توزيع الاجهادات في أداة ثلم اسفينية الشكل

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خلاصة المقالة :

لقد بينت التحليلات المبنية على أساس نظرية اللدونة أنه خلال عمليه الثلم بواسطة اداة اسفينية الشكل، يتعرض سطحي مماسي الاسفين الى اجهاد عمودي منتظم ، واجهاد آلي منتظم. ويقدم هذا البحث دراسة نظرية حول توزيع الاجهادات في الاسفين عند تعرضه لمثل اجهادات التماس هذه. ويظهر الحل النهائي على شكل معادلات تكاملية وقد استخدمت في ذلك طريقة «ترانتر» التي تعتمد على محولات « ملن » التكاملية.

STRESS DISTRIBUTION IN A WEDGE INDENTOR

by
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Summary :

Plasticity analysis for orthogonal wedge indentation of plastic rigid material suggests that the faces of the wedge will be subjected to uniformly distributed pressure and tangential stress over the contact length. In the present work the stress distribution at the tip of an infinitely deep wedge is obtained when the wedge is subjected to such a stress distribution. The problem simulates the industrial scoring process for the manufacturing of easy open can tops. The solution is obtained in integral form using a method proposed by Tranter, which is based on the application of Mellin Integral Transform.

Notation:

P_0	normal pressure	
K	shear yield strength	
E	modulus of elasticity	
τ	shear stress	
T_0	frictional shear stress	
μ	coefficient of friction	
σ_θ , σ_r	normal stresses	polar co-ordinates
$T_{r\theta}$	shear stress	polar co-ordinates
γ	radial co-ordinate	polar co-ordinates
σ_x , σ_r	normal stresses	cartesian co-ordinates
τ_{xy}	shear stress	cartesian co-ordinates
α , ψ , θ	angles	cartesian co-ordinates

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1 – INTRODUCTIN :

The industrial Scoring* process presents a number of unsolved problems related to the wear or fracture of the tool tip. For these problems to be solved it will be necessary to obtain detailed information on the stress distribution in the indenting tool under surface loadingsimilar to those encountered in the scoring process. In industry it has been usual to effect scoring of the can ends by the use of a Crank press in which a scoring tool is fixed rigidly to a ram which is caused to penetrate can ends. The process, is therefore,approximately a plane strain indentation problem. The most conventional tool profile used for scoring aluminium can ends was the trapezoidal wedge [1] having an included angle of 50° and a flat width of 0.002 in to 0.007 in. Tools with such profile were found, in practice, to have a relatively short life when used for scoring ferrous metals such as tin-plate and to be subject to fracture at the corners. It has therefore, been replaced in the last few yearsby sharp-edged wedges[2] having a total included angle of 90 . This, however, proved also to be not very satisfactory [1]

The primary object of the present work is to obtain basic information on the nature of stress distribution in the region where fatigue-crack nucleation or surface wear occurs. For this reason the general equations for stress distributions throughout a sharp-edged wedge are evaluated for boundary conditions approximating to the loads on sharp-edged wedge indenter. The distributions of stress are represented in terms of trigonometric integrals with the aid of a method proposed by Tranter [3] which is based on the application of the Mellin Integral Transforms. Full description of this method is contained in references[3] and[4] and, therefore, details are not repeated here.

2- STRESS BOUNDARY CONDITIONS IN SHARP–EDGED WEDGE INDENTATION :

The process to be considered is one in which a flat strip or sheet of plastic–rigid material is indented by means of a tool. The tool spans the width of the strip and the width/thickness ratio of the strip is large enough for edge effects to be neglected The flow is confined to planes perpendicular to the tool and the deformation is assumed to take place under plane - strain conditions . As the tool advances into the material , two stages of the operation can be distinguished:

(a) Surface indentation.
(b) Deep Penetration.

Surface indentation is an elastic-plastic mode of deformation and at this stage the bulk of the indented material is elastic. However, after a certain critical depth the plastically stressed region reaches the foundation. This stage is defined as Deep-penetration. Hill, Lee and Tupper [5] initiated a slip-line field solution for the surface indentation of semi-infinite block of plastic rigid material by frictionless acute angled wedge . This field is shown in figure[1] and is characterised by two isosceles triangles and a fan. The configuration is geometrically similar at every stage and the field merely changes in size as the deformation proceeds. This filed suggests that the normal pressure acting on the surfaces of the tool, will be uniformly distributed over the contact length and is given by the equation [5]

$$p = 2k (1 + \psi)$$

* "Scoring" is the industrial name for the indentation process of metal can ends .

where 'K' is the shear yield stress of the indented material. The angle' ψ ' of the slip-line field is related to the wedge semi-angle' α ' by the relation[5]:

$$\frac{h}{d} = \{ \cos\alpha - \sin(\alpha - \psi) \}^{-1}$$

Thus, the magnitude of the uniformly distributed pressure acting on the wedge faces depends on the tool geometry and the yield stress of the material and is not affected by the variation of the depth of indentation.

Grunzweig, Longman and Petch[6] have given the solution for plane strain surface indentation of a plastic rigid material by a rough (Coulomb friction) wedge. The slip-line field of this solution is shown in figure (2) and the relationship between $\alpha, \psi, \lambda, \mu$ and c which are given by Grunzweig[6] can be found from the requirements of continuing geometrical similarity and incompressibility. The normal compressive stress 'p' and the shearing stress 'τ' acting on the wedge are given by the relations:

$$p = k(1 + 2\psi + \sin 2\lambda)$$

$$\tau = k \cos 2\lambda$$

Consequently the coefficient of friction is given by :

$$\mu = \cos 2\lambda / (1 + 2\psi + \sin 2\lambda)$$

A limit occurs in this field when $(\alpha - \lambda) > \pi/4$, since the yield criterion should not be violated in the rigid region near the point B, Figure 2. If $\alpha > \pi/4$ then the indenter will be covered with a 90° wedge shaped dead metal cap. The slip-line field for this case has been presented and calculated by Johnson, Mahtab and Haddow[7] It should be noted, however, that for wide angled wedges elastic effects become important. This has been discussed by Mulhearn [3] and March[9], whereas Hirst and Howse[10] idealised the process of blunt-wedge indentation by the elastic expansion of cylind rical cavity in an infinite medium.

It is therefore, possible to conclude that the theoretical plasticity analysis suggessts that for frictionless acute angled wedge indentation the normal pressure acting on the wedge face is uniformly distributed and fo rough acute angled wedge indentation (Coulomb friction) both normal pressure and tangential stresses are uniformly distributed. However, two conditions are to be satisfied;

a- The included angle of the wedge must be acute.
b- The ratio E/Y for the indented material must be high (i.e. plastic – rigid material

In the following work, Tranter's method is used to find the stress distributions in an infinite wedge when subjected to such boundary conditions. The method could be applied directly to the problem of finding the stresses in the wedge for uniform distribution of normal pressure and shear stress. However, the algebra involved in reducing the Mellin inversion integrals to trigonometric form is considerably simplified by first solving for uniformly distributed normal pressure and then for uniformly tangential boundary stresses.
The solution for the stress distributions in wedge indentation is then obtained by superposition of the two solutions. The procedure is represented schematically in Figure (3).

3– THE SOLUTION

A– Application of Uniform Normal Pressure to The Faces of an Infinite Wedge :

Using polar coordinates the distribution of stress in an infinite wedge of semiangle 'α' when its faces are each subjected to a uniform pressure p_0, acting on both faces of the wedge for a distance A measured from the vertex, was found to be represented by the following equations :

$$\frac{\pi r}{2\,AP_0}(\sigma_\theta - \sigma_r) = \frac{\sin\alpha\cos\theta}{2\alpha + \sin 2\alpha} - \int_0^\infty P(u)\sin\left[u\ln\left(\frac{A}{r}\right)\right]du \qquad \ldots(1)$$

$$\frac{\pi r}{2AP_0}(\sigma_\theta + \sigma_r) = \frac{-\pi\sin(\alpha)\cos(\theta)}{2\alpha + \sin(2\alpha)} + \int_0^\infty [P(u) - uQ(u)]\sin\left[u\ln\left(\frac{A}{r}\right)\right] -$$

$$- [Q(u) + uP(u)]\cos\left[u\ln\left(\frac{A}{r}\right)\right]\frac{du}{1+u^2} \qquad \ldots(2)$$

$$\frac{\pi r}{AP_0}\tau_{r\theta} = \int_0^\infty R(u)\cos\left[u\ln\left(\frac{A}{r}\right)\right]du \qquad \ldots\ldots\ldots(3)$$

where the functions P (u), Q (u) and R (u) are given by

$$(u\sin 2\alpha + \sinh 2\alpha u)\,P(u) = \sin(\alpha - \theta)\cosh(\alpha + \theta)u + \sin(\alpha + \theta)\cosh(\alpha - \theta)u$$

$$(u\sin 2\alpha + \sinh 2\alpha u)\,Q(u) = \cos(\alpha - \theta)\sinh(\alpha + \theta)u + \cos(\alpha + \theta)\sinh(\alpha - \theta)u$$

$$(u\sin 2\alpha + \sinh 2\alpha u)\,R(u) = \sin(\alpha - \theta)\sinh(\alpha + \theta)u - \sin(\alpha + \theta)\sinh(\alpha - \theta)u$$

The values of the stress components were evaluated numerically for a wedge angle $2\alpha = 60$ and $2\alpha = 90$ Simpson formula was used for high ratios of A/r. Computations were stopped very close to the wedge tip where the ratio A/r tends to infinity giving indeterminate stress at the apex of the wedge. All the integrals were computed over equal intervals of θ from $\theta = \alpha$ to $\theta = -\alpha$ and the stress distribution in the form of $(\sigma_\theta - \sigma_r)$, $(\sigma_\theta + \sigma_r)$ and $\tau_{r\theta}$ were evaluated. This enabled the construction of contours of equal maximum shear stress (Isochromatics) and the results are shown in Figurs (4) and (5). The contours of equal directions of principal stresses (Isoclinics) for a (90° wedge are presented in Figure (6) and it enabled the construction of the Isostatic pattern presented in Figure (7) Several theoretical results are of interest. An isotropic region is seen to extend across the wedge to join the two points at the two faces of the wedge where $r = A$. Above this isotropic region the maximum shear stress increases sharply and reaches its peak value on the axis of symmetry at a distance equal to $A(\sin\alpha + \cos\alpha$ from the apex.

Below the isotropic region, the order of isochromatic increases continuously toward the apex. It is clear from the inspection of Figure (7) that within this region both radial and tangential stresses are compressive.

The radial stress distribution on the faces of a wedge were evaluated numerically from either of equations (1) or (2) for wedge angles of $2\alpha = 60^\circ$ and $2\alpha = 90^\circ$ and the results are shown in Figures (8) and (9). It can be seen that a finite stress discontinuity of magnitude P exists at $r = A$. In practice, such stress discontinuity would not be expected. However, it is reasonable to assume that the radial stress drops rapidly at a small distance within $r = A$

The variation of maximum shear stress across different sections perpendicular to this axis of symmetry are also indicated in Figures (8) and (9). The variation of radial and tangential stress along the axis of symmetry are also represented in these figures, and because of symmetry the shear stress $\tau_{r\theta}$ is equal to zero. Except for a small region at the apex the radial stresses σ_r are compressive all along the axis of symmetry. Within the small region at the wedge apex the tangential stresses tend to be infinite. This physically unrealistic result represents a breakdown in this present type of solution within a small region enclosing the apex. However, in practice this region mav be regarded as a zone of incipient plastic deformation.

III- Application of Uniform Shear Stress to the Faces of an Infinite Wedge :

[illegible] the use of Tranters method, the stress distribution in an infinite wedge loaded on both Faces on the segment $0 < r < A$ by uniform shear stress was found to be represented by the following equations :

$$\frac{\pi r}{2AT_0}(\sigma_\theta - \sigma_r) = \frac{\pi \cos\alpha \cos\theta}{2\alpha + \sin 2\alpha} - \int_0^\infty [\,z(u) - u\,Y(u)\,]\left[\sin\left(u \ln \frac{A}{r}\right) - u \cos\left(u \ln \frac{A}{r}\right)\right] \frac{du}{1+u^2}$$

$$\frac{\pi r}{2AT_0}(\sigma_\theta + \sigma_r) = \frac{-\pi \cos\alpha \cos\theta}{2\alpha + \sin 2\alpha} + \int_0^\infty \left[\,[\,z(u) + Y(u)\,] \sin\left(u \ln \frac{A}{r}\right) + [\,-u\,z(u) + Y(u)\,] \cos\left(u \ln \frac{A}{r}\right)\right] \frac{du}{1+u^2}$$

$$\frac{\pi r}{AT_0}\tau_{r\theta} = \int_0^\infty [\,\theta(u) + uX(u)\,]\left[\cos\left(u \ln \frac{A}{r}\right) + u \sin\left(u \ln \frac{A}{r}\right)\right] \frac{du}{1+u^2}$$

where

$[illegible] = \cos(\alpha + \theta)\cosh(\alpha - \theta)u + \cos(\alpha - \theta)\cosh(\alpha + \theta)u$

$[illegible] = \sin(\alpha - \theta)\sinh(\alpha + \theta)u + \sin(\alpha + \theta)\sinh(\alpha - \theta)u$

$[illegible] = \cos(\alpha - \theta)\sinh(\alpha + \theta)u - \cos(\alpha + \theta)\sinh(\alpha - \theta)u$

$[illegible] = \sin(\alpha + \theta)\cosh(\alpha - \theta)u - \sin(\alpha - \theta)\cosh(\alpha + \theta)u$

These equations for the stress distribution were again evaluated for wedge angles of $2\alpha = 60^\circ$ and $2\alpha = 90^\circ$. Figure [illegible] shows the [illegible] lines produced by this system of loading. High order of maximum shear stress is seen to occur around the points on the wedge face at $\frac{r}{A} = 1$. The neutral axis originates at the apex and coincides with the axis of symmetry. The features of this isochromatic pattern are similar to those observed in photoelastic experiments with point loading at the apex[11].

III – Superposition of the Normal Pressure and Shear-stress Solutions to Simulate the Stress Boundary Conditions on a Scoring Tool:

The normal pressure and shear stress solutions are superimposed to simulate the stress boundary conditions on a Wedge indenter. However, from plasticity analysis (Ref 6) it is evident that with increasing ratio of shear stress to normal pressure a limit occurs when the frictional stress T_c reaches the shear yield stress K of the indented material. For a wedge of 10° included angle, this limit is reached at a ratio of 0.359, whereas for a wedge of 40 the limit is reached at a ratio of 0.27. Wide-angled wedges are limited by the formation of a dead metal nose (Ref. 6). Therefore, to simulate the indentation process a ratio of shear stress to normal pressure equal to 0.2 would appear to be fairly representative of wedge indentation process. This ratio is high enough to show the influence of frictional stresses and low enough not to exceed the fore-mentioned limitations. In this section, a solution for combined shear stress and normal pressure in the ratio 0.2 is obtained by superposition of previous solutions for P_0 and

$T_0 = \frac{1}{5} P_0$ acting over a distance 'A' from the wedge apex .

Figure (11) shows the resulting isochromatic lines obtained for the case of a 90 wedge under the combined effect of normal pressure and shear stress. An interesting feature of this pattern is the presence of maximum shear stresses along the wedge face at r = A. The effect of frictional stress is also to shift the position of neutral axis towards the apex.

Figures (12) and (13) show the distribution of rectangular stress components in a 60 and 90 wedges respectively. The effect of frictional stress is to induce more compressive stress to both σ_x and σ_y without much effect on the shape of the curves.

4– CONCLUSIONS

Previous investigators showed that during the indentation process of semi – infinite plastic rigid material by sharp wedge indenter, the faces of the wedge will be subjected to uniform normal and tangential stresses. In this paper the stress distribution in a wedge when subjected to such boundary conditions has been examined. It is possible from the present theoretical solution to predict the influence of frictional stresses on the distibution of stresses in the wedge. Comparison of theoretical results with photoelastic results will be presented in later work.

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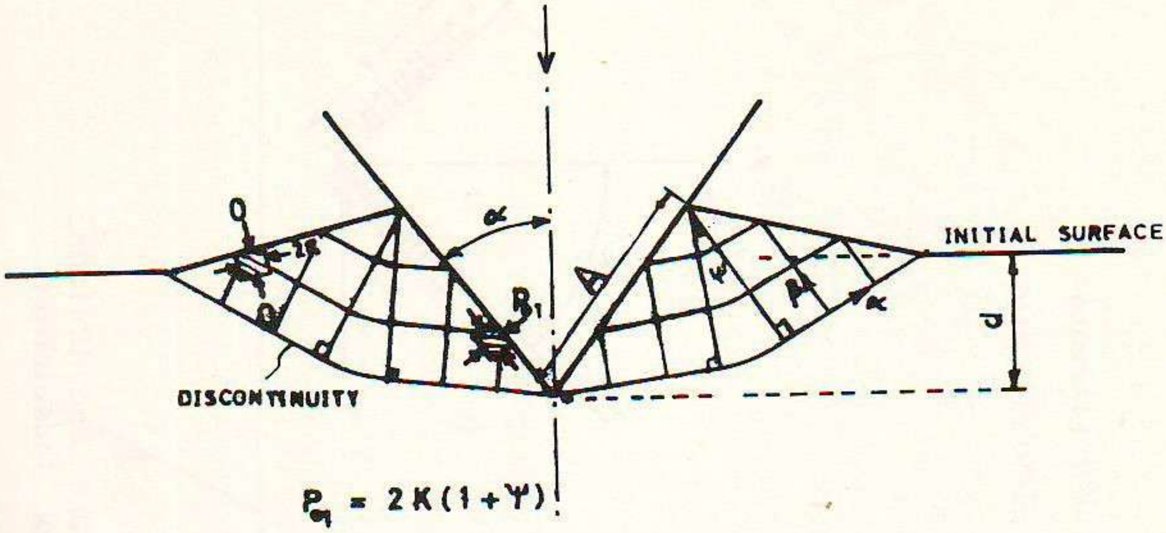

Fig.1 FRICTIONLESS WEDGE INDENTATION

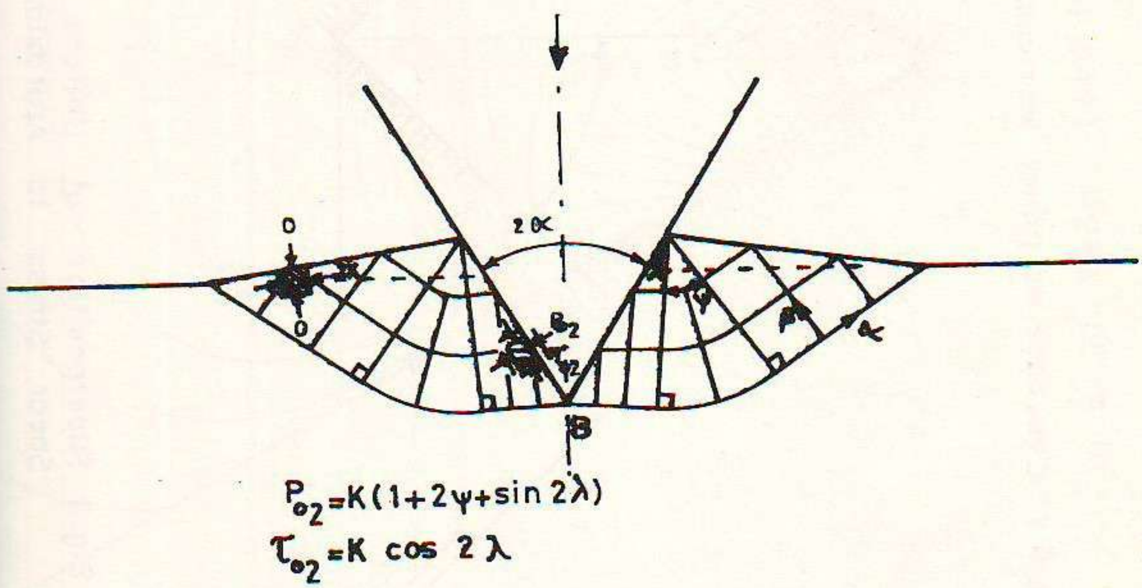

Fig. 2 ROUGH (COULOMB FRICTION) WEDGE INDENTATION

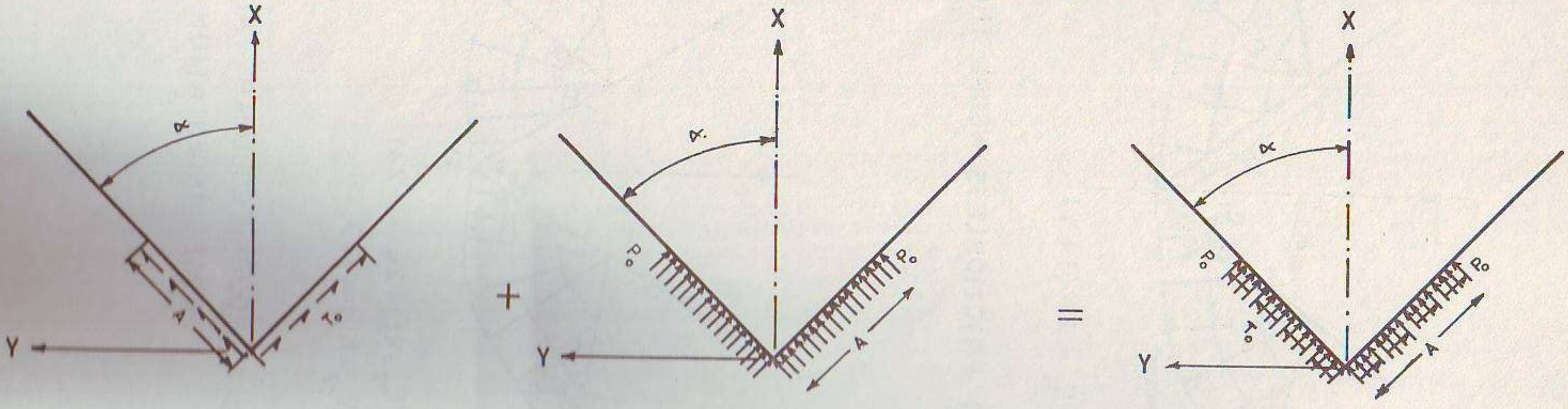

Fig. 3 Superposition of Uniform Normal Pressure And Uniform Shear Stress to Approximate Rough Wedge Indentation.

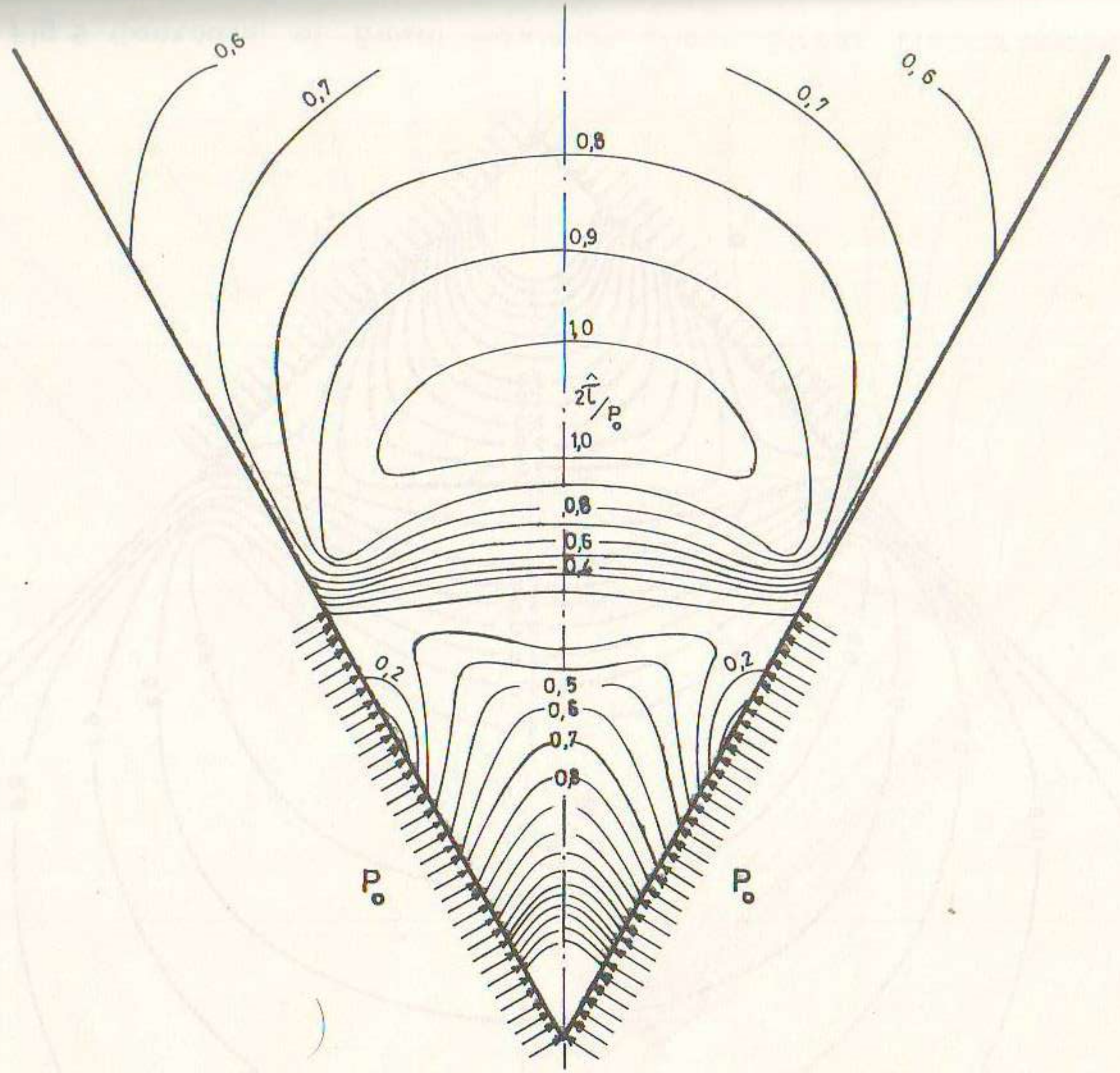

Fig. 4 Contours of Equal Maximum Shear Stress (Isochromatics) in a 60° Wedge, Loaded by Uniform Normal Pressure.

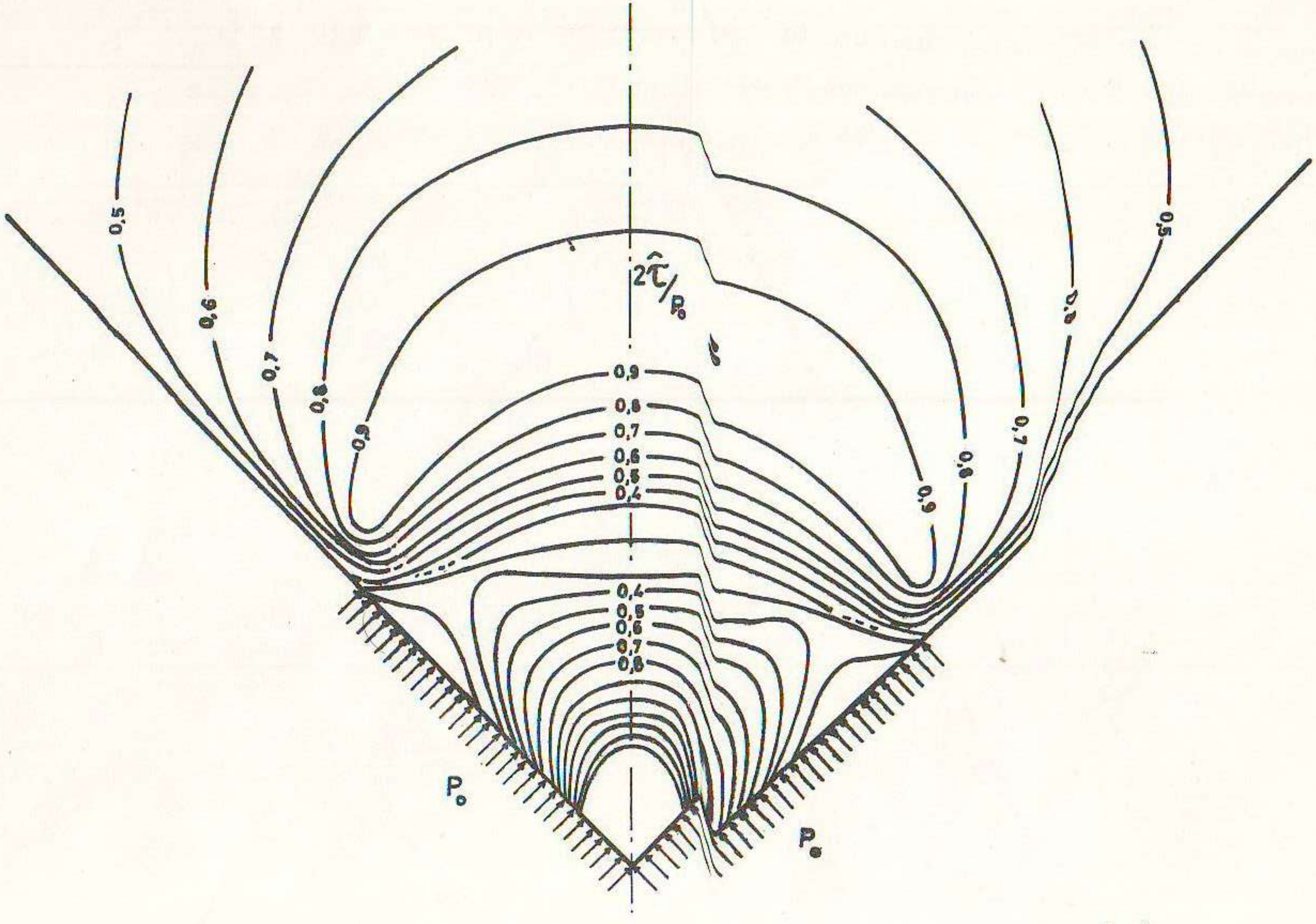

Fig. 5 Contours of Equal Maximum Shéar Stress (Isochromatics)
in a 90° Wedge, Loaded by Uniform Normal Press...

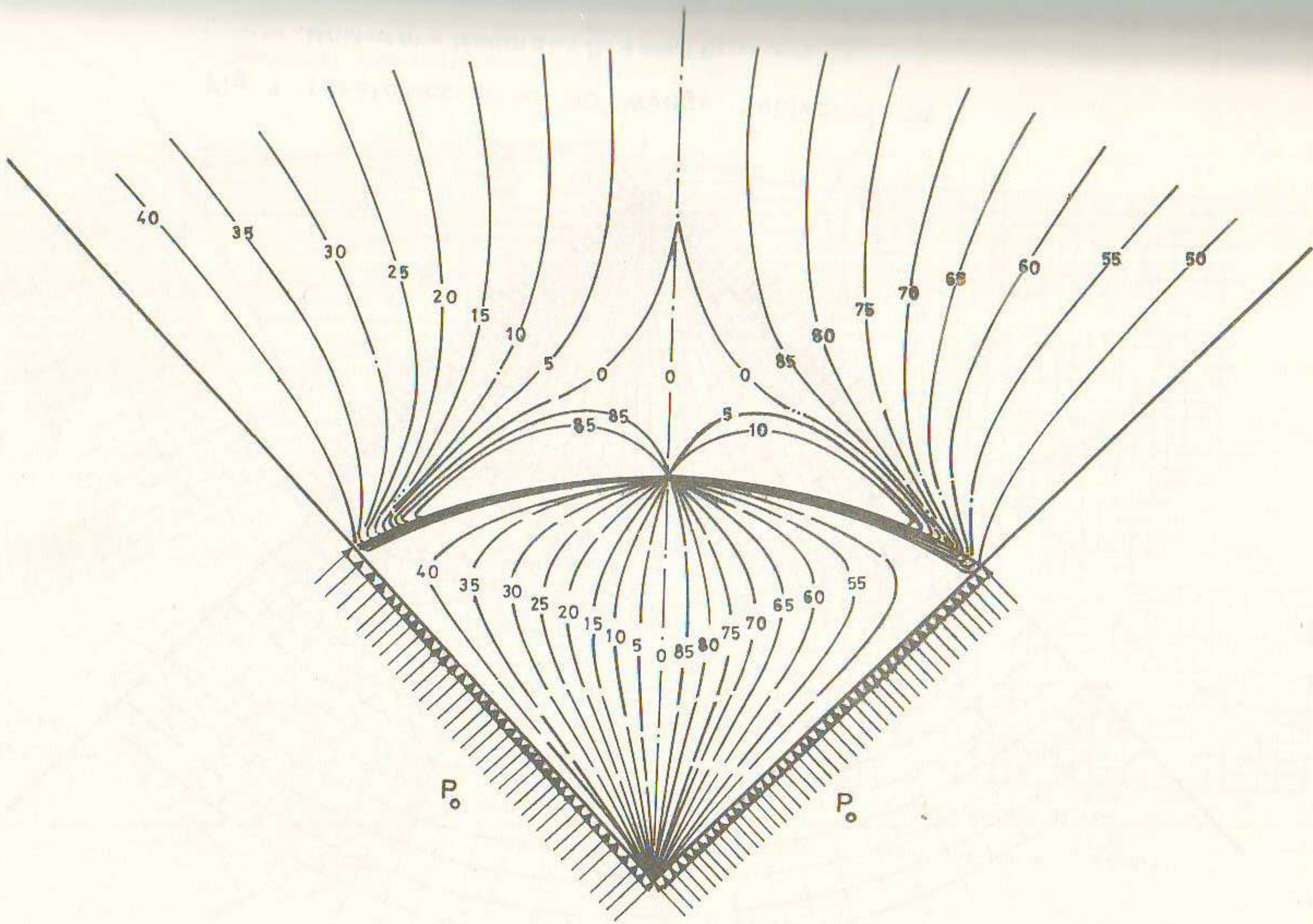

Fig. 6 Isoclinics in a 90° Wedge Subjected to Uniform Normal Pressure

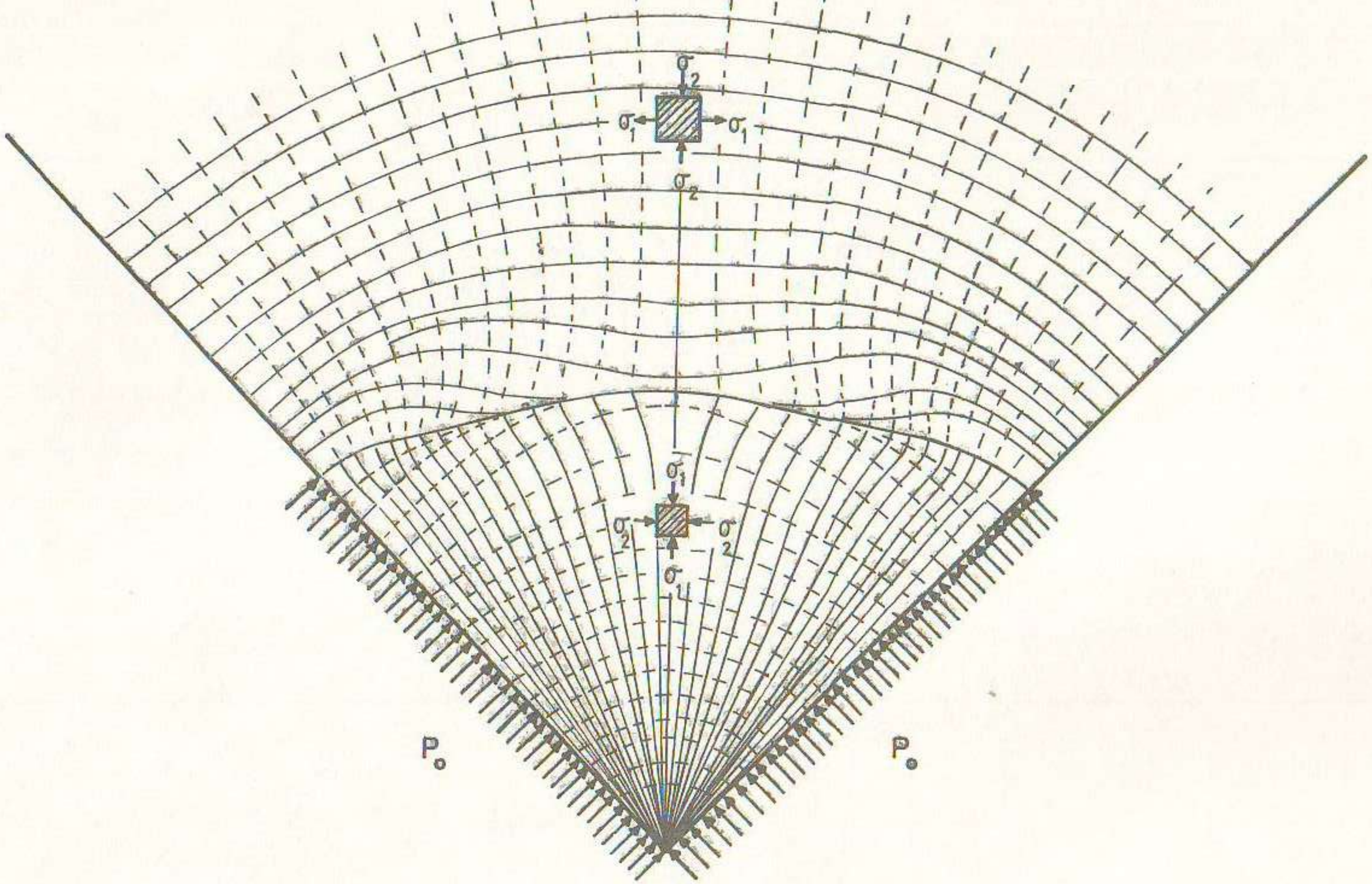

Fig. 7 Isostatics in a 90° Wedge Subjected to Uniform Normal Pressure.

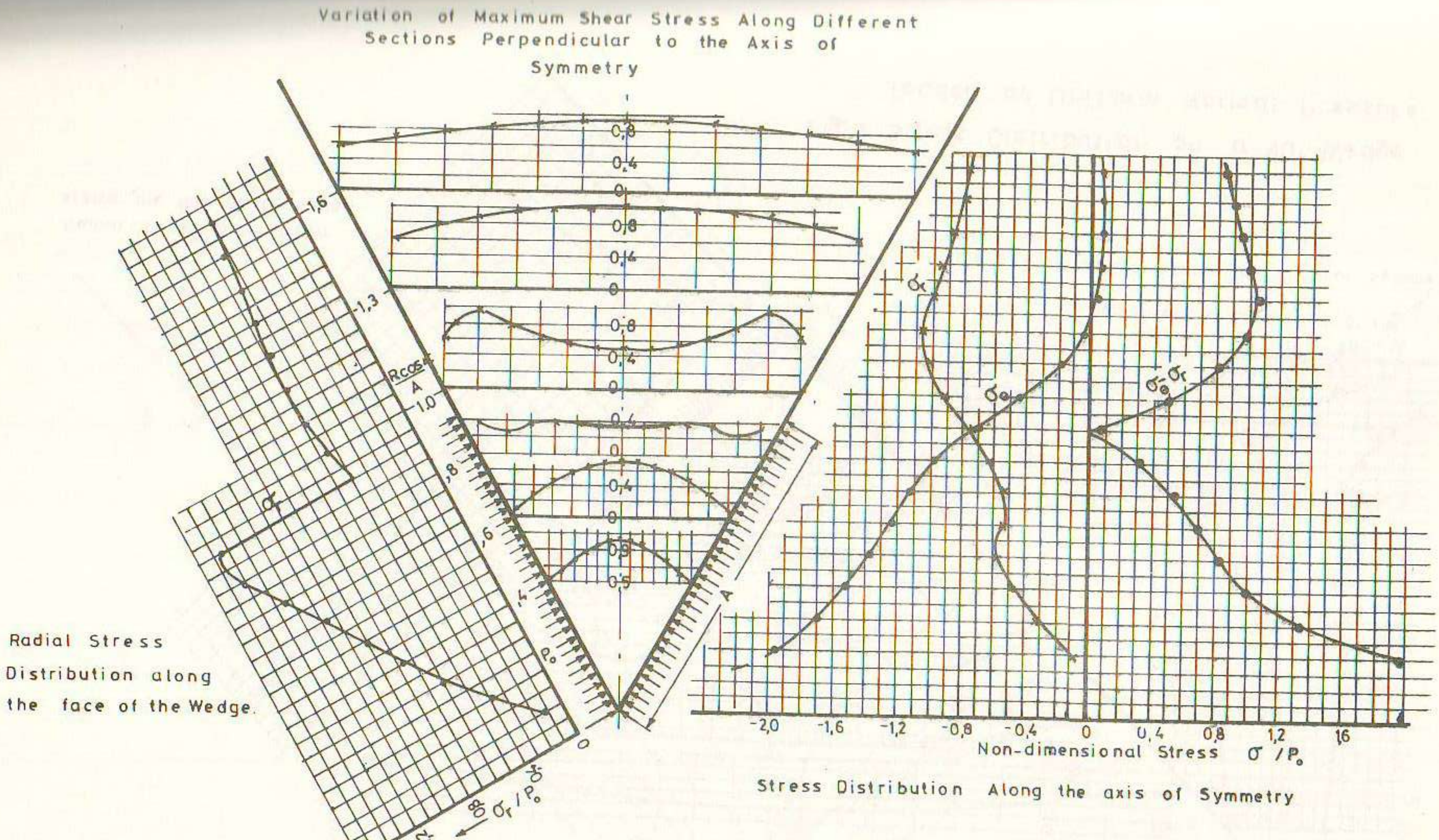

Fig. 8 Stress Distribution on a 60 Wedge Loaded by Uniform Normal Pressure.

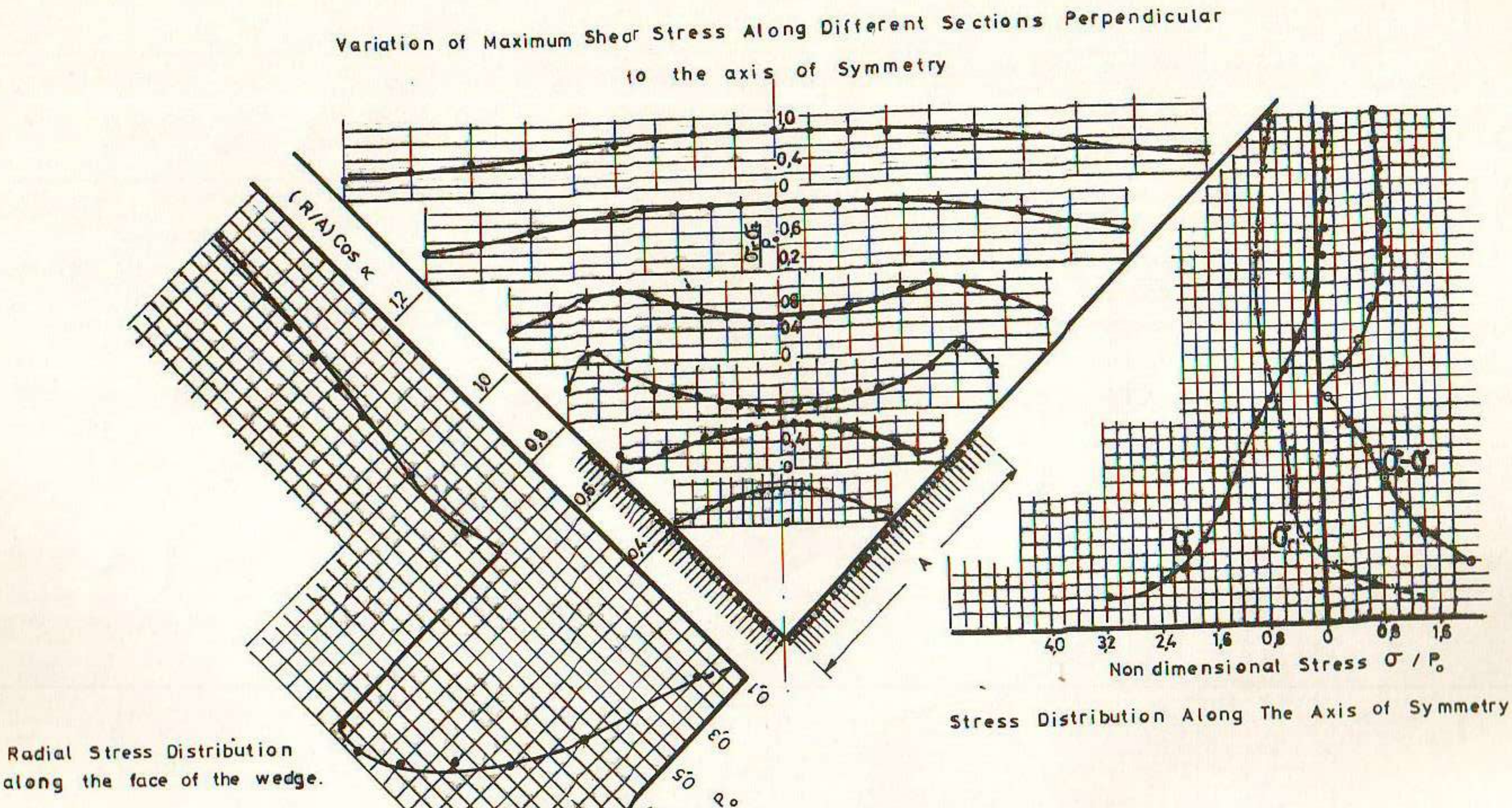

Fig. 9 Stress Distribution on a 90° Wedge loaded by Uniform Normal Pressure.

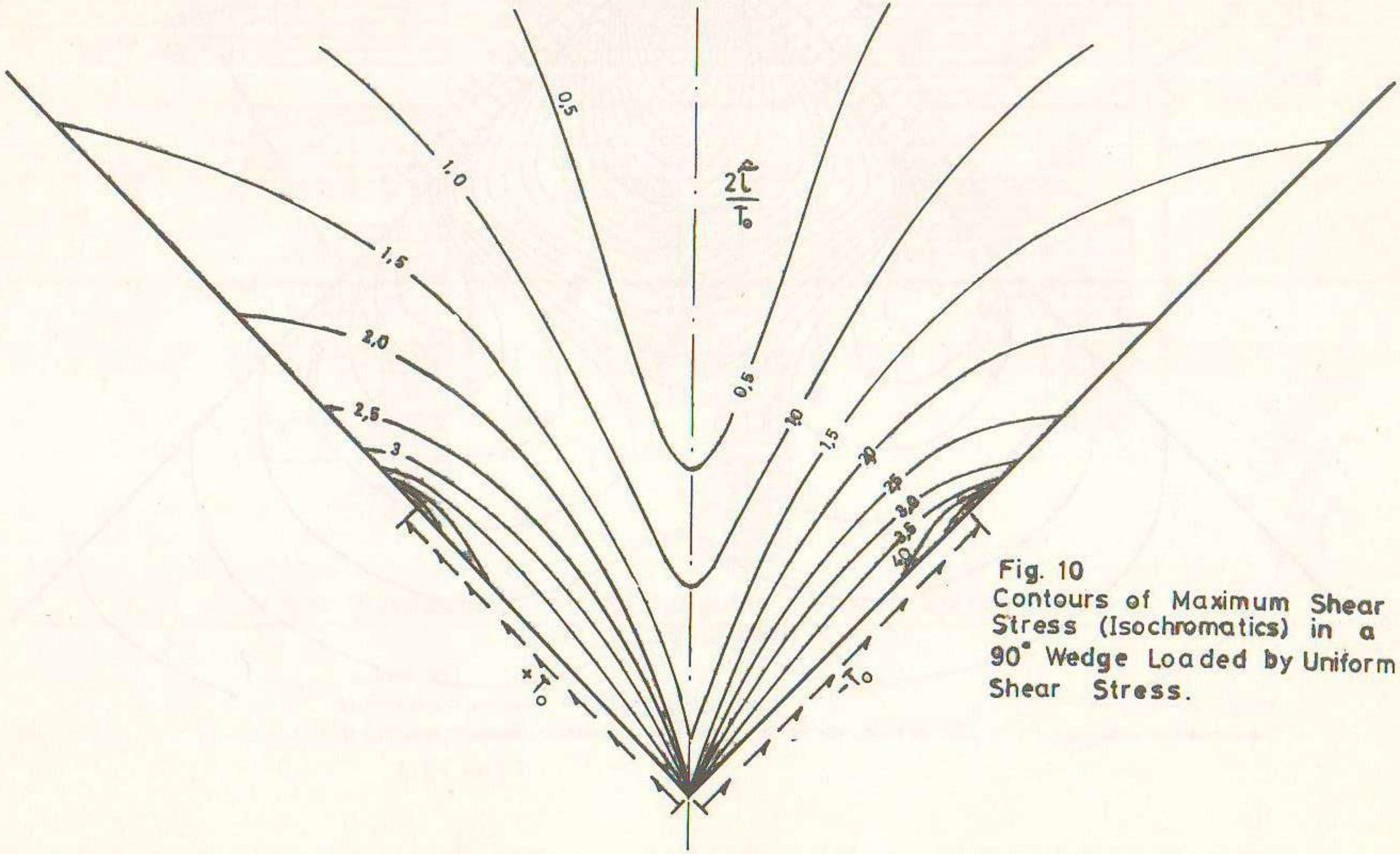

Fig. 10
Contours of Maximum Shear Stress (Isochromatics) in a 90° Wedge Loaded by Uniform Shear Stress.

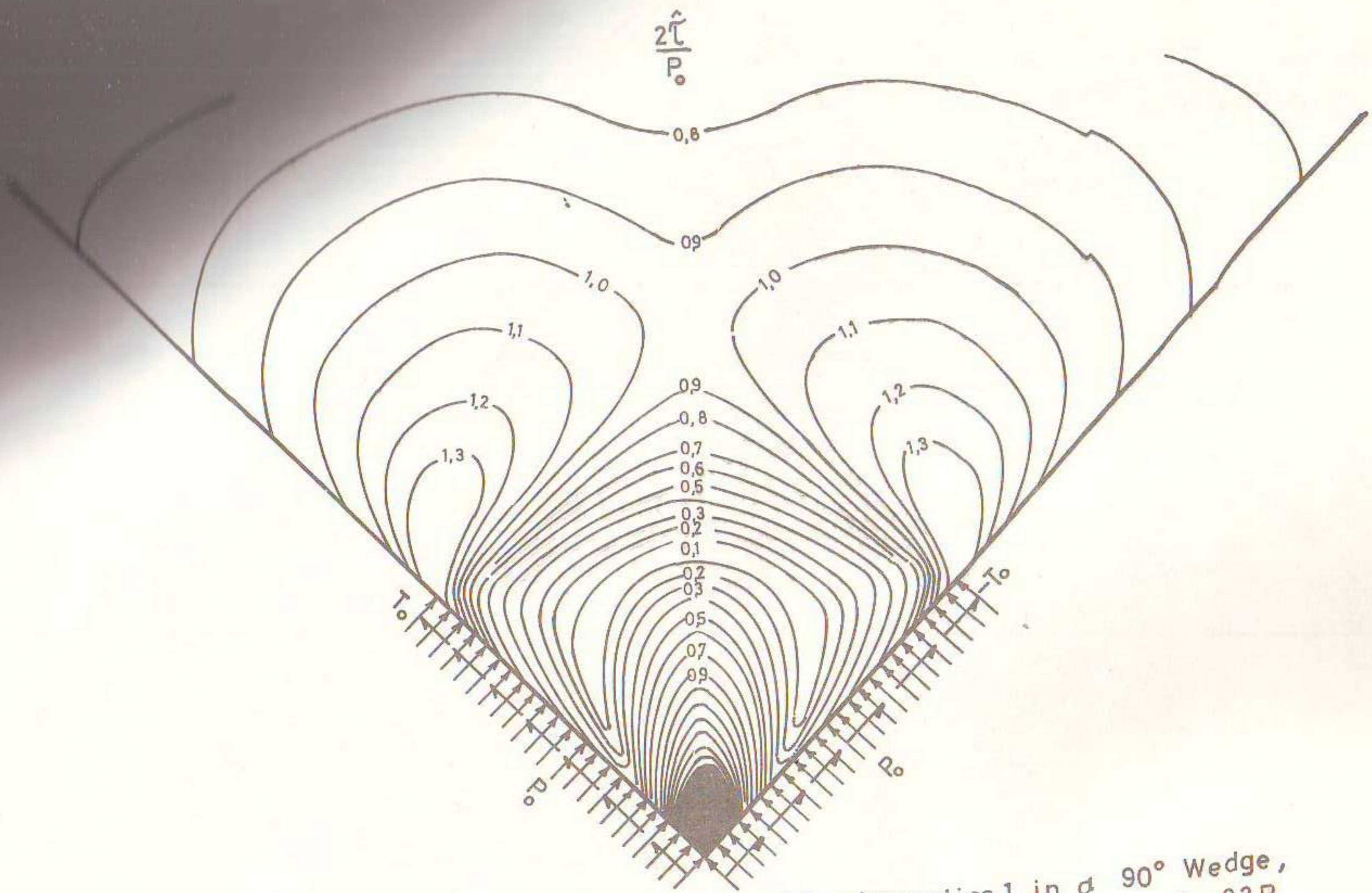

Fig.11 Contours of Equal Maximum Shear Stress [Isochromatics] in a 90° Wedge, Loaded by Uniform Normal Pressure P_o and Uniform Shear Stress $T_o = 0,2\,P_o$

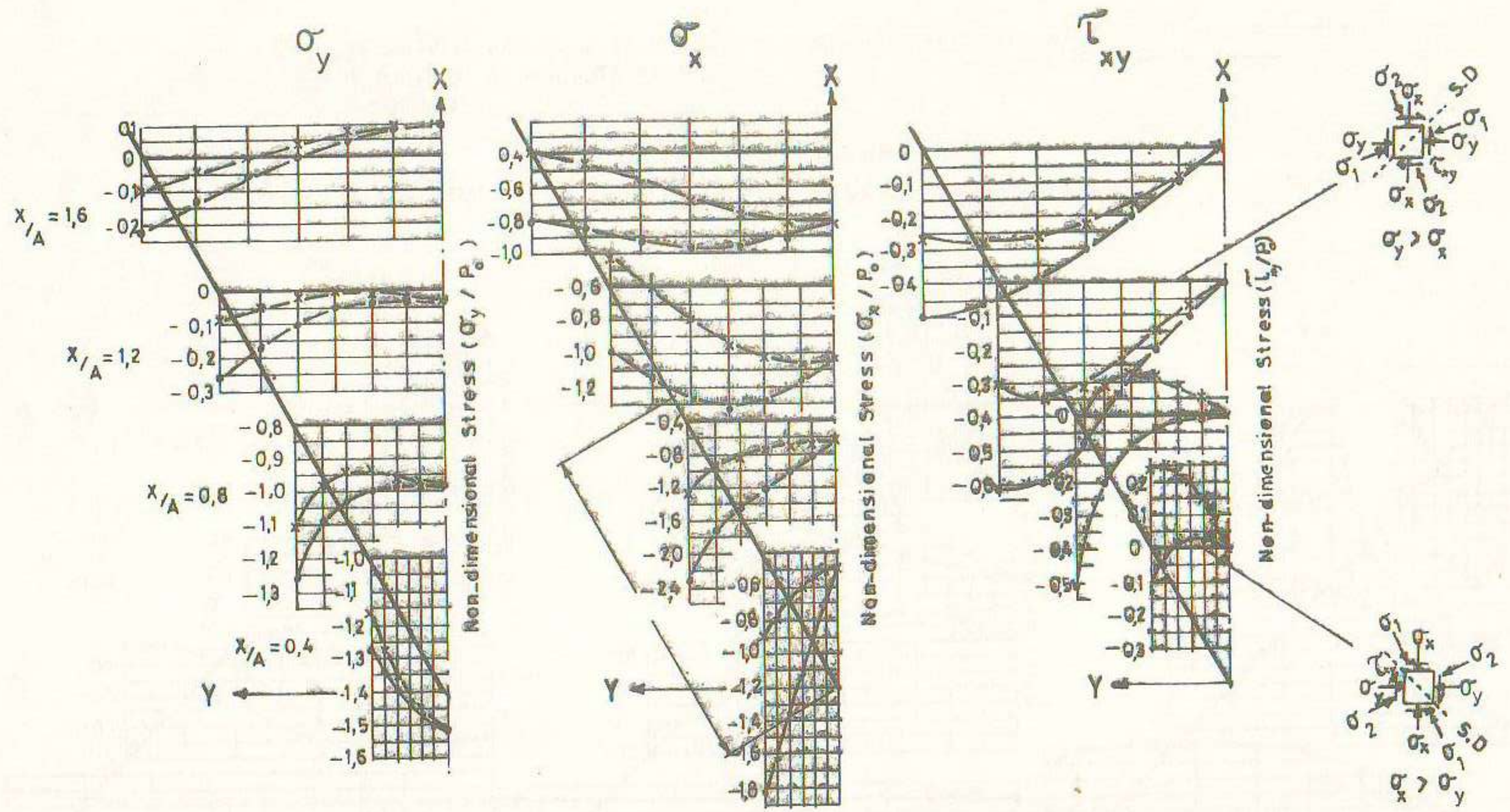

Fig 12 Distribution of Rectangular Stress Components in a 60° Wedge

LOADED BY:

a) Uniform Normal Pressure 'P_o' ——x——x——x——

b) Uniform Normal Pressure and Uniform Shear Stress 'T_o' ——●——●——●

[$T_o = 0,2\,P_o$]

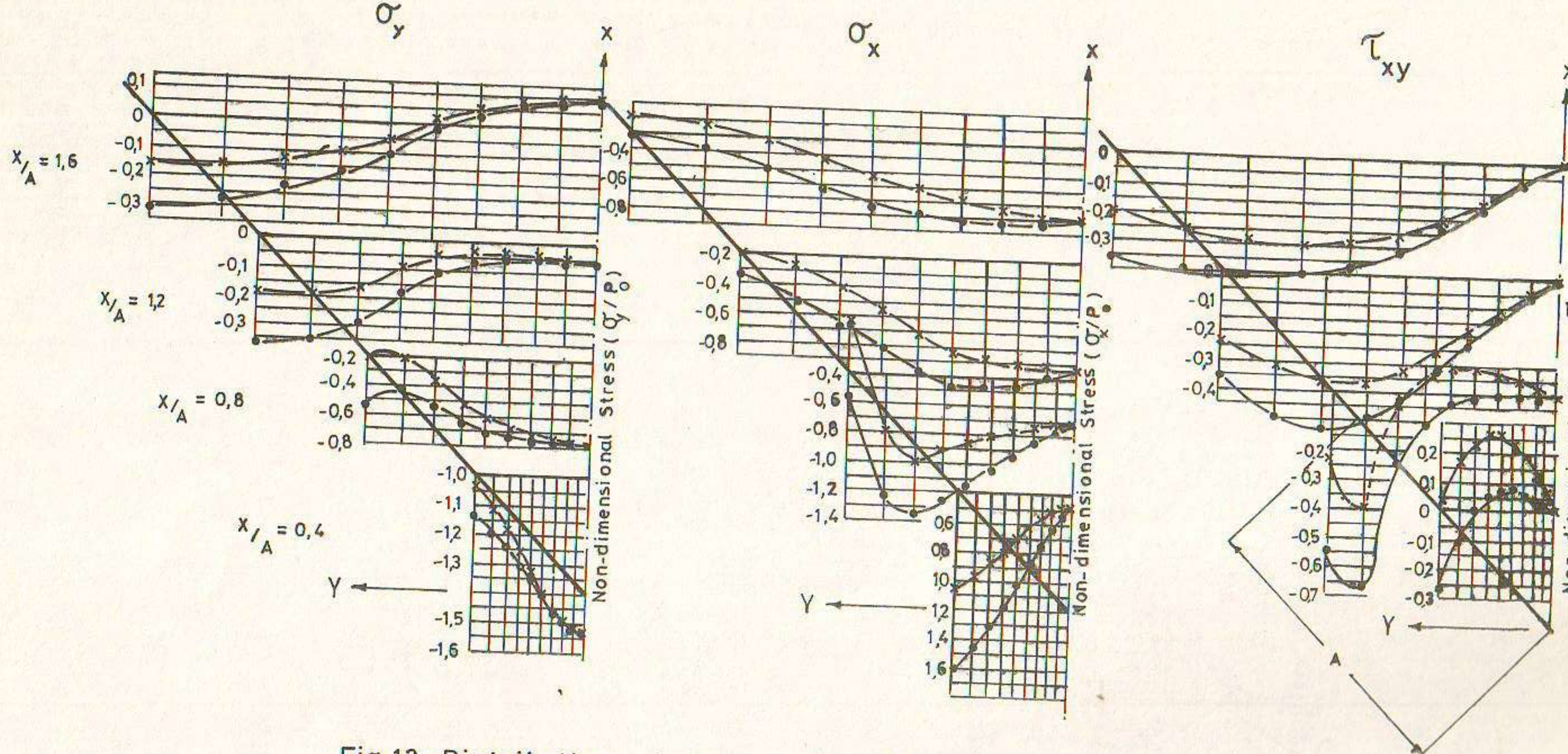

Fig.13 Distribution of Rectangular Stress Components in a 90° Wedge.

LOADED BY :

a) Uniform Normal Pressure 'P'

b) Uniform Normal Pressure 'P_o' and Uniform Shear Stress 'T_o'