [G V + P C V] = K \ [N' I + P M I] \]

(2)

Substituting for K from equation (22), bearing in mind, for balanced operation,

\[ \sum_{i=a}^{c} F_i = 0 \]
where $F$ may be $i$, $v$ or $\Psi$, we get:

For direct axis:

$$(G - g) \cdot V_d + (C - c) (pV_d - wV_q)$$

$$= (N' - n') I_d + (M - m) (pI_d - wI_q)$$

(3)

For quadrature axis:

$$(G - g) \cdot V_g + (C - c) (wV_d + pV_q)$$

$$= (N' - n') I_q + (M - m) (wI_d + pI_q)$$

(4)

Re-writing equations (3) and (4) in matrix form, and after collecting voltage terms in one side, and current terms in the other sides, we get:

\[
\begin{bmatrix}
(G-g) + p(C-c) \\
-p(C-c) \\
(w(C-c) + p(C-c)) \\
\end{bmatrix} \begin{bmatrix}
V_d \\
V_g \\
\end{bmatrix} = \begin{bmatrix}
(N-n') \\
-N' - n' \\
\end{bmatrix} I_d + \begin{bmatrix}
-N(M-m) \\
N'N - n'M-m) \\
\end{bmatrix} I_q
\]

From the last equation, we get:

\[
\begin{bmatrix}
V_d \\
V_g \\
\end{bmatrix} = \begin{bmatrix}
(G-g) + p(C-c) \\
-N(M-m) \\
\end{bmatrix} I_d + \begin{bmatrix}
(N-n') \\
-N' - n' \\
\end{bmatrix} I_q
\]

where:

\[
A = (G - g), \quad B = (C - c)
\]
\[ e = (N' - n') \]
\[ f = (M - m) \]
\[ K = w (C - c) \]
\[ h = w (M - m) \]

Re-writing from the appendix equation (30):

\[
\begin{bmatrix}
V_d \\
V_q
\end{bmatrix}
= \begin{bmatrix}
r_a & 0 & i_d \\
0 & r_a & i_q
\end{bmatrix}
\begin{bmatrix}
-w \\
-1
\end{bmatrix}
\begin{bmatrix}
1 \\
0
\end{bmatrix}
\begin{bmatrix}
\Psi_d \\
\Psi_q
\end{bmatrix}
- \begin{bmatrix}
p \\
w_d
\end{bmatrix}
\]

Substituting for the voltage matrix in this last equation from (5), and collecting the flux terms in one side, we obtain:

\[
\begin{bmatrix}
-w \\
-1
\end{bmatrix}
\begin{bmatrix}
0 \\
1
\end{bmatrix}
\begin{bmatrix}
\Psi_d \\
\Psi_q
\end{bmatrix}
- \begin{bmatrix}
p \\
w_b
\end{bmatrix}
\]

\[
= \frac{1}{(A+pB)^2 + K^2}
\begin{vmatrix}
(A+pB)(r_a A + e) & (Ke - Ah) \\
+ p(r_a B + f) & + p(KJ - Bh) \\
+ K(r_a K + h) & \end{vmatrix}
\begin{bmatrix}
i_d \\
i_q
\end{bmatrix}
\]

(6)

Re-writing equation (29) from appendix, then:

\[
\begin{bmatrix}
i_d \\
i_q
\end{bmatrix}
= \begin{bmatrix}
1/x_d' & 0 & \Psi_d \\
0 & 1/x_q' & \Psi_q
\end{bmatrix}
- \begin{bmatrix}
-x_{aD} - \frac{x_{af} x_d}{x_f} \\
+ F_q
\end{bmatrix}
\begin{bmatrix}
i_d \\
i_q
\end{bmatrix}
\]

Substituting for the current matrix in equation (6) from this last equation, and multiplying both sides by \( (A + pB)^2 + K^2 \), we get:

\[
\begin{vmatrix}
0 & -1 & \Psi_d \\
w & -p w_b & \Psi_q
\end{vmatrix}
\]

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\[
\begin{align*}
&\begin{bmatrix}
\kappa(r_a + h) + (A + pB)(r_a A + c) \\
+ p(r_a B + f)
\end{bmatrix} \\
&\begin{bmatrix}
(Ah - \kappa) + p(Bh - Kf)
\end{bmatrix} \\
&\begin{bmatrix}
K(r_a h + (A + pB).
+ (r_a A + c) + p(r_a B + f)
\end{bmatrix}
\end{align*}
\]
\[
\begin{bmatrix}
1/x_d' \\
D
\end{bmatrix} = \begin{bmatrix}
\Psi_d \\
E_d'
\end{bmatrix} - \begin{bmatrix}
\Psi_q \\
E_d''
\end{bmatrix} = \begin{bmatrix}
\frac{x_{af} x_{ID}}{x_f} \end{bmatrix} I_d
\]

Collecting homogeneous terms and rearranging, we get:

\[
\begin{align*}
\left(\frac{m}{\lambda} + pw_b U\right) x_d' &= \Psi_d \\
\left(\frac{m}{\lambda} + pw_b U\right) x_q' &= \Psi_q \\
\left(\frac{n}{\lambda} - m\right) x_d'' &= E_d' \\
\left(\frac{n}{\lambda} + m\right) x_q'' &= E_d''
\end{align*}
\]

where

\[
M = K(r_a h + (A + pB). (r_a A + c) + p(r_a B + f)
\]
\[
= P^2 B(r_a B + f) + p B(r_a A + c) + A(r_a B + f) + A((A + c) + K(r_a h + n)
\]
\[
n = h (A + pB) - K(c + pB) = p(Bh - Kf) + Ah - \kappa h
\]
\[
\]

Substituting from appendix equations (25b), (24) and (27b) for \(E_d', I_d\) and \(D_d\) into equation (7a), we get:

\[
\begin{bmatrix}
\left(-\frac{m}{\lambda} + pw_b U\right) x_d' \\
\left(\frac{n}{\lambda} - m\right) x_d''
\end{bmatrix} = \begin{bmatrix}
\left(\frac{m}{\lambda} + pw_b U\right) x_q' \\
\left(\frac{n}{\lambda} + m\right) x_q''
\end{bmatrix} \Psi_d \]

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Re-arranging this last equation, we get:

\[
\begin{vmatrix}
\frac{m}{x_d} + pwbu & (wu - n) & \frac{pmb}{x_q} \times \frac{m}{x_d} + pwbu & \frac{mx_d}{x_q} \times \frac{m}{x_d} + pwbu & \frac{mx_d}{x_q} \\
& & & & \\
(nu - \frac{n}{x_d}) & (\frac{m}{x_u} + pwbu) & \frac{pmb}{x_u} \times (\frac{m}{x_u} + pwbu) & \frac{mx_d}{x_u} \times (\frac{m}{x_u} + pwbu) & \frac{mx_d}{x_u} \\
\end{vmatrix}
\]

\[
\begin{vmatrix}
\Psi_d \\
\Psi_q \\
\Psi_\phi \\
\Psi_Q \\
\end{vmatrix}
= \begin{vmatrix}
0 \\
0 \\
\end{vmatrix}
\]

where \( b = (\frac{x_{af} \times m}{x_f} - x_{ai}) \)

Now, let us accomplish the coefficient matrix by adding 3 rows, and hence it becomes a square matrix, whose inverse, generally (if nonsingular) exists. These rows, as the author proposes, can be derived by differentiating twice the appendix equations (36), (37) and (38). Thus equation (8) now can be changed into:
\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\frac{m}{x^p_d} + pw_{b,u} & \frac{(wu - \frac{n}{x^p_q})}{x^p_q} & \frac{mp}{w_{b,D} x^p_d x^q} \frac{\frac{\frac{a}{x_D}}{x^p_d x^q} x^p_d x^q}{w_{b,D} x^p_d x^q} & \frac{-nx_{aQ}}{x^p_d x^q} x^p_d x^q & \frac{mx_{af}}{x^p_d x^q} x^p_d x^q & \psi_d \\
\hline
\frac{(wu - \frac{n}{x^p_D})}{x^p_d} & \frac{m}{x^q_q} + pw_{b,u} & \frac{np}{w_{b,D} x^p_d x^q} & \frac{nx_{af}}{x^p_d x^q} x^p_d x^q & \psi_q \\
\hline
\frac{p^2 r_D}{a} & 0 & \frac{p^3}{w_b} + \frac{p^2 r_D}{a} \left( x^p_d x^q - x^p_d x^q \right) & 0 & \frac{p^2 r_D}{a} \times \psi_Q \\
\hline
(x^p_D x^p_d - x^p_d x^p_d) & 0 & \frac{p^2 r_Q x^p_D}{(x^p_D x^q - x^p_d x^q)} & 0 & \frac{p^3}{w_b} \\
\hline
\frac{p^2 r_Q x^p_D}{(x^p_D x^q - x^p_d x^q)} & 0 & \frac{p^3}{w_{b,D}} x^p_d x^q & \frac{p^2 r_Q x^p_D}{(x^p_D x^q - x^p_d x^q)} & \psi_f \\
\hline
\frac{p^2 x^p_D}{(x^p_D x^q - x^p_d x^q)} & 0 & \frac{p^3}{w_{b,D}} x^p_d x^q & \frac{p^3}{w_{b,D}} x^p_d x^q & \frac{p^1 x^p_Q x^p_D}{x^p_Q x^p_D} & \psi_D \\
\hline
\end{array}
\]

\[= [0 \ 0 \ 0 \ 0 \ 0 \ p^2 P_{fd}]^T \] (9)
where \( a = x_d x_f x_D + x_a^D x_f - x_{aF} x_D - x_{dF} x_d \)
equation (9) can be put in the matrix concise form:

\[
\begin{pmatrix} P^3 W_1 + P^2 W_2 + P W_1 + W_0 \end{pmatrix} Y = F \tag{10}
\]

where:

\[
W_1 = \begin{pmatrix}
  w_b B^2 & 0 & B(r_a B + f) & x_{af} x_{dD} - x_{aD} & 0 & 0 \\
  0 & w_b B^2 & 0 & 0 & 0 & 0 \\
  0 & 0 & 1 & w_b & 0 & 0 \\
  0 & 0 & 0 & 1 & w_b & 0 \\
  0 & 0 & x_{fa} (x_{aD} - x_d x_D) & w_b r_D (x_{aD} x_{aF} - x_d x_{dD}) & x_{af}^T d & x_f \\
\end{pmatrix} \tag{11}
\]
\[
W_2 = \begin{align*}
& \frac{B(r_a B + f)}{x_d} & wB^2 & \frac{x_{af}^2}{x_f} - \frac{x_{aD}^2}{x_f} - \frac{1}{w_b r_f D x_d} \\
& \frac{B r_a B + f}{x_q} & wB^2 & \frac{x_{af}^2}{x_f} - \frac{x_{aD}^2}{x_f} - \frac{1}{w_b r_f D x_d} \\
& + 2AB w_b & wB^2 & \frac{x_{af}^2}{x_f} - \frac{x_{aD}^2}{x_f} - \frac{1}{w_b r_f D x_d} \\
& \frac{r_D(x_D x_{ad} - x_{aD} x_{af})}{a} & r_D & \frac{x_D x_{ad} - x_{aD} x_{af}}{a} \\
& \frac{r_D(x_D x_{ad} - x_{aD} x_{af})}{a} & r_D & \frac{x_D x_{ad} - x_{aD} x_{af}}{a} \\
& \frac{r_Q x_{aQ}}{(x_{aQ} - x_{aq} x_{Q})} & 0 & \frac{r_Q x_{aQ}}{(x_{aQ} - x_{aq} x_{Q})} \\
& \frac{r_Q x_{aQ}}{(x_{aQ} - x_{aq} x_{Q})} & 0 & \frac{r_Q x_{aQ}}{(x_{aQ} - x_{aq} x_{Q})} \\
& \frac{x_{af} x_{aQ}}{1 x_{aD} x_{af} - x_{d} x_{ld}} & 0 & \frac{x_{d} x_{af}}{(x_{d} x_{af} - x_{aD} x_{af})} \\
& \frac{x_{af} x_{aQ}}{1 x_{aD} x_{af} - x_{d} x_{ld}} & 0 & \frac{x_{d} x_{af}}{(x_{d} x_{af} - x_{aD} x_{af})} \\
\end{align*}
\]
\[
\begin{align*}
\mathcal{W} &= (1 + \beta \gamma) V \\
&= \frac{p_x^x}{(\xi - \eta) \beta x} + \frac{\partial_x^x \beta x}{\eta - \eta \beta x} + \frac{p_x^x \gamma^x q_x}{(\xi \eta) \beta x} + \frac{b_x^x}{(1 + \beta \gamma) V} + \frac{b_x^x}{(\xi \eta) \beta x} + \frac{p_x^x}{(\xi \eta) \beta x} + (\tau \gamma + \tau \gamma') q_x + \omega b V^2
\end{align*}
\]
\[ W_0 = \begin{array}{c|c|c|c|c}
\frac{1}{x'_d} \text{tr}(r_a + c) + \\
\frac{w(A^2 + K^2)}{K(r_a K + h)} & \frac{w(A^2 + K^2)}{x''_q} & 0 & \frac{x_a Q}{x'_q x''_q} (Kc - Ah) \\
\frac{(Ke - Ah)}{x'_d} & 0 & 0 & \frac{x_a Q}{x'_q x''_q} (Kc - Ah) + K(r_a K + h) \\
\end{array} \]

\[ Y = \begin{bmatrix} \Psi_d & \Psi_q & \Psi_D & \Psi_Q & \Psi_f \end{bmatrix}^T \]  

\[ F = \begin{bmatrix} 0 & 0 & 0 & 0 & p^2E_{kd} \end{bmatrix}^T \]
In solving the investigated system equation (9), throughout this paper, we shall discuss its solution with the discard of the effect of the excitation regulator. i.e. $P_Ef_d$ will be replaced by zero. However, inclusion of excitation regulator's effect will be discussed in a third paper.

Thus equation (10) becomes:

$$ (p^2W_3 + p^2W_1 + pW_1 + W_0) Y = 0 $$

(15)

To solve this last equation, let us proceed as follows:-

Let

$$ Y_1 = Y $$

$$ Y_2 = Y_1 = p Y $$

$$ Y_3 = Y_2 = p^2 Y $$

and from equation (15)

$$ Y_4 = Y_3 = p^3 Y $$

$$ = -W_1^{-1}(W_2 Y_3 + W_1 Y_2 + W_0 Y_1) $$

(16d)

The set of equations (16a) up to (16d) can be rearranged into the form:

$$ \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -W_3^{-1}W_2 & -W_3^{-1}W_1 & -W_3^{-1}W_0 & -W_3^{-1}W_4 \end{pmatrix} \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} $$

(17)

where

$Y_1$, $Y_2$ and $Y_3$ designate $5 \times 1$ matrices.

0, I designate zero and identity matrices of 5th order respectively.

Equation (17) can be re-written in the concise form:

$$ PX = AX $$

(18)

where

$$ X = \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} $$ i.e. a 15th order column matrix.

$$ A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ -W_1^{-1}W_0 & -W_1^{-1}W_1 & -W_1^{-1}W_2 \end{pmatrix} $$

Owing to the fact that the diagonal elements involve zeros, hence a convergent solution by Runge-Kutta method is practically impossible.

Solution of equation is readily known (4) and is:

$$ X = e^{tA} X_0 $$

(19)

where $X_0$ is the matrix $X$ at $t = 0$.

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Substituting from the appendix equation (42) for the transition matrix $e^{tA}$, we finally get the solution:

\[
X = -\left[ \frac{e^{-\lambda_1 t}}{(\lambda_1 - \lambda_2)\ldots(\lambda_1 - \lambda_{15})} \{A + \lambda_1 I\} \ldots \Omega + \lambda_{15} I \right. \\
+ \frac{e^{-\lambda_2 t}}{(\lambda_2 - \lambda_1)(\lambda_2 - \lambda_3)\ldots(\lambda_2 - \lambda_{14})} \{A + \lambda_2 I\} \{A + \lambda_1 I\} \ldots \\
\left. + \frac{e^{-\lambda_3 t}}{(\lambda_3 - \lambda_1)(\lambda_3 - \lambda_2)\ldots(\lambda_3 - \lambda_{14})} \{A + \lambda_3 I\} \ldots \{A + \lambda_3 I\} \right] X_0
\]  

(20)

where $\lambda_1, \lambda_2, \ldots, \lambda_{15}$ are the eigen values of $-A$.

Once finding out the time variation of the flux linkage (and its first and second derivative) matrix $X$, then; back substitution from equation (20) into the appendix equation (29a); $i_d$ and $i_q$ are calculated. Again, substituting into into equation (30) $V_d$ and $V_q$ are calculated. and consequently the machine terminal voltage is calculated by aid of equation (34i).

Voltage and current any point along the line are calculated by aid of the formula (from the accompanying paper):

\[
\begin{bmatrix}
V \\
I
\end{bmatrix}
= e^{-tA}
\begin{bmatrix}
V \\
I_0
\end{bmatrix}
\]

where $0$ designates sending end.

**NUMERICAL EXAMPLE**

Using the above mentioned algorithm, a computer run is made with the data given in Table I, referred to 500 kV and 1236 MVA base.

<table>
<thead>
<tr>
<th>Gen. stator values</th>
<th>Total reacts. of rotor wdgs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>p.v.</td>
<td></td>
</tr>
<tr>
<td>$X_d$</td>
<td>$X_q$</td>
</tr>
<tr>
<td>1</td>
<td>0.67</td>
</tr>
<tr>
<td>$X'_d$</td>
<td>0.33</td>
</tr>
<tr>
<td>$X''_d$</td>
<td>0.23</td>
</tr>
<tr>
<td>$X_q$</td>
<td>0.003</td>
</tr>
<tr>
<td>$r_a$</td>
<td>1.11</td>
</tr>
<tr>
<td>$X_f$</td>
<td>0.995</td>
</tr>
<tr>
<td>$X_D$</td>
<td>0.61</td>
</tr>
<tr>
<td>$X_Q$</td>
<td></td>
</tr>
</tbody>
</table>

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Mutual reacts. of rotor wdgs  
Rotor circuit resistances  

<table>
<thead>
<tr>
<th>(X_{af})</th>
<th>(X_{rD})</th>
<th>(X_{aD})</th>
<th>(X_{aQ})</th>
<th>(r_f)</th>
<th>(r_D)</th>
<th>(r_Q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.87</td>
<td>0.87</td>
<td>0.87</td>
<td>0.52</td>
<td>0.0004</td>
<td>0.018</td>
<td>0.0063</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time const</th>
<th>Line Parameters (Km) (\times 10^{-6})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_d)</td>
<td>(T_g)</td>
</tr>
<tr>
<td>8.54</td>
<td>0.31</td>
</tr>
</tbody>
</table>

Figure (1) shows the computed values for the per unit generator terminal voltage \(v_b\), receiving end voltage \(v_R\), line sending end current variations for the case of sudden switching off of line remote end breadder when the system initially with the following conditions:

- Receiving end load = 1 + j0.3 P.U
- Receiving end voltage = 0.955 P.U
- Sending end voltage = 1.035 P.U
- Generator terminal voltage = 1.045 P.U.
- Shunt reactor reactive power compensation = 0.5 P.U.

CONCLUSION:
Throughout this paper formulae are derived to link the fast transient line switching domain with the synchronous machine electromagnetic transient time domain. Although the excitation regulator effect is not investigated in the subject paper, yet the derived equations are the natural entrance for this target. however, the influence of excitation regulator upon the overall line switching and synchronous generator’s electromagentic (stator) transient behaviour is reserved for a future paper.

APPENDICES

I. SYNCHRONOUS MACHINE EQUATIONS

According to Park equations [1, 2 and after proper selection of transformation matrices such that power should be invariant anywhere in the system [ we have:

\[
\begin{pmatrix}
F_d \\
F_q \\
F_o
\end{pmatrix} = K \begin{pmatrix}
F_a \\
F_b \\
F_c
\end{pmatrix}
\]
\[
\begin{bmatrix}
\cos \theta & \cos (0 - \frac{\pi}{4}) & \cos (0 + \frac{\pi}{4}) \\
\sin \theta & \sin (0 - \frac{\pi}{4}) & \sin (0 + \frac{\pi}{4}) \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{bmatrix}
\begin{bmatrix}
F_a \\
F_b \\
F_c
\end{bmatrix}
\]

where \( F \) designates \( v, i, \) and \( \Psi^d \).

d.q.o. designate direct, quadrature and zero components respectively.

\( a, b, c \) designate phase values.

Conversely

\[
\begin{bmatrix}
F_a \\
F_b \\
F_c
\end{bmatrix}
= \mathbf{K}^{-1}
\begin{bmatrix}
F_d \\
F_q \\
F_o
\end{bmatrix}
\]

Applying transformation matrices (22) and (23) it had been proved elsewhere [7] that the following relationships are derived:

\[
\begin{align*}
I_d &= x_d q_d + x_a f_d + x_e D_d \\
I_q &= x_q q_q + x_a q_Q Q \\
I_o &= x_o q_o \\
I_t &= x_a f_d + x_f f_f + x_D D_d \\
I_D &= x_a D_d + x_f f_f + x_D D_d \\
I_Q &= x_a Q_q + x_Q q_Q \\
V_d &= -r_a q_d - (1/w_b) \Psi_d - w \Psi_q \\
V_q &= -r_a q_q - (1/w_b) \Psi_q + w \Psi_d \\
V_t &= \sqrt{V_d^2 + V_q^2} \\
V_o &= -r_o q_o - (1/w_b) \Psi_o \\
V_f &= r_f q_f + (1/w_b) \Psi_f \\
C &= r_D q_D + (1/w_b) \Psi_D \\
O &= r_Q q_Q + (1/w_b) \Psi_Q
\end{align*}
\]
Multiplying out both sides of equation (24d) by \(\frac{x_{af}}{x_f}\) then subtracting from equation (24a), we get:

\[
\Psi_d - \left(\frac{x_{af}}{x_f}\right)\Psi_f = \left(\Psi_f - \frac{x_{af}}{x_f}\right)\Psi_d + \left(\frac{x_{af}}{x_f}\right)\frac{x_{af}}{x_f} \cdot \frac{x_{af}}{x_f} \cdot \left(\Psi_f - \frac{x_{af}}{x_f}\right) - \frac{x_{af}}{x_f} \cdot \frac{x_{af}}{x_f} \cdot \left(\Psi_f - \frac{x_{af}}{x_f}\right)
\]

But

\[
\Psi_d - \frac{x_{af}}{x_f} = \Psi_d
\]

= Direct axis transient reactance and

\[
\frac{x_{af}}{x_f} \cdot \Psi_f = E'_q
\]

Thus, equation (25) becomes, after rearrangement:-

\[
x_{d}'' = (\Psi_d - E'_q) + \left(\Psi_f - \frac{x_{af}}{x_f}\right)\frac{x_{af}}{x_f} \cdot \left(\Psi_f - \frac{x_{af}}{x_f}\right)
\]

Again, multiplying out both sides of equation (24f) by \(\frac{x_{af}}{x_Q}\) then subtracting from equation (24b), we get:

\[
\Psi_q - \left(\frac{x_{af}}{x_Q}\right) \langle \Psi_q \rangle = \left(\Psi_q - \frac{x_{af}}{x_Q}\right)\frac{x_{af}}{x_Q} \cdot \left(\Psi_q - \frac{x_{af}}{x_Q}\right)
\]

and

\[
x_{q}'' = x_q - \frac{x_{af}}{x_Q}
\]

= Quadrature axis subtransient reactance.

and let us define:

\[
E_d = \left(\frac{x_{af}}{x_Q}\right) \cdot \Psi_f
\]

Thus, equation (27) becomes:

\[
\Psi - E_d = x_{q}'' \Psi_q
\]
From equations (26) and (28), we can write down:

\[
\begin{bmatrix}
\frac{i}{x} & 0 \\
0 & 1 / x''
\end{bmatrix}
\begin{bmatrix}
\psi_d \\
\psi_q
\end{bmatrix}
= \begin{bmatrix}
\frac{x_a l f x_D}{x_a D} \\
- E_d''
\end{bmatrix}
\begin{bmatrix}
i_d \\
- i_q
\end{bmatrix}
\]

Substituting into equation (29) for \(i_d\) from equation (24l), and for \(E_d''\) from equation (27b) and for \(E_q\) from equation (25b) we get:

\[
\begin{bmatrix}
i_d \\
i_q
\end{bmatrix}
= \begin{bmatrix}
\frac{1}{x_d'} & 0 \\
0 & \frac{1}{x_q''}
\end{bmatrix}
\begin{bmatrix}
\psi_d \\
\psi_q
\end{bmatrix}
\]

\[
\begin{bmatrix}
(x_{af} \Psi_f / x_f) + (x_{aD} - \frac{x_{af} x_D}{x_f}) p \Psi_D / w_D \\
(x_{aQ} / x_Q) \psi_Q
\end{bmatrix}
\]

Also, rewriting equations (24g), (24h) in matrix form, then:

\[
\begin{bmatrix}
\psi_d \\
\psi_q
\end{bmatrix}
= A_L \begin{bmatrix}
i_d \\
i_q
\end{bmatrix} + A_M \begin{bmatrix}
\psi_d \\
\psi_q
\end{bmatrix} + A_R \begin{bmatrix}
\psi_d \\
\psi_q
\end{bmatrix}
\]

where

\[
A_L = - r_a \cdot I_{2 \times 2}
\]

\[
A_M = - w
\]

\[
A_R = - 1 / w_b \cdot I_{2 \times 2}
\]

To find out explicit expressions for machine's current, then, from the set of appendix equations (24a) up (24f) excluding (24c), we have:
Since relations between $i_d$, $i_q$, and machine fluxes have been implicitly obtained when investigating the machine's terminal constraints, then we shall the $i_d$, $i_q$, and $i_f$ – fluxes relationships.

Thus, after executing (31a), we get:

$$i_D = \frac{(x_d^2 - x_{af}^2)\Psi_D + (x_{D}^2x_{af} - x_{d}^2x_{D})\Psi_D + (x_{d}^2 - x_{af}^2)\Psi_f}{(x_d^2x_{D}^2 + x_{af}^2x_{D}^2 - x_{d}^2x_{af}^2 - x_{D}^2x_{af}^2)}$$

(32)

To avoid complication, $i_D$ in the expression of $i_f$ will be substituted from (24L) and $i_q$ in the expression of $i_Q$ will be substituted from (24m). Thus, after eliminating $i_d$ from equations (24a) and (24d), we get:

$$i_f = \frac{1}{(x_{D}^2x_{af} - x_{d}^2x_{D}^2)} \left\{ x_{aD} \Psi_d - x_d \Psi_D \right\} + \frac{(x_{aD}^2 - x_{d}^2x_{D})}{w_h \cdot r_D} \cdot p \Psi_D$$

(33)

Substituting for $i_Q$ from equation (24b), we get:

$$i_q = \frac{1}{x_q} (\Psi - x_{aQ}^2) i_q$$

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Substituting from this last equation into (24 L), we get:

\[
\Psi_Q = \frac{x_{aq}}{x_q} (\Psi_q - x_{aq}^2 Q) + x_{aq}^2 Q \\
= \frac{x_{aq}}{x_q} \Psi_q + i_Q (x_{aq}^2 Q - \frac{x_{aq}}{x_q}) \\
\text{Thus } i_Q = \frac{x_{aq}}{x_q} \left( \Psi_Q - (x_{aq}^2 Q - \frac{x_{aq}}{x_q}) \Psi_q \right) \\
= \frac{(x_{aq}^2 Q - \frac{x_{aq}}{x_q}) \Psi_q}{(x_{aq}^2 Q - \frac{x_{aq}}{x_q})} \\
\tag{34}
\]

Multiplying out both sides of equation (24K) by x_{aq}/r_f we get:
Lastly substituting for i_Q from equation (34) into (24m) we get the equality:

\[
\begin{bmatrix}
\frac{r_f}{x_{aq}^2 Q - \frac{x_{aq}}{x_q}} \\
\frac{x_{aq}^2 Q - \frac{x_{aq}}{x_q}}{x_{aq}^2 Q - \frac{x_{aq}}{x_q}}
\end{bmatrix}
\begin{bmatrix}
-x_{aq}^2 Q + x_q + \frac{\dot{p}}{w_b r_f Q} (x_{aq}^2 Q - \frac{x_{aq}}{x_q}) \Psi_Q
\end{bmatrix} = 0 \\
\tag{38}
\]

II. SOLUTION OF A MATRIX DIFFERENTIAL EQUATION:
To evaluate the solution of the equation:

\[
p X = A X
\]

The solution is generally given by:

\[
X = e^{tA} X_0 = e^{-(t - A)} X_0 , X_0 = e^{-tA} X_0 \\
\tag{39}
\]

where X_0 is the vector X at t = 0, and A' = -A.

To calculate the transition matrix e^{-t A'}, the eigen values of the state matrix A' are calculated. Let these be \lambda_1, \lambda_2, \ldots, \lambda_{15}. This, with the aid of sylvester formula, the expansion of the exponential function is given by:

\[
e^{-t A'} = \sum_{k=1}^{n} P(\lambda_k) Z_k(A') \\
\tag{40}
\]

where

\[
Z_k(A') = \prod_{k \neq r} (\lambda' - \lambda_r I) \\
\prod_{k \neq r} (\lambda_k - \lambda_r)
\]
\[ p(\lambda_K) = e^{-\lambda_K t} \quad (41b) \]

where \( \lambda_K \) are the eigen values of \( A' \), \( K, \ r = 1, 2, \ldots, 15 \) and \( I \) is the unit matrix of order 15.

Interpreting equation (40) for the 15th order state matrix \( A' \), we get:

\[
e^{-tA'} = \frac{e^{-\lambda_1 t}}{(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_{15})} \left\{ A' - \lambda_1 I \right\} + \frac{e^{-\lambda_2 t}}{(\lambda_2 - \lambda_1)(\lambda_2 - \lambda_{14})} \left\{ A' - \lambda_2 I \right\} + \ldots
\]

\[
+ \frac{e^{-\lambda_{14} t}}{(\lambda_{14} - \lambda_{15})(\lambda_{14} - \lambda_{13})} \left\{ A' - \lambda_{14} I \right\}.
\]

\[
\left\{ A' - \lambda_1 I \right\} \ldots \left\{ A' - \lambda_{15} I \right\} + \ldots
\]

\[
+ \frac{e^{-\lambda_{15} t}}{(\lambda_{15} - \lambda_1)(\lambda_{15} - \lambda_{14})} \left\{ A' - \lambda_{15} I \right\}.
\]

REFERENCES

Figure (1): Time response of gen. terminal voltage $v_b$, gen. current $i_b$ and receiving volt $v_R$. 

(a) Detail ofatched zone in(a)