دراسة المبازل المختلفة الاعماق بواسطة طرق تحليلية

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تم دراسة مشكلة المبازل المختلفة الاعماق في هذا البحث . وقد تم تقديم المشكلة بنمط رياضي خطي ومن ثم اوجد لــه حلاً « تحليلياً » باستخدام سلسلة فورير. وقد تم تقديم الحل بشكل مخططات بيانية فيها المتغيرات بدون ابعاد.

STUDY ON BI-LEVEL DRAINAGE PROBLEM BY ANALYTICAL METHODS

BY

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SUMMARY

The bi-level drainage problem is studied in this paper. The problem is represented by a linearized theoretical model and the same is solved by an analytical procedure making use of Fourier series expansions. The different boundary and initial conditions are included in the problem formulation. The problem is studied in detail and the problem formulation. The problem is studied in detail and the computed results are presented in nondimensional graphical form and discussed.

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The objective of subsurface drainage is the orderly removal of excess water from the land to provide sufficient air diffusion within the root zone of crops, or to prevent the accumulation of salt due to excessive evaporation. The major aim is to lower the water content of the upper soil layers so that air can penetrate more easily to the plant roots. Generally, all nonaquatic plants are damaged if the soil is allowed to remain water logged. However, it is necessary to distinguish between long-term effects of adverse aeration and the effects of temporary flooding. In the first case, there is a change in the environment which effectively restricts the metabolic activity and development of the root system of the plants. In the second case, the effects of a short term oxygen defficiency or excess of carbon dioxide are the main injurious factors.

Subsurface drainage lowers the water table and thereby reduces the water content in the soil above it. In addition, the consequence of drainage may include a change in the quality of soil-water. The costs of the subsurface drainage system can play an important role in the economic feasibility of the irrigation projects. The term bi-level drainage refers to a subsrface drainage system where drains are at two different depths on an alternating basis. Since the drains are at two different levels in the bi-level drainage system the costs involved in the projects are considerably reduced compared to the drainage system where all the drains are at normal deep drain lines. In order to evaluate the capacity of a given bi-level drainage system to satisfy the necessary design criterion, it is advantageous to develop a procedure to study the water table elevations for agiven set of design conditions.

A review on the literature on this topic indicated a lack of analytical procedures to study the bi-level drainage system. DeBer and Chu (1.2) developed a theoretical model to study the fall of a water table under bi-level drainage situation. Their studies are limited to steady state situations and to falling water table problem. For the latter case they have assumed a relation which is valid for short time intervals when the water table drops at the same rate at all points.

In this paper, an analytical procedure is developed to study the bi-level drainage problem in both steady state and unsteady state situations. The problem is represented by a linearized theoretical model and the solution is obtained by using Fourier series expansions. The problem is studied for different conditions and the computed results are presented and discussed.

THEORETICAL BACKGROUND

The geometry of the bi-level drainage problem is illustrated in Fig. 1. The soil is assumed to be homogeneous and isotropic and the Dupuit–Forcheimer assumptions (3) are assumed to be valid for the problem under consideration. With the above mentioned assumptions the basic governing equation for one dimensional horizontal flow results in Boussinesq equation (4.5) given by

$$\frac{\partial}{\partial x}\left(h^* \frac{\partial h^*}{\partial x} \right) = \frac{f}{k} \frac{\partial h^*}{\partial t} \qquad \ldots (1)$$

where $h^*(x, t)$: height of water table above the impermeable layer.

- x : horizontal space coordinate, positive towads the right.
- t : time coordinate
- k : hydraulic conductivity of the soil
- f : drainable porosity of the soil.

Because of the nonlinearity of the above equation a limited number of exact solutions, for particular problems only, are available (3,6,7). Assuming hydrostatic pressure distribution, Murray and Monkmeyer (6,7) solved this equation by integrating the Laplace equation over its depth of flow. Analytical solution of Eq. (1) can be obtained by linearizing it as presented subsequently.

LINEARIZED THEORETICAL MODEL

The linearized partial differential equation representing the flow of water of bi-level drainage system can be written as

$$\frac{K\,D}{f}\ \frac{\partial^2 h}{\partial x^2} = \frac{\partial h}{\partial t} \qquad \dots (2)$$

where , h (x,t) : height of water table above the deeper drain center line (x axis)

D average depth of drainable section

$\frac{K\,D}{f}$: aquifer diffusivity

The linearization of this form has been extensively used in modelling drainage of agricultural lands (3,7) and studying water table fluctuations under varying conditions. The average depth of drainable section for the problem under consideration can be expressed (referring to Fig. 1) as

$$D = d_2 + \frac{H}{2} \qquad \dots (3)$$

The initial condition is

$$h(x, o) = H \qquad (4)$$

and the boundary conditions are

$$h(0, t) = 0 \qquad (5)$$

$$h(L, t) = d_1 - d_2 = d_0 \qquad \dots (6)$$

For the steady state condition, h (x,t) satisfying Laplace equation can be written as

$$h_o(x) = \frac{d_o x}{L} \qquad \dots (7)$$

and the solution of the unsteady state can be expressed as

$$u(x, t) = h(x, t)\ h_o(x) \qquad \dots (8)$$

where u (x,t) represents an assumed function to be evaluated and the corresponding conditions to be satisfied by this function are .

$$u(x, o) = H - \frac{d_o x}{L} \quad \ldots (9)$$

$$u(o, t) = 0 \quad \ldots (10)$$

$$u(L, t) = 0 \quad \ldots (11)$$

Substituting for h (x,t) from Eq. (8) into Eq. (2) the differential equation for u (x,t) can be written as

$$a \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \quad \ldots (12)$$

subject to the conditions given by Eqs. (9) (11) and 'a' stands for aquifer diffusivity.

SOLUTION PROCEDURE

Suppose u (x,t) has a Fourier series in x for each specified $t > 0$ it can be expressed as

$$u(x, t) = \sum_{n=1}^{\infty} B_n(t) \sin \frac{n\pi x}{L} \quad \ldots (13)$$

A sine series is chosen as it would satisfy the boundary conditions given by Eqs. (10) and (11). Substituting Eq. (13) into Eq. (12) and differentiating term by term the result can be written as

$$-a \frac{\pi^2}{L^2} \sum_{n=1}^{\infty} n^2 B_n(t) \sin \frac{n\pi x}{L}$$

$$= \sum_{n=1}^{\infty} B_n^1(t) \sin \frac{n\pi x}{L} \quad \ldots (14)$$

where B_n^1 denotes derivatives with respect to t. Equating the coefficients and integrating the resulting equation it can be shown that

$$B_n(t) = C_n e^{\frac{a\pi^2 n^2}{-L^2} t} \quad \ldots (15)$$

where c_n is a coefficient to be evaluated. After substituting B_n from Eq. (15) into Eq. (13) the expression for u (x,t) can be written as

$$u(x, t) = \sum_{n=1}^{\infty} C_n e^{-\frac{a\pi^2 n^2}{L^2} t} \sin \frac{n\pi x}{L} \quad \ldots (16)$$

Applying the condition given by Eq. (9) and evaluating the Fourier coefficients in the sine series expansion of $H - \frac{d_0 x}{L}$ in the interval $0 < x < L$ it can be shown that

$$C_n = \frac{2}{n\pi} [H \{ 1 - (-1)^n \} + (-1)^n d_o] \qquad \dots (17)$$

Substituting the value of C_n from Eq. (17) into Eqs. (16) and (18), the required solution can be written as

$$h(x,t) = \frac{d_o x}{L} + \frac{2}{H} \sum_{n=1}^{\infty} \frac{1}{n} [H \{ 1 - (-1)^n \}$$

$$+ (-1)^n d_o] e^{-\frac{a\pi^2 n^2 t}{L^2}} \text{Sin} \frac{n\pi x}{L} \qquad \dots (18)$$

ANALYSIS OF RESULTS

In order to study the problem in detail, it is convenient to introduce the nondimensional parameters defined by

$$\bar{h} = \frac{h(x,t)}{H} \qquad \dots (19)$$

$$\bar{x} = \frac{x}{L} \qquad \dots (20)$$

$$\bar{d}_o = \frac{d_o}{H} \qquad \dots (21)$$

$$\bar{t} = \frac{K D \pi^2}{f L^2} t \qquad \dots (22)$$

Using these parameters Eq. (18) can be written in a nondimensional form as

$$\bar{h} = \bar{d}_0 \bar{x} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} [1 - (-1)^n + (-1)^n \bar{d}_0] e^{-n^2 \bar{t}} \text{Sin}\, n \pi \bar{X} \qquad (23)$$

subject to the conditions $0 < \bar{X} < 1$, $0 < \bar{d}_0 < 1$, $0 < \bar{t} < \infty$ and $0 < \bar{h} < 1$. At any time the position ofthe extremum $\bar{h}$ is given by $\frac{\partial \bar{h}}{\partial \bar{x}} = 0$. From Eq. (23) this condition can be written as

$$\bar{d}_0 + 2 \sum_{n=1}^{\infty} [1 - (-1)^n + (-1)^n \bar{d}_0] e^{-n^2 \bar{t}} \cos n \pi \bar{x} = 0 \qquad (24)$$

The computed results using Eq. (23) are shown in Figs. 2–8. In Figs. 2–4 the variation of $\bar{h}$ with $\bar{x}$ for fixed values of $\bar{t}$ = 0.2, 0.4,..... 1.5 and for prescribed values of $\bar{d}_0$ = 0.25, 0.5 and 0.75 are presented. In all these three figures it can be seen that initially $\bar{h}$ is uniform ($\bar{t} = 0$) for $0 < x < 1$ and as $t \rightarrow \infty$, h will have a linear form for $0 < x < 1$. When $\bar{t} = 0$ the series on the right hand side of Eq. (24) would converge to $-\bar{d}_0$ making the equation identically true. This happens because $\bar{h} = 1$ when $\bar{t} = 0$ corresponding to the prescribed initial condition namely $h = H$. When $\bar{t} \rightarrow \infty$ Eq. (24) takes the form $\cos n\pi\bar{x} = -\bar{d}_0$ making $\bar{x}$ indeterminate. This is because of the fact that the value of $\bar{h}$ indicates steady state condition when $\bar{t} \rightarrow \infty$ and $\bar{h} = \bar{d}_0\ \bar{x}$, a linear form. As the value of $\bar{d}_0$ increases from 0.25 to 0.75 the value of $\bar{h}$ increases indicating the phenomenon that the height of the water table increases as the vertical distance between the deeper drain line and the shallower drain line increases.

The variations of the height of water table with time for given problem are shown in Figs. 5–8. The position of water table for fixed values of $\bar{x}$ = 0.2, 0.5 and 0.8 for prescribed values of $\bar{d}_0$ = 0.25, 0.5 and 0.75 are shown in Fig. 5–7. It can be noted that the values of $\bar{x} = 0.2$ and $\bar{x} = 0.8$ are corresponding to the values in the range of deep, shallow drain conditions respectively and the value $\bar{x} = 0.5$ is corresponding to the middle of the drain span. At $\bar{x} = 0.2$ the height of water table is nearly identical for all values of $\bar{d}_0$ until $\bar{t}$ equals approximately 0.8 and the discrepancy is negligibly small until $\bar{t} = 1.0$. Even though $\bar{t} \rightarrow \infty$, the differences between water table heights for various $\bar{d}_0$ values are small. As $\bar{x}$ increses the range of $\bar{t}$ in which the above mentioned discrepancy is small, is decreasing until the difference between the water table heights for various $\bar{d}_0$ values appreciably appear at the beginning of the drainage cycle itself as shown in Fig. 7 corresponding to values at $\bar{x} = 0.8$. It is also interesting to note that each curve corresponding to specified value of $\bar{x}$ is having particular pattern and it is different from the curves corresponding to other $\bar{x}$ values as shown in Fig. 8.

In general, as time increases the height of the water table drops down as it can be observed from Figs. 2-4 also. This indicates that the moisture content of the upper soil decreases as the time increases which is a required condition for plant growth.

CONCLUSION

An analytical solution procedure is presented to study the bi-level drainage problem. The problem is studied in detail and the computed results are presented in a nondimensional form. These plotted results can be made use of for the design of bi-level drainage systems before installation and for the design of a modification to the existing drainage system if the same is under-designed.

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APPENDIX II - NOTATION

The following symbols are used in this paper.

B_n	: Fourier Coefficients as defined by Eq. (15)
C_n	: Coefficients given by Eq. (17)
D	: average depth of drainable section
H	: vertical distance between deep drain and initial water table
K	: hydraulic conductivity of the soil.
L	: drain spacing
a	: aquifer diffusivity
d_1	: height of shallon drain from impermeable bed.
d_2	: height of deep drain from impermeable bed.
d_0	: vertical distance between two drains
f	: drainable porosity.
h (x , t)	: height of water table above the deeper drain center line
$h_0(x)$	: height corresponding to steady state condition
t	: time
x	: horizontal space coordinate positive towards right .
u (x,t)	: function denoting unsteady state condition defined by Eq. (8)
h^* (x. t)	: height of water table above the impermeable bed .
$\bar{d}_0$	: nondimensional parameter defined by Eq. (21)
$\bar{h}$	: nondimensional parameter defined by Eq. (19)
$\bar{t}$	: nondimens onal parameter defined by Eq . (22)
$\bar{X}$	: nond mensional parameter defined by Eq . (20)

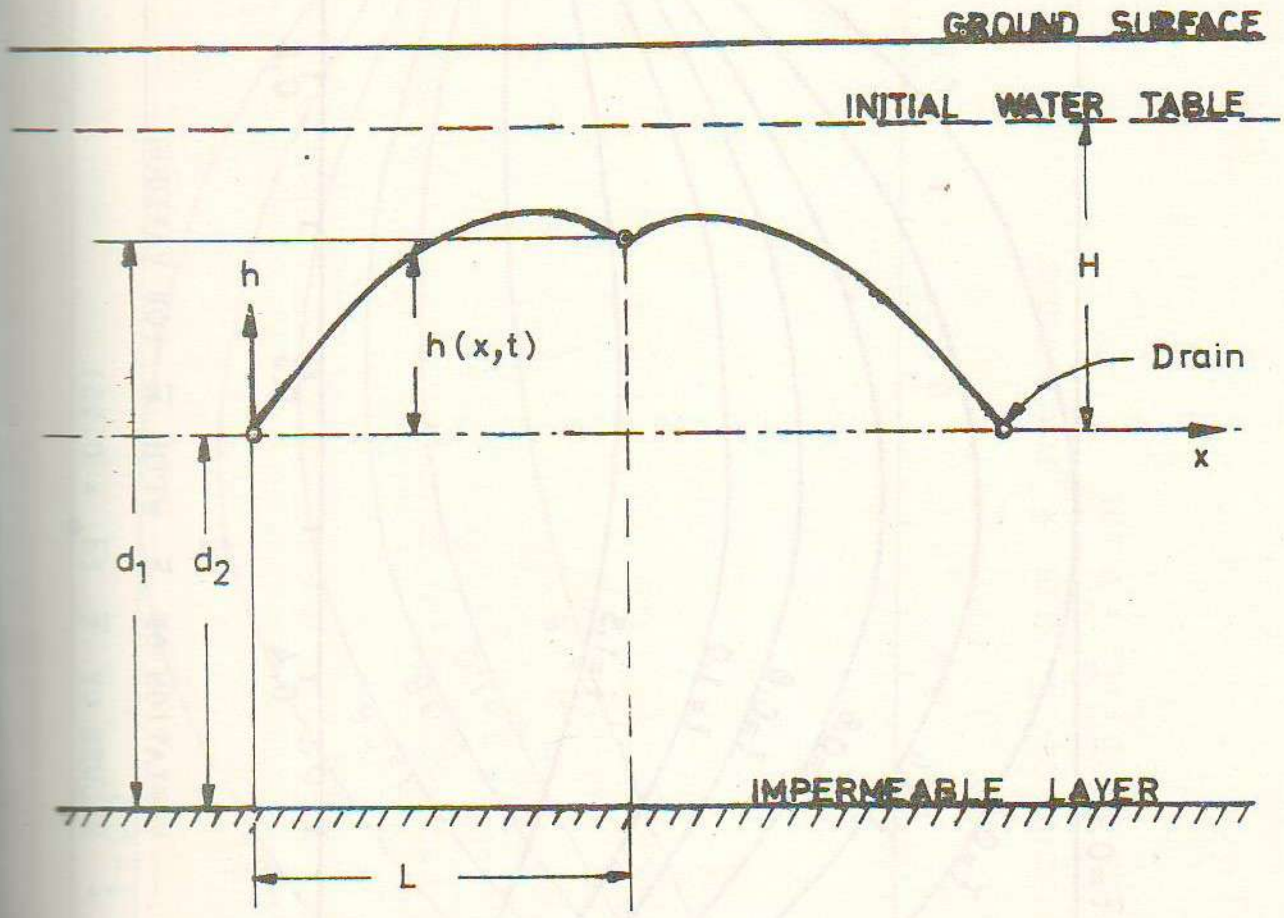

FIG.1.- DEFINITION SKETCH

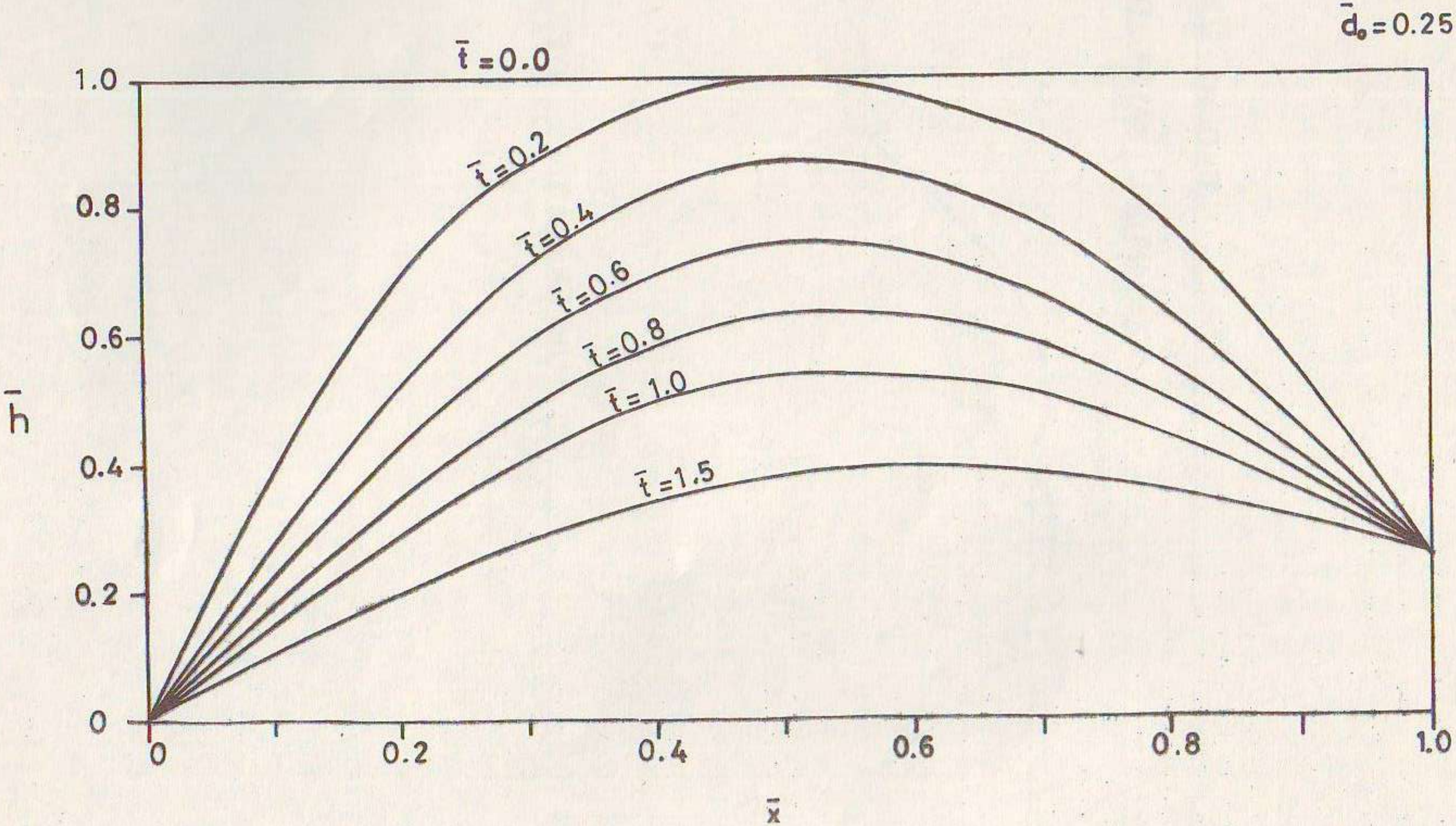

FIG.2.- VARIATION OF $\bar{h}$ WITH $\bar{x}$ FOR VARIOUS VALUES OF $\bar{t}$ ($\bar{d}_o = 0.25$)

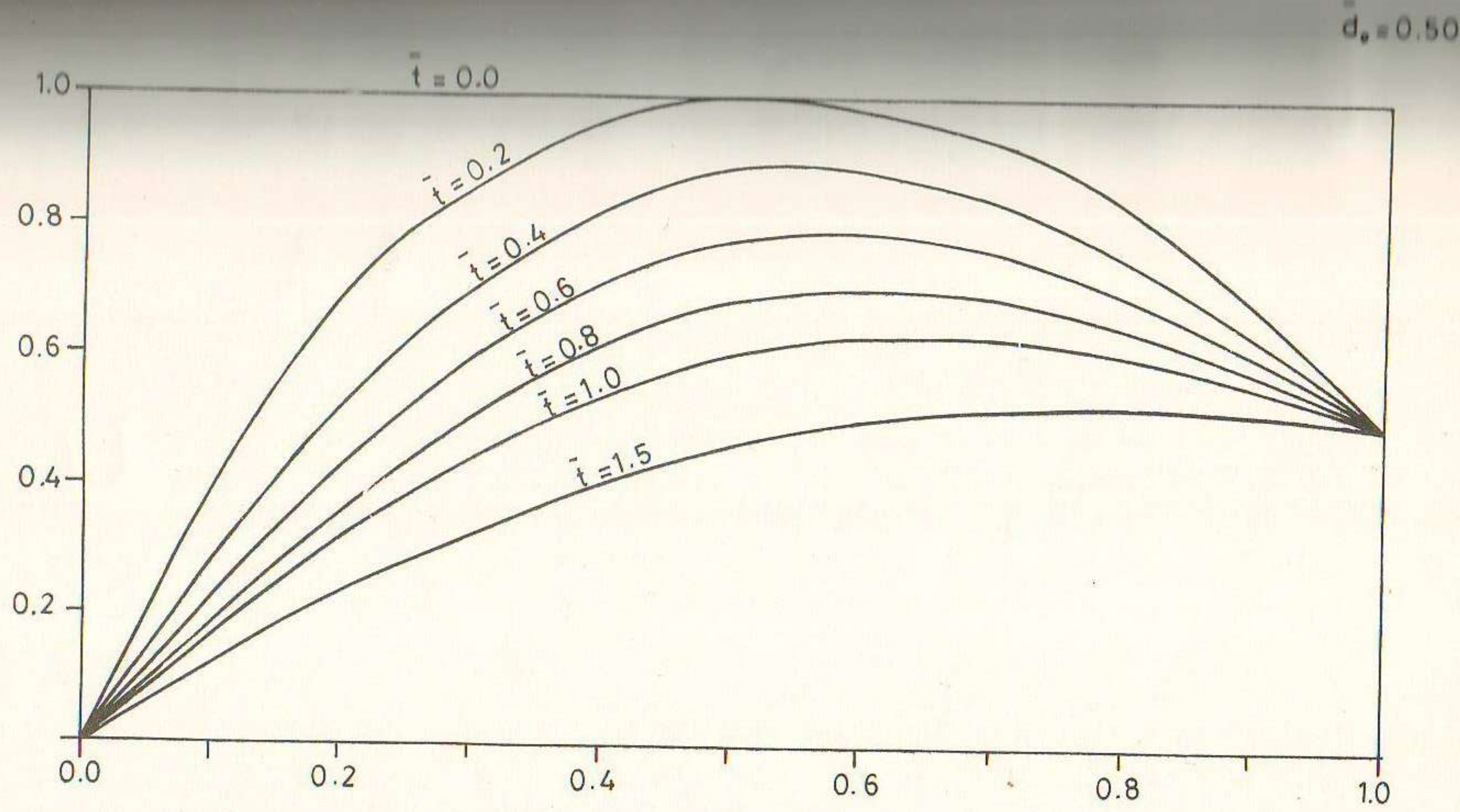

FIG.3.- VARIATION OF $\bar{h}$ WITH $\bar{x}$ FOR VARIOUS VALUES OF $\bar{t}$ ($\bar{d}_o = 0.50$)

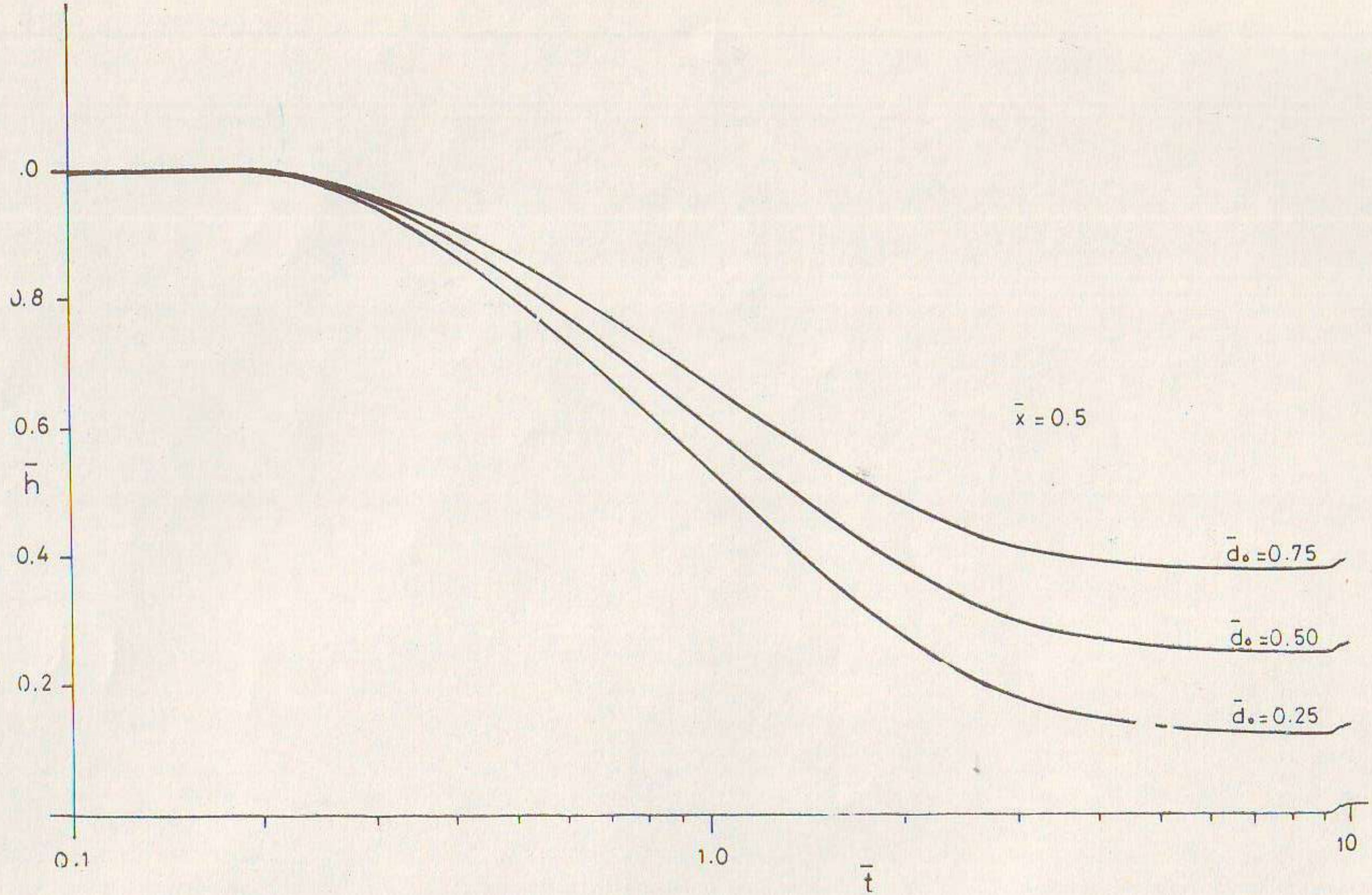

FIG.6.- VARIATION OF $\bar{h}$ WITH $\bar{t}$ FOR VARIOUS VALUES OF $\bar{d}_o$ ($\bar{x}$ = 0.50)

تعميم طريقة المعامل لتتبع الفيضانات خلال خزانات السدود

الدكتور غازي المشهداني

كلية الهندسة- جامعة الموصل

ملخص البحث :

تحوي الدراسة على محاولة رياضية لتعميم طريقة المعامل لتتبع الفيضانات خلال خزانات السدود وذلك بافتراض ان العلاقة بين الخزن المؤقت والتصريف الخارج من الخزان علاقة لاخطية. وبناء على ذلك تقترح الدراسة طريقة تجريبية رياضية للتعامل مع المسألة.

GENERALISATION OF COEFFICIENT METHOD FOR ROUTING A FLOOD THROUGH A RESERVOIR

By
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ABSTRACT

The coefficient method has been generalised by assuming a nonlinear relationship between storage and outflow. An empirical procedure for the solution of general equation has been developed. The method of solution was found to yield reasonable results.

INTRODUCTION

Coefficient method[1,2] of reservoir routing is well known. It is easy to apply since there is no need of initial curves to be prepared as is required in other graphical procedures . The method is based on the general continuity equation as used in other techniques, but, further, assumes that storage is directly proportional to the outflow . An equation can thus be derived which gives the outflow at a given time step as a function of average inflow, outflow at previous time step and the constant of propoprtionality, K, of storage outflow relationship. The assumption that storage is directly proportional to the outflow is usually not true in a real field problem, hence it is customary to use, K, as a variable (Chow 1964) the values of which can be determined for various segments of storage versus discharge curve, although in so doing the ease with which coefficient method can be applied is lost. The present paper is an attempt to modify the coefficient method by assuming a non-linear relationship between storage and outflow .

Development of the method

Assuming a non – linear relationship between storage, S, and outflow, Q, one can write

$$S = KQ^n \qquad \text{.......(1)}$$

where ,n, and ,K, are constants which can be determined by usual regression technique given in equations(2) and (3) if some outflow values are known for the corresponding storage values.

$$n = \frac{\Sigma(\ln S - \overline{\ln S})(\ln Q - \overline{\ln Q})}{\Sigma(\ln Q - \ln Q)^2} \qquad \text{............ (2)}$$

$$\text{and } K = \overline{\ln S} - n\,\overline{\ln Q} \qquad \text{............ (3)}$$

The continuity equation for reservoir routing is written as follows :

$$\frac{1}{2}(I_i + I_{i+1})\,t - \frac{1}{2}(Q_i + Q_{i+1})\,t = S_{i+1} - S_i \qquad \text{........ (4)}$$

where suffix i and i + 1 represent the state of inflow I, outflow Q and storage S before and after a chosen time step t. Combining eq. (1) and (4) it can be shown that

$$\frac{2K}{t} Q^n_{i+1} + Q_{i+1} = (I_{i+1} + I_i) + \left(\frac{2K}{t} Q^n_i - Q_i\right) \qquad \text{........ (5)}$$

The only unknown in eq. (5) is, Q_{i+1}, which can be solved by trial and error.

The solution of eq. (5) by trial and error although straight forward is time consuming, such that an empirical solution procedure which gives reasonable results has been developed.

General procedure for solution

Eq. (5) can be rewritten as :

$$Q_{i+1} = \left[\frac{(I_{i+1} + I_i) + \left(\frac{2K}{t} Q^n_i - Q_i\right)}{\frac{2K}{t} + Q^{1-n}_{i+1}}\right]^{\frac{1}{n}} \qquad \text{................. (6)}$$

In eq. (6) if a small time step is chosen the value of $\frac{2K}{t}$ is much larger than Q_{i-1}, hence even if the value of Q_{i+1}^{1-n} in the denominator is replaced by an approximate value, it does not affect the result significantly. Thus, the value of Q_{i+1}^{1-n} in denominator of eq. (6) is replaced by $(\bar{Q}_{i+1})^{1-n}$ the value of which can again be calculated from eq. (6) by replacing the unknown term Q_{i+1}^{1-n} in the denominator by its value at previous time interval as is shown in eq. (7) and (8). Thus an approximate solution of eq. (5) can be written as :

$$Q_{i+1} = \left[\frac{(I_{i+1} + I_i) + \left(\frac{2K}{t} Q_i^n - Q_i \right)}{\frac{2K}{t} + (\bar{Q}_{i+1})^{1-n}} \right]^{\frac{1}{n}} \qquad \ldots\ldots\ldots\ldots\ldots (7)$$

where,

$$(\bar{Q}_{i+1}) = \left[\frac{(I_{i+1} + I_i) + \left(\frac{2K}{t} Q_i^n - Q_i \right)}{\frac{2K}{t} + Q_i^{1-n}} \right]^{\frac{1}{n}} \qquad \ldots\ldots\ldots\ldots (8)$$

Two numerical examples have been solved by following the above procedure, the solutions have been compared with results obtained from other procedures.

Details of numerical example

Ist Example :- The value of inflow flood hydrograph is shown in table 1 for Ist example. The storage outflow relationship for this example can be written as :

$$S = 302200\ Q^{0.82055}.$$

Thus the value of $k = 302200$ and $n = 0.82055$ a time step of 12 hours (= 12 × 3600 sec) has been chosen for analysis.

The values of $\bar{Q}_{i-1}$ and Q_{i-1} along with corresponding values of Q_{i+1} calculated by step by step method (Varshney, 1977) are given in table 1. It can be seen that the two values are very close to each other. The values of Q_{i-1}, calculated in Col. (4) has also been found to satisfy eq. (5).

IInd Example :- The second example is based on the data taken from Bekhme Reservoir (Urban, 1967) on Greater Zab river in North of Iraq. The storage outflow relationship for this case is given in table 2 and can be represented by

$$S = 419529.86\ Q^{0.8476593} + 1.95 \times 10^{9}.$$

Thus the value of K and n are 419529.86 and 0.8476593. The value of time step chosen here again is 12 hours. The outflow hydrograph for this example has also been calculated by conventional* coefficient method using variable value of K (details are shown in table 2). The values of outflow, Q_{i+1}, as calculated from eq. (7) here again has been found to satisfy eq. (5).

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* The conventional coefficient method means herein the coefficient method in general use as $Q_{i+1} = Q_i + \left(\frac{2t}{2K + t} \right) \left(\frac{I_{i+1} + I_i}{2} - Q_i \right)$

Table 1. Results of Example 1 : —

Time hours	Inflow m^3/sec	Approx . values Q_{i+1}	Outflow Q_{i+1} m^3/sec	Values of Q_{i+1} by step by step method
0	0	0	0	0
12	127.5	14.77	14.77	14.2
24	350.4	95.70	80.11	71.0
36	736.0	253.18	243.96	241.0
48	1700.0	725.36	695.80	695.0
60	1050.0	1020.03	1003.67	977.0
72	732.0	951.16	951 74	905.0
84	510.0	795.71	801.78	780.0
96	325.0	625.94	632.36	651.0
108	198.5	469.22	474.81	481.0
120	99.3	337.73	342.12	340.0
132	42.5	233.81	237.03	227.0
144	0	155.73	157.96	142.0
156		102.06	103.49	99.3
168		69.08	69.92	
180		47.99	48.50	

Table 2. Values of , K , and Coefficient , C , for Conventional Coefficient method .

Outflow m^3 / sec	Storage m^3	K	Coefficient $C = \frac{1}{\left(K / \Delta t + \frac{1}{2} \right)}$
0	1.95×10^9	140000	0.2673
1000	2.09×10^9	130000	0.28496
2000	2.22×10^9	115000	0.31625
4000	2.45×10^9	95000	0.3705
6000	2.64×10^9	75000	0.4473
8000	2.79×10^9	70,000	0.4176
10000	2.93×10^9		

Table 3. Results of example 2.

Time hours	Inflow 1 m^3/sec	Approx . values Q_{i+1} eq. (8)	Outflow Q_{i+1} eq. (7)	Outflow by conventional coefficient method method
48	1000	1000	1000	1000
60	1250	1038	1037	1033
72	2000	1218	1214	1202
84	3500	1705	1691	1643
96	7000	2900	2861	2671
108	10200	4951	4877	4546
120	8000	6488	6435	6233
132	5600	6576	6572	6487
144	4500	5986	6003	5844

156	3950	5328	5346	5244
168	3600	4759	4774	4700
180	3300	4286	4299	4237
192	3100	3899	3909	3853

Discussion and Conclusions :

It can be seen that the method requires no graphs to be plotted . as errors due to graphical plotting . choice of scale etc . are eliminated . Besides . the method appears to be sufficiently general and can be easily programmed. For the two examples worked out in the present case the results obtained are comparable to those obtained by other methods . The values calculated by Eq . (7) satisfied eq. (5) reasonably well . even the approximate values worked out by eq . (8) give results which are close to that of eq . (7) .

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Nomenclature :

S = Storage
Q = Outflow
K = Constant of proportionality in storage outflow relationship
n = Exponent of outflow in storage outflow erlationship
I = Inflow
i and i+1 are suffixes indicating state of event before and after a time step t
t = time step
In S = average value of natural logarithm of storage
In Q = average value of natural logarithm of outflow values
Q = Approximate value of outflow $\bar{Q}$.