MATHEMATICAL ANALYSIS OF THE PERFORMANCE OF CYLINDRICAL – PARABOLIC SOLAR CONCENTRATOR

BY
R.A.M. AL – Ashoor
School of Control & Systems
Univerisy of Technology.

https://doi.org/10.30684/etj.1983.1.8
2412-0758/University of Technology-Iraq, Baghdad, Iraq

ملخص البحث :

يتناول البحث التحليلات النظرية لاداء المركزات الشمسية ذات القطع المكافيء الأسطواني الشكل . مع خصوصية في التركيز على التطبيقات العملية لهذه المركزات في مدى درجات الحرارة المتوسطة (اقل من ١٠٠° م) حيث يتم استخدام المستقبلات بكثرة . ان معاملات مناسبة تصف اداء المركزات قد تم تعريفها وقد تم تخمين تأثير كل متغير على الاداء الكلي للمركزات .
ولكي توضع هذه التحليلات بشكل عام حيث يمكن استخدامها لكافة الابعاد . وضعت كافة النتائج بصيغة غير بعديـــة
كما ان نتائج التحليلات وضعت بهيئة مجموعة رسوم . من الممكن استخدامها بسهولة في تصميم المركزات الشمسية ذات القطع المكافيء .

Mathematical Analysis of the Performance of Cylindrical - Parabolic Solar Concentrator

R·A·M· AL -Ashoor
School of Control & Systems
Unverisy of Technology.

Abstract

The paper gives a theoretical analysis of the performance of cylindrical parabolic solar concentrators. Particular attention is given to the application of this type of concentrator in the intermediate temperature range(<100°C) where wide receivers are used.

Suitable indicies that describe the performance of the concentrator are defined and evaluated. The effect of individual parameter on total concentrator performance is investigated. In order to put these analyses to a universal use the non – dimensional approach of mathematical formulation is used .

The results of the analysis are presented in a set of graphs which can be used easily for solar concentrator design

Introduction

The solar collector is the item of equipment which converts solar energy to some other useful form of energy. Of the solar thermal collectors in use two types are dominent

1) The Flat Plate Collector .
2) The Solar Concentrator

The flat plate collector receives both direct and diffused energy without any concentration, this makes these types of collectors suitable for low temperature applications .

On the other hand, solar concentrators concentrate the received energy by its large surface area on a small absorber area. It has higher efficiency per unit area in comparison with flat plate collectors and a wider range of temperature operation .

Parabolic type concentrators have been studied in some details by many authors[(1–3)] and the theoretical performance is derived using the assumption of a uniform - intensity solar disk and that the sun's rays are received in the form of a cone With an apparnt diameter of the sun constant at 32° min of angle .

The present work is a study of the performance of cylindrical parabolic concentrators when used for relatively low temperatures. An integral expression is derived for the power concentrated by the collector in terms of the dominent parameters affecting the collector performance. The results are presented in a non–dimensional format such that the effect of each individual parameter on the system performance can easily be seen .

Assumptions :

The analysis of power received by the receiver is made under the following assumptions :

1) The cone angle of the sun rays is zero .
2) The reflecting surface is continuous perfect parabola .
3) Reflectivity is constant all over the reflecting surface and is independent of the incident angle .
4) The centre line of the receiver coincides with the focal line of the parabolic cylinder .

Power concentrated and the Rim angle

Referring to Fig . (1) the solar power concentrated , dP , by an elemental area dA , for a position angle θ is given as :[(1)]

$$dP = I\rho \cos(\theta / 2)\, dA \qquad (1)$$

Where I is the solar intensity in K w m²

ρ is the reflectivity of the reflecting surface . From . (1) the elemental area dA is deduced as

$$dA = L\, dS \qquad (2)$$

where L = length of the solar concentrator in meters and S is the rim length of the parabola in meters. From Fig . (1) and by using polar coordinates the elemental rim length dS is

$$dS = \left[r^2 + \left(\frac{dr}{d\theta} \right)^2 \right]^{1/2} d\theta \qquad (3)$$

also the parabola equation in polar coordinates is

$$r = 2f / (1 + \cos\theta) \qquad (4)$$

by differentiating Eq (4) w .r.t. θ we get : –

$$\left(\frac{dr}{d\theta} \right)^2 = r^2 \frac{\sin^2\theta}{(1 + \cos\theta)^2} \qquad (5)$$

Substituting Eq (5) for $\frac{dr}{d\theta}$ in Eq (3) and simplifying we get:

$$dS = r \left(\frac{2}{1 + \cos\theta} \right)^{1/2} d\theta$$

the above equation can be re - written in terms of $\theta / 2$ as

$$dS = r \sec(\theta / 2)\, d\theta \qquad \ldots \quad (6)$$

Now by substituting Eq (6) for ds in Eq (2). dA can be determind as

$$dA = Lr \sec(\theta / 2)\, d\theta \qquad \ldots \quad (7)$$

The power received by elemental area dA thus can be obtained by substituting Eq (7) for dA in (1) therefore

$$dP = I\rho\, Lr\, d\theta$$

and using the value of r from Eq (4) in above Eq.

$$dP = \frac{2I\rho Lf}{1 + \cos\theta}\, d\theta \qquad \ldots \quad (8)$$

The total power concentrated can be obtained by integrating Eq (8) w. r. t. θ between zero and the rim angle θm thus

$$P = 4 I \rho f L \tan(\theta m / 2) \qquad \ldots \quad (9)$$

Eq. (9) gives the total power concentrated on the absorber surface. which for the purpose of the above analysis is considered as a line passing through the focal line of the concentrator.

The total power concentrated therefore is affected by the following parameters.

1) Solar radiation intensity. I
2) Reflectivity of the reflector surface. ρ
3) Focal length of parabolic concentrator. f
4) Length of the parabolic concentrator. L
5) Rim angle θm.

Dimensionless Power Pd .

Analysis of Eq (9) shows that the power concentrated is linearly proportional to the first four parameters, these parameters are included under a new variable which is called **Dimensionless Power** , Pd

$$Pd = P / I \rho f L = 4 \tan (\theta m / 2) \qquad \ldots (10)$$

This expression gives a direct relationship between power and the rim angle θm whatever the variation in other parameters . Fig. (2)
shows the variation of Pd with θ m .

Effect of Width of the Receiver

Under practical conditions part of the reflecting surface does not receive any radiation since it is shaded by the receiver as shown in Fig. (3) Referring to Fig (3), (ψ) will be known as the shading angle and the width of the receiver can be found as

$$w = 2f \tan \psi/2$$

$$\text{therefore} \quad \psi = 2 \tan^{-1} (w / 2f) \qquad \ldots (11)$$

for a given f, Ψ can vary between 0° and 90° such that w extends from 0 up to 2f.

The width of the concentrator from Fig (3) is

$$D = x \Big|_{\theta = \theta_m} = 2f \tan (\theta m / 2) \qquad \ldots (12)$$

The ratio between the width of the concentrator to that of the receiver is defined here as the width ratio and denoted by R, where

$$R = \frac{D}{W} = \frac{\tan (\theta m / 2)}{\tan (\psi / 2)} \qquad \ldots (13)$$

for all values of ψ from $\psi = 0°$ to $\psi = 90°$

Fig (4) shows the width ratio variation with the shading angle for various rim angles θ m .

By taking the shading angle into account in Eq (9) we can give power concentrated P_{nt} as

$$P_{nt} = 4 I \rho f L (\tan (\theta m / 2) - \tan (\psi / 2)) \qquad \ldots (14)$$

The above equation also can be written in a dimensionless form,

$$\text{. thus } P_{ntd} = 4 (\tan (\theta m / 2) - \tan (\theta m / 2)) \qquad \ldots (15)$$

Using Eq. (13), P_{ntd} can be written as:

$$P_{ntd} = 4 \tan (\theta m / 2) (1 - 1 / R)$$

This is plotted in Fig (5) and it can be seen that the net power concentrated P_{ntd} increases rapidly for values of R less than 10 and it remains fairly constant at values of R greater than 10 .

Concentration Efficiency :

Concentration efficiency is defined as

$$= \frac{\text{Power received by the receiver}}{\text{Power incident on the planar area of the concentrater}}$$

from Fig (3) and Fig (1) and by using Eq. (12) the planar area of the concentrator is

$$A_p = 4\,f\,L\tan(\theta m/2) \qquad \ldots (16)$$

and the total power received by this area is

$$P_r = 4\,I\,f\,L\tan(\theta m/2) \qquad \ldots (17)$$

where I = normal-incident radiation in KW/m^2

for a line absorber concentrator the concentration efficiency depends on reflectivity and uniformity of the cylinderical conentrator . i.e.

$$\zeta = \rho E$$

where E is the dispersion factor. By introducing the shading effect the concentration efficiency can be written as

$$\zeta = \rho E\,\frac{(\tan(\theta m/2) - \tan(\psi/2))}{\tan(\theta m/2)}$$

and by using Eq. (13)

$$\zeta = \rho E(1 - 1/R) \qquad \ldots\ldots\ldots (18)$$

Now by, taking ρ E as constant parameters the variation of the concentration efficiency with R is shown in Fig. (6), this confirms the results obtained earlier that for values of R greater than 40 the effect of the receiver width on the efficiency is minimal.

Variation of Surface Area with Rim Angle

The area of the reflecting surface, A, is an important parameter in the design of the solar concentrator. Referring to Fig. (1) the elemental area of the reflecting surface is :-

$$dA = L\,ds$$

by substituting Eq. (4) for r in.(6), dS can obtained as

$$dS = \frac{2f}{1+\cos\theta}\sec(\theta/2)\,d\theta$$

and in terms of $\theta/2$

$$dS = f\sec^3(\theta/2)\,d\theta \qquad \ldots\ldots(19)$$

By integrating both sides w.r.t. θ from $(O - \theta_m)$ and multiplied by 2 one can find S as :

$$S = 2f(\tan(\theta m/2)\sec(\theta m/2) + L_n(\tan(\theta m/2) + \sec(\theta m/2))) \qquad \ldots\ldots\ldots\ldots (20)$$

and the collector area thus is

$$A = 2\,Lf(\tan(\theta m\ 2)\sec(\theta m\ 2) + L_n(\tan(\theta m\ 2) + \sec(\theta m\ 2))) \qquad \ldots\ldots\ldots\ldots (21)$$

In the dimensionless form A can be written as

The plot of variation of A with θm is shown in Fig. (7). Here the surface area increases rapidly for rim angles greater than 90°. From Fig (2) greater power can be obtained for $\theta m > 90^\circ$. Reducing the rim angle to values less than 90° results in a saving in the reflecting surface. This also reduces the power concentrated. An engineering compromise usually is made hereby using Fig. (8) which is a plot of rate of increase of reflecting surface w.r.t. rim angle $\frac{dA_d}{d\theta_m}$ and a plot of rate of increase of the power concentrated $\frac{dP_d}{d\theta_m}$ w.r.t. θm The graph shows that for $\theta m > 90^\circ$ $\frac{dP_d}{d\theta_m}$ increases more than $\frac{dP_d}{d\theta_m}$ for the same value of $d\theta_m$.

The Overall Collector Efficiency

Collector efficiency is defined as the ratio of the power concentrated by the concentrator on the absorber surface to the power absorbed by the fluid passing through the receiver. i.e.

$$\mu = \frac{\text{Energy Absorbed}}{\text{Energy Concentrated}}$$

In a medium range of temperature operation under atmospheric pressure and temperature less $100^\circ C$ the energy absorbed by the liquid flowing through the absorber can be given as

$$m \cdot C_p \cdot \Delta t \quad \text{........} \; K_j / hr \; \text{..............} (22)$$

where $\dot{m}$ = flow rate of the fluid (Kg / hr)

C_p = specific heat of the fluid (Kj / Kg. C°)

t = difference in temperature between I /O of abeorber tube.

The net power concentrated is

$$= \frac{m \, C_p \, \Delta t}{4 \, I \rho f \, L (\tan \theta m/2) - \tan (\psi/2) \, 3600} \quad \text{...........} (23)$$

Eq. (23) presents the efficiency of the concentrator in terms of the above defined parameters .

Conclusions

To design parabolic cylinderical concentrators in the medium temperature range, certain values such as the rim angle, focal length surface area and etc. have to be selected. The analysis and the graphs presented in this paper enable the designer to select the dimensions required for the optimum concentrator optimaly . The analyses are performed in a dimensionless form as to decouple the effect of other parameters when study is made on the effect of a specific parameter on the overall system.

Reference

1 – K. Hassan, M·F· El – Refaie
"The oretical Performance of Cylinderical Parabolic Solar Concentrator" Solar Energy 15 – 1973

2 – D·L· Evano
"On the Performance of Cylinderical Parabolic Concentrator with Flat Absorbers"
Solar Energy , Vol 19 – 1977

3 – Parmpal Singh and L·S· Cheema.
" Performance and Optimization of Cylinderical Parabolic Collectors ".
Solar Energy. Vol. 18 – 1976

List of Symbols Used

A	Surface area of the reflector	m^2
A_d	Dimesionless area of the reflector	
A_p	Planar area of the reflector	m^2
R	width ratio	
P	Power concentrated	KW
P_d	Dimensionless power	
P_{nt}	Net power received by the receiver	
P_{ntd}	Dimensionless net power received by the receiver	
P_{sh}	Shading power	KW
P_r	Power received by the concentrator surface	KW
θ	Position angle	degrees
θ_m	Rim angle	degrees
S	Rim length of the parabola	m
f	Focal length of the reflector	m
L	Length of the concentrator	m
I	Intensity of solar radiation	KW/m^2
ρ	Reflectivity of the reflecting surface	
W	Width of the receiver	m
ψ	Shade angle	dagrees
D	Aperture	m
ξ	Concentrator efficiency	
μ	Collector efficiency	
E	Dispersion factor	

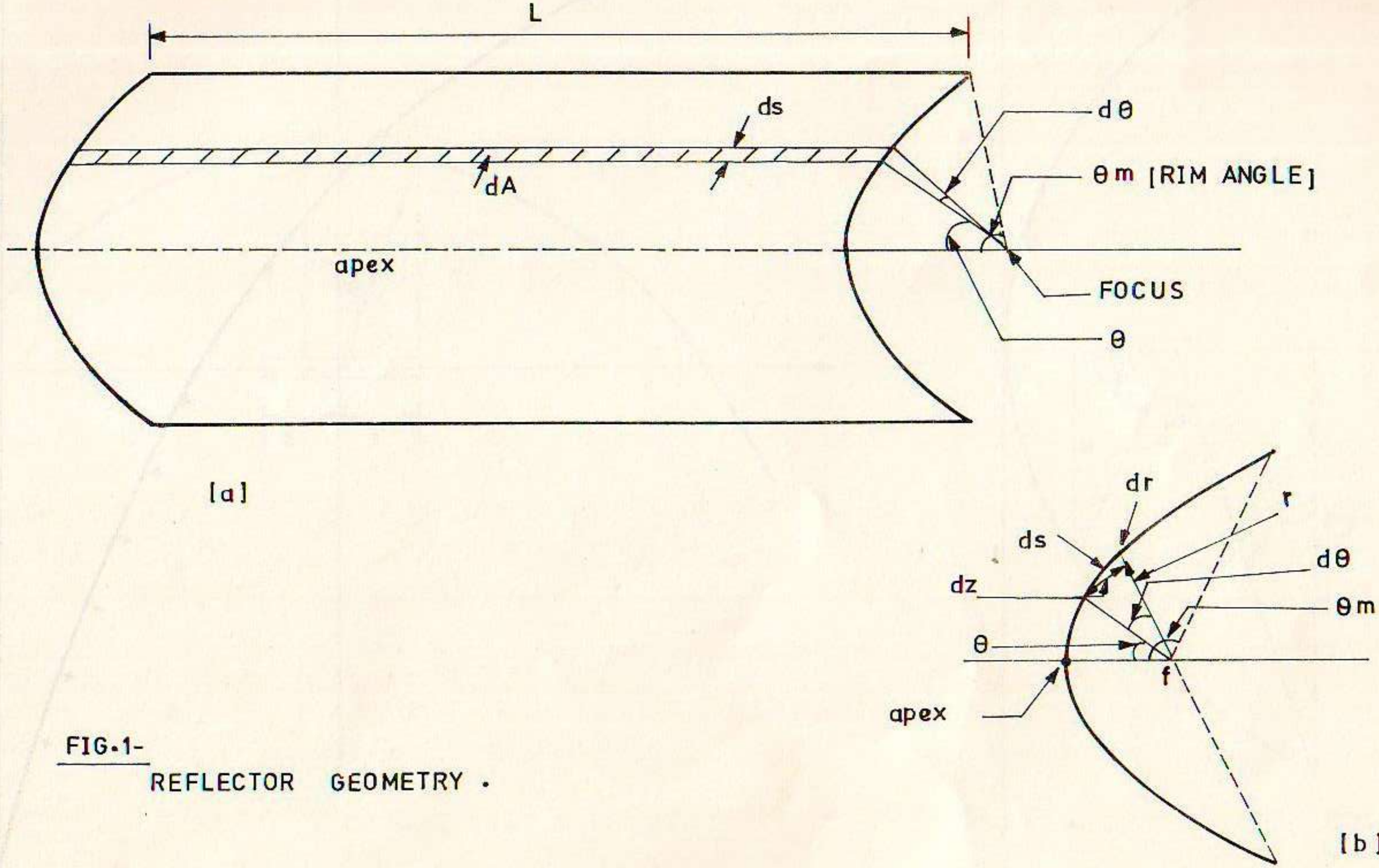


FIG.1- REFLECTOR GEOMETRY .

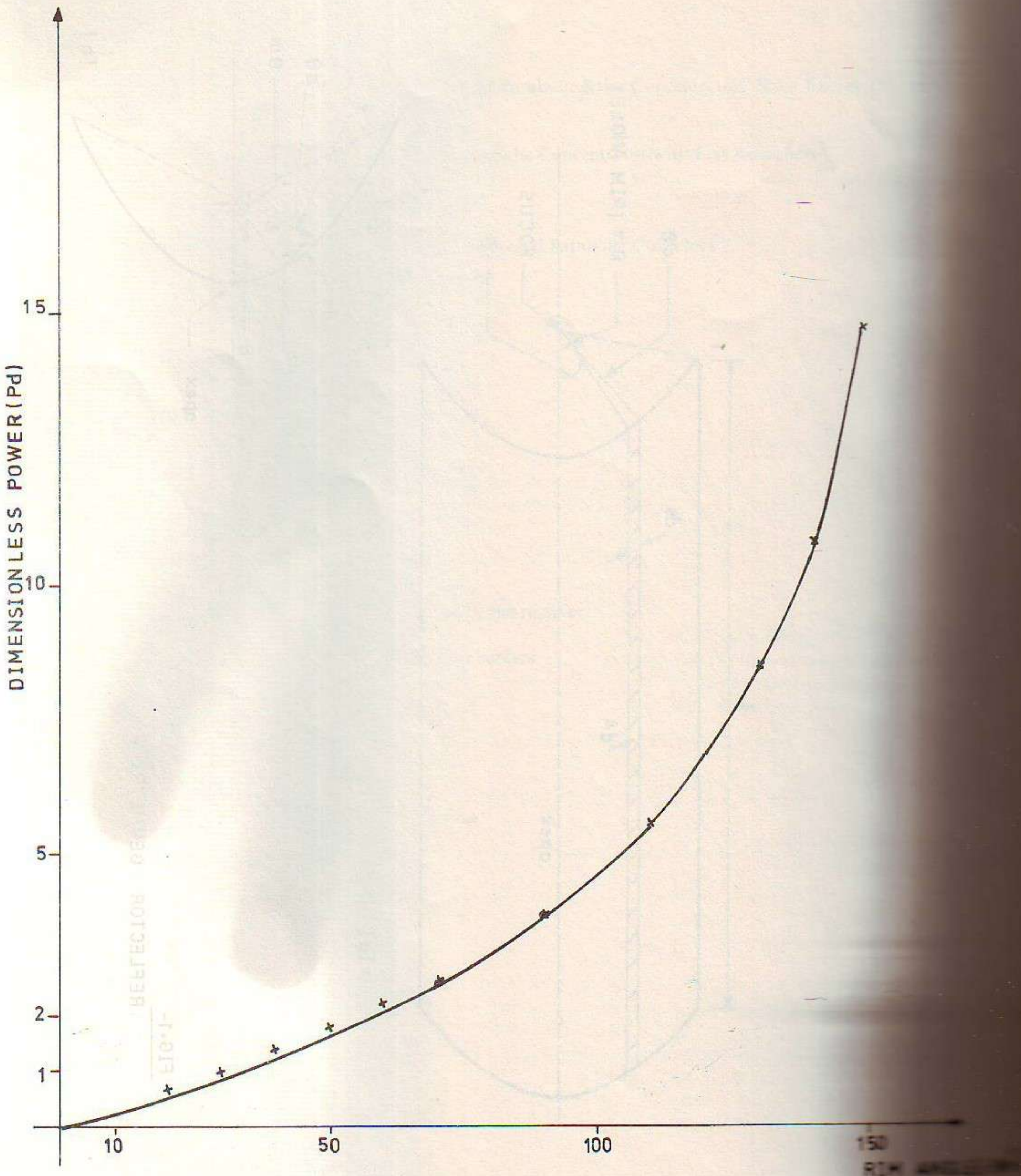
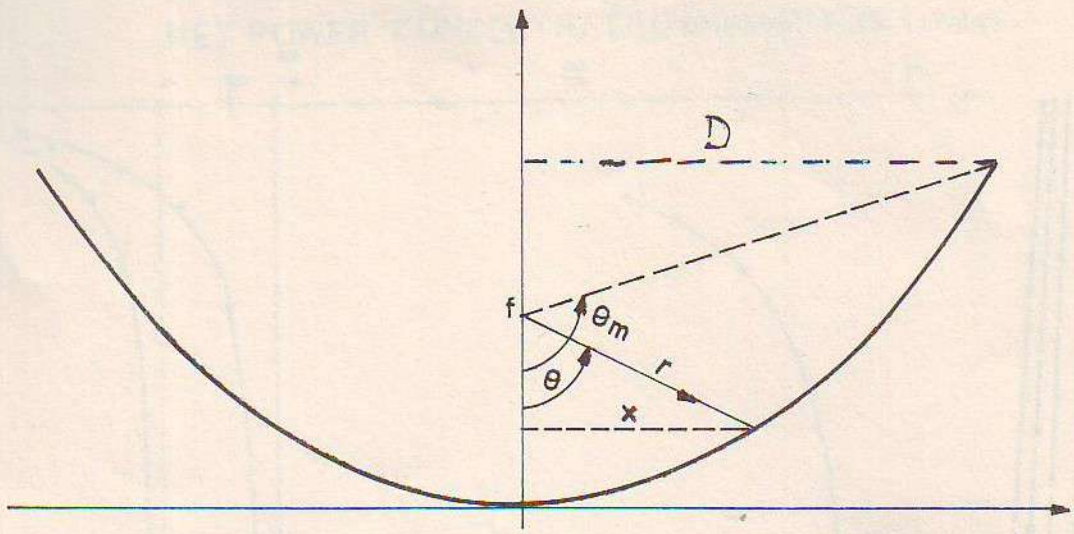


FIG.2- VARIATION OF DIMENSIONLESS POWER WITH THE RIM ANGLE θm .



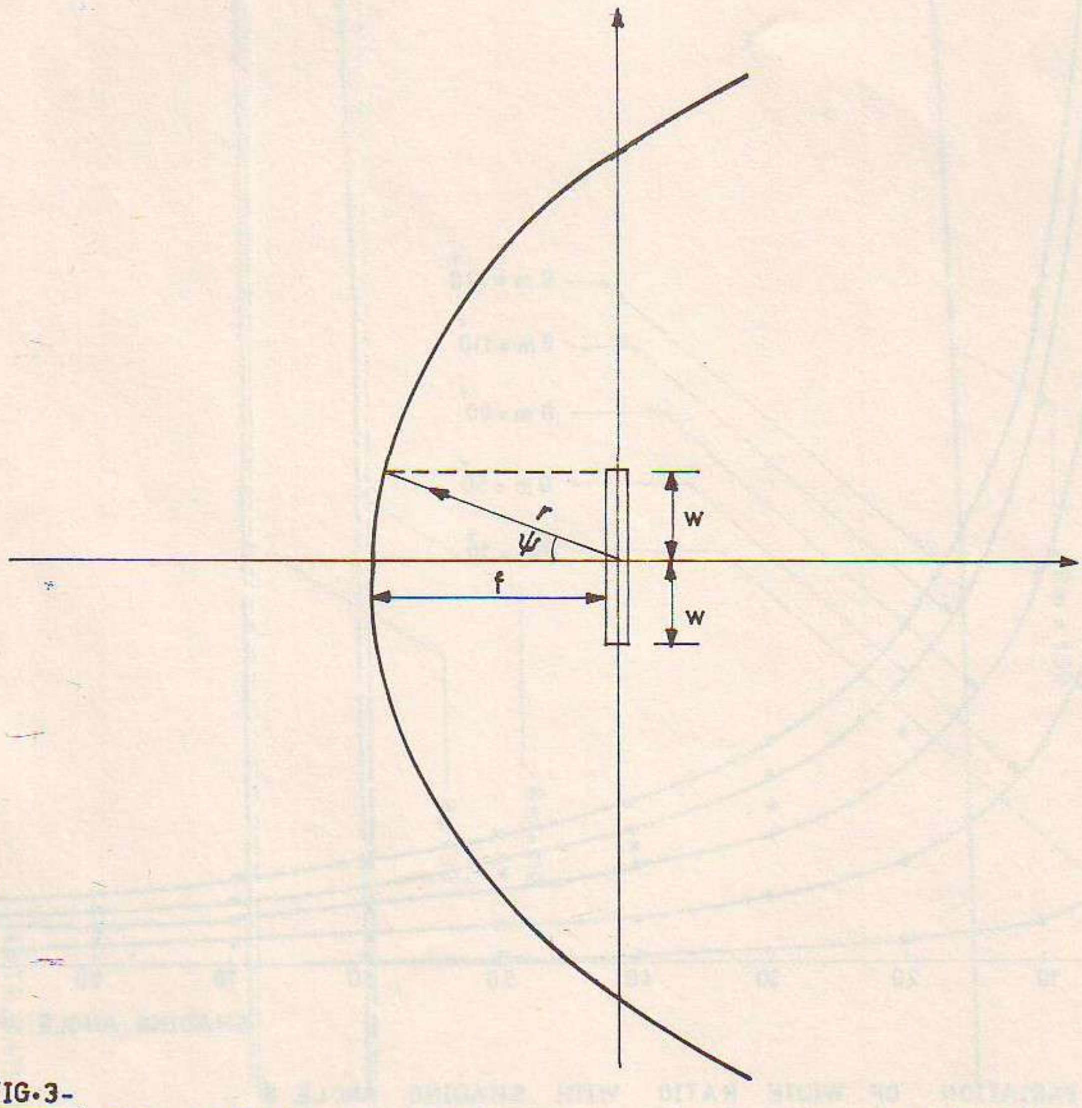


FIG.3- THE EFFECT OF WIDTH RATIO ON POWER CONCENTRATED .

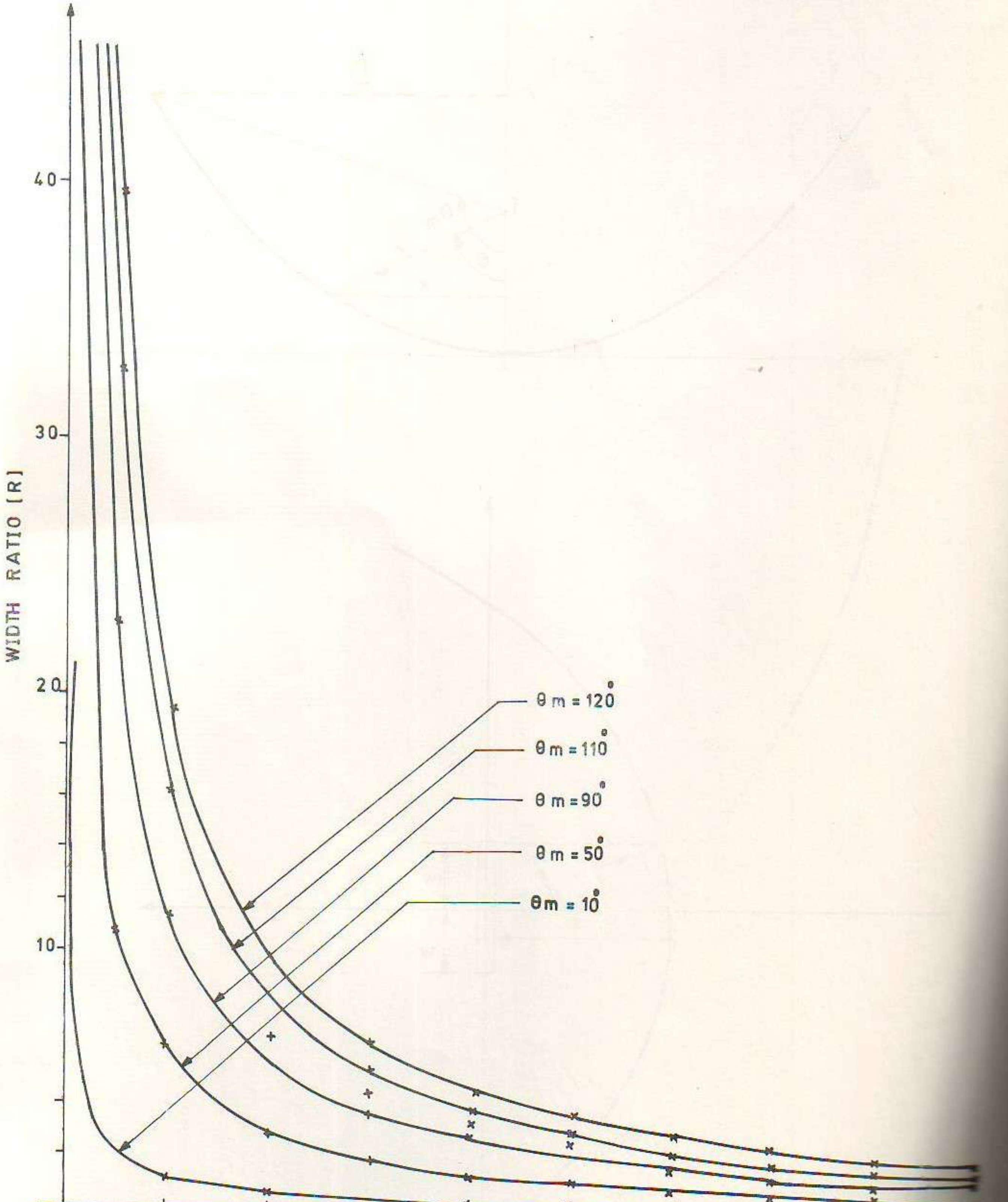


FIG.4. VARIATION OF WIDTH RATIO WITH SHADING ANGLE ψ .

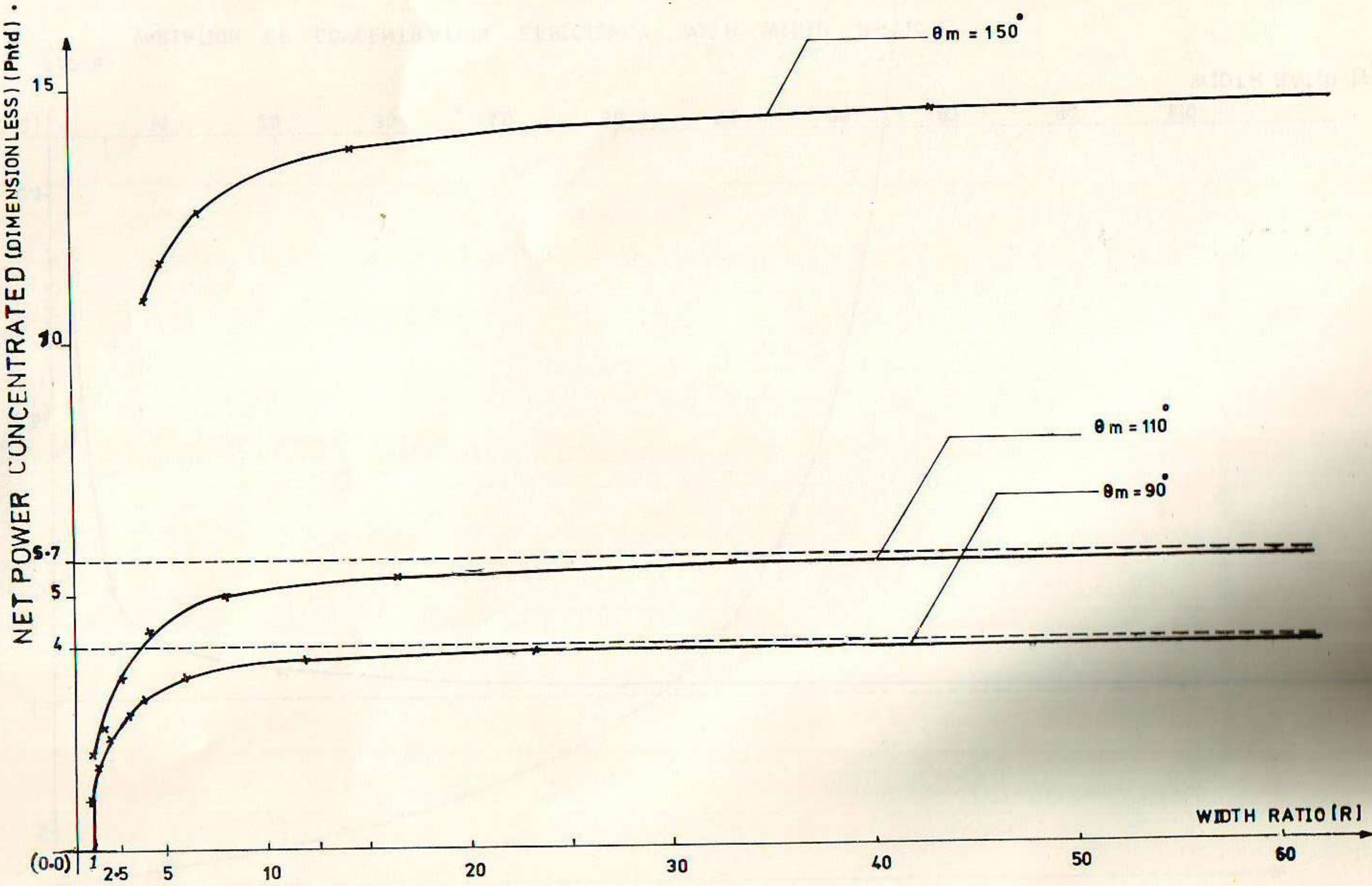


FIG.5- VARIATION OF NET POWER RECIEVED WITH WIDTH RATIO FOR VARIOUS θm .

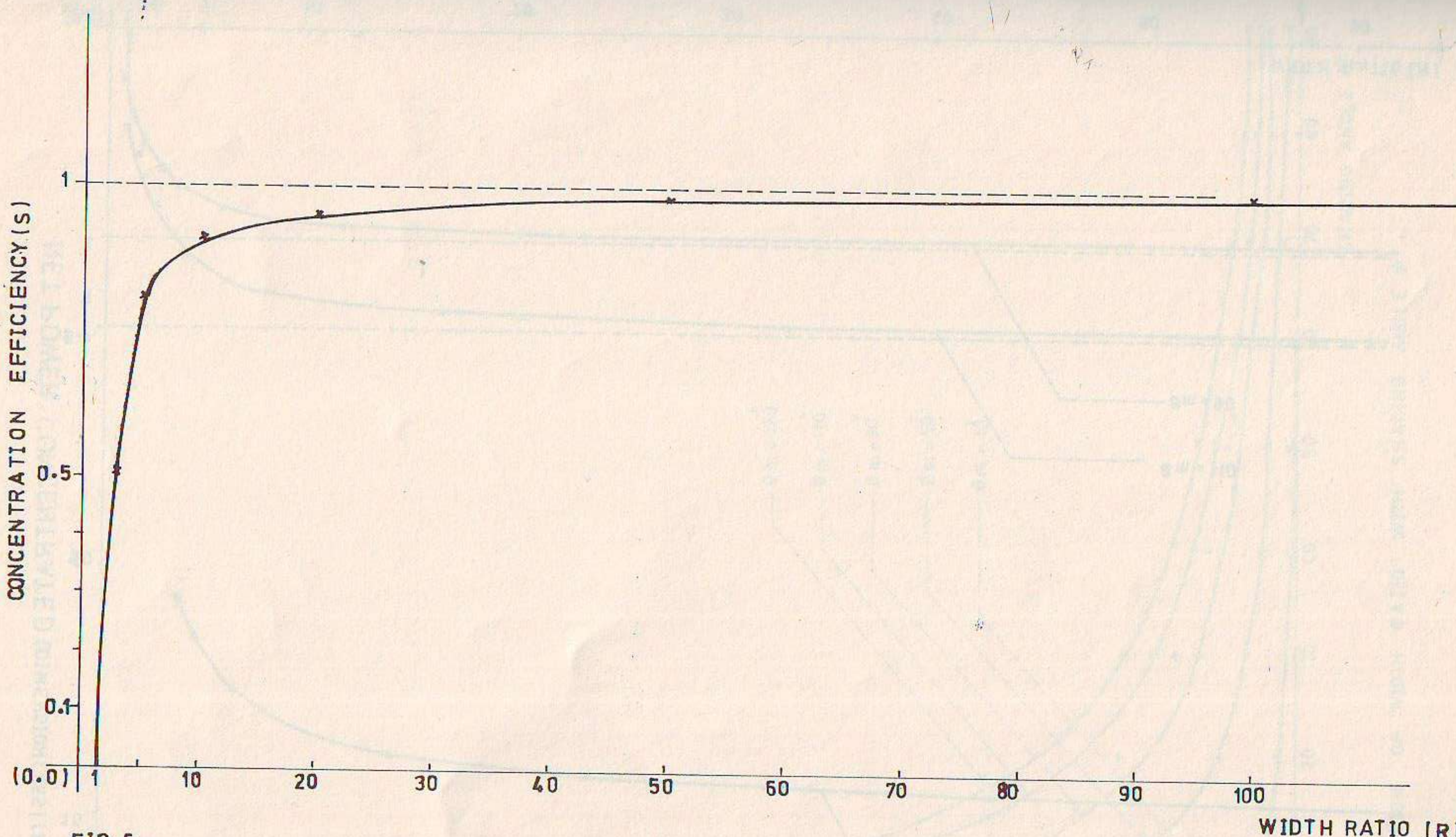


FIG.6- VARIATION OF CONCENTRATION EFFICIENCY WITH WIDTH RATIO.

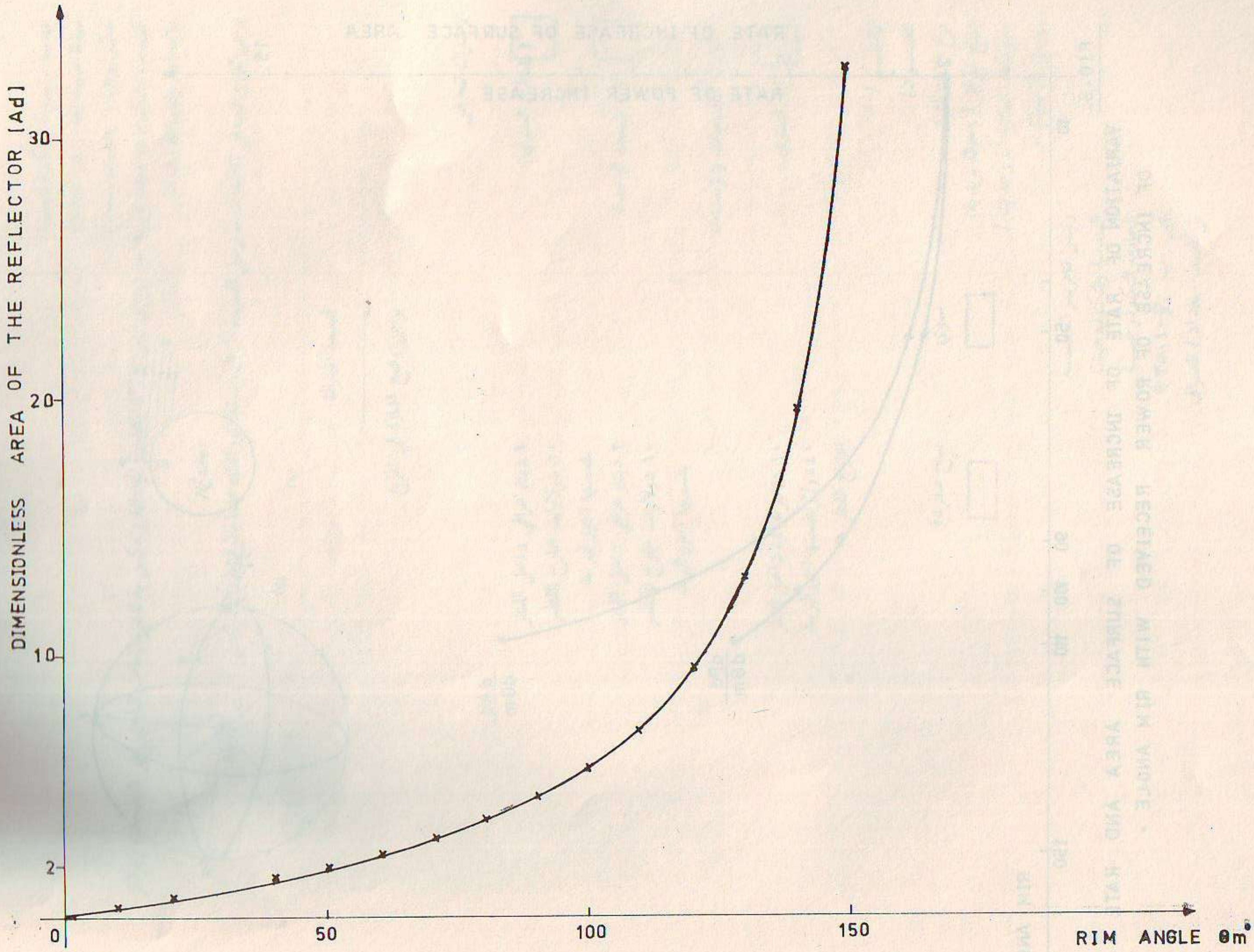


FIG.7- VARIATION OF THE AREA OF THE REFLECTOR WITH THE THE RIM ANGLE .

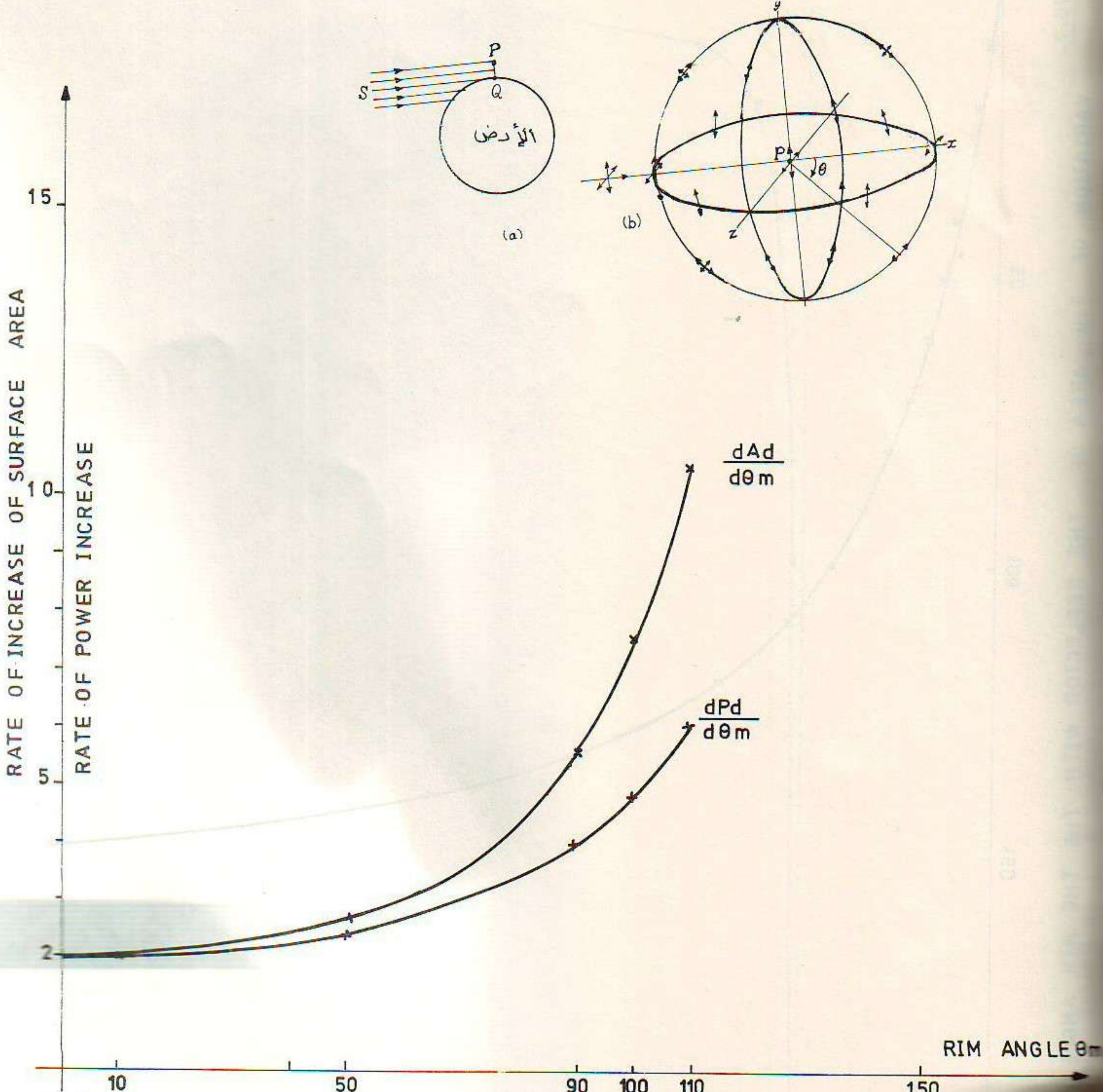


FIG.8_ VARIATION OF RATE OF INCREASE OF SURFACE AREA AND RATE OF INCREASE OF POWER RECEIVED WITH RIM ANGLE.