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Formation Control of Mobile Robots Using Sliding Mode Control Based on Conditional Servocompensator

Abstract- *In this paper, the problem of controlling a group of mobile robots is considered; each one operates with the nonlinear nonholonomic under actuated dynamics. A coordinated control scheme is designed based on leader-follower(s) method to achieve prescribed formation maneuvers. This objective is fulfilled using the approach of sliding mode controller based on conditional servocompensator, which will bring the system error trajectories into a positively invariant set (boundary layer) close to the origin. Then, a special form of servocompensator conditional integrator, which will be active only inside the boundary layer, will regulate the trajectories to the origin in finite time. Compared to the traditional integral which will deteriorate the performance of the feedback system, the conditional version will have a very insignificant effect on the performance. The simulation results show that the designed controller is able to achieve the objective efficiently with very reasonable control actions.*

Keywords: *Mobile Robot, Sliding Mode Controller.*

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1. Introduction

Formation control represents one of the major problems in the Multi-agent control systems field. In this problem, it is required to design a controller (either centralized or decentralized) to define a coordination of an ensemble of agents. In other words, it is required that the agents to maintain certain inter-agent distances to form specified geometrical shapes [1,2]. Maintaining certain shape while moving has many advantages; for example it can reduce the system cost, increasing the robustness and efficiency of the system and providing redundancy while moving, and also provides flexibility and reconfiguration capability [3-6]. Formation control has been utilized in various applications; like transportation of large dangers objects, surveillance mapping, search, rescue, or large data acquisition .[7]

One of the most popular schemes to deal with the problem of formation control is the leader-follower(s) scheme [1,8]. The basic idea of this scheme is that to designate a leader robot; either one of the formation group, or a fictitious leader to be the responsible for guiding the formation. One important advantage of this approach is the simplicity of the approach, because the leader’s motion directs the group as reference trajectory, and the internal formation stability is induced based on individual agents’ control laws [7].

There are a variety of control design methods have been explored and tested in literature; including for example, the use of feedback linearization [1], dynamic feedback linearization [9], and high gain observer [10]. In ref. [11], the authors designed a formation controller using an approach known as terminal sliding mode control, which involves using non-smooth sliding surfaces. This sacrificing of the smoothness of the sliding surfaces came into the advantage of achieving faster convergence time compared to regular sliding mode approach. However, there is a price from using such technique is that the singularities in the state space. To remedy this problem, the authors proposed an approach to partition the state space into two regions, one of them the controller is bounded and the other will be its complement .

In our work, we use the approach of designing a continuously-implemented sliding mode controller based on conditional servocompensator to design the coordination controller. The conditional servocompensator is introduced in 2005 by Seshagiri and Khalil [12], where in their work a special form of internal dynamics (internal model) is implemented to act with a continuously sliding mode controller. In general, continuous sliding mode control will derive the system state variables towards a positively invariant set around the origin, which represents the boundary layer. The conditional servocompensator will be active

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only inside this layer, so that the internal model states and the steady-state control action will be relatively small. The benefit behind that the servocompensator will have a very slight effect on the transient performance of the system under control. In other words, the controller will recover the performance of a discontinuous sliding mode controller and will render a zero-error equilibrium with a continuous version of sliding mode control. This actually is a great benefit compared to the issues resulted due to the discontinuous term of the sliding mode like chattering behavior, which considered undesirable from the practical point of view. Yet the controller will preserve the ability to control the system with the same robustness properties as the discontinuous version .

In this work, this idea is been invested to design a continuously-implemented sliding mode formation control law and the results are compared with the ones obtained in the work of ref. [11]. The comparison show comparable results in terms of performance and stability with reasonable control actions. With the benefit of this work to have two privileges over the method of ref. [11], which are: in this work we don't need to have non-smooth surface manifolds, we will just use a linear surfaces form, therefore we don't expect to see singularities on the state- space , and the 2nd important privilege is that we can obtain continuous sliding mode controller in finite time instead of the asymptotic convergence like the one in.[11]

The remaining of the paper is outlined in the following way; in the next section the derivation of the mobile robot agent dynamics is provided. In section 3, the derivation of classical discontinuous version of the controller is presented. Section 4, shows how to modify the control laws and sliding manifolds to achieve the control objective with continuous sliding mode controller version. The simulation results is shown in section 5, finally, section 6, ends with the paper conclusions.

2. Mobile Robot Agent Dynamics

In this section, the model dynamics of each mobile robot agent subjected to nonholonomic constraint is derived. This nonholonomic constraint is considered due to the condition of no slip in the normal direction to the robot path [11], this will simply means that the motion of each wheel is restricted to the longitudinal direction with the linear velocities of wheels [13].In other words, no motions occurs in robot lateral coordinates. In Fig. (1), the free-body diagram of the mobile robot is presented. The center of robot mass horizontal and vertical position coordinates (in meters) with respect to the inertial frame are denoted by the variables and, respectively. The orientation angle is determined by the variable θ (in radians).

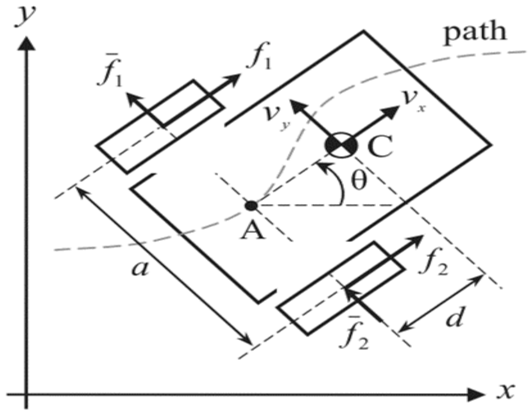

Figure 1: Mobile robot free-body diagram [11].

The mobile robot model considered here have the differential scheme, where it is derived using two wheels on the left and right; each one is actuated using independent motor. The projections of the Newtonian equations of motion onto the body-fixed frame with center of mass **C** are given by [11];

$$\sum F_x = m\dot{v}_x(t) - m\dot{\theta}(t)v_y(t) = f_1(t) + f_2(t) \quad (1)$$

$$\sum F_y = m\dot{v}_y(t) - m\dot{\theta}(t)v_x(t) = \bar{f}_1(t) + \bar{f}_2(t) \quad (2)$$

$$\sum M_{C_z} = I\ddot{\theta}(t) = \frac{L}{2}\big(f_2(t) + f_1(t)\big) - d(\bar{f}_1(t) + (\bar{f}_2(t)) \quad (3)$$

Where;

v_x and v_y: are the projections of the velocity of the center of mass onto the body-fixed frame;

m: is the mass of robot;

I: is the robot's moment of inertia about the axis orthogonal to the plane passing through center point **C**;

a : is the robot's axle length;

d : is the distance between mid-axle point A and the center of mass point **C**;

$f_1$andf_2: are the corresponding longitudinal friction forces acting left and right wheels, respectively; and $\bar{f}_1$ and $\bar{f}_2$: are the corresponding lateral friction forces acting left and right wheels, respectively.

The nonholonomic constraint equation is given by;

$$v_y = \dot{\theta}(t)d \qquad t \geq 0 \quad (4)$$

Differentiation of Eq. (4) with respect to time gives;

$$\dot{v}_y = \ddot{\theta}(t)d \qquad t \geq 0 \quad (5)$$

By substituting Eq. (5) back into Eq. (2), yields;

$$md\ddot{\theta}(t) + m\dot{\theta}(t)v_x(t) = \bar{f}_1(t) + \bar{f}_2(t) \quad (6)$$

By equating Eq. (6) with Eq. (3); results

$$(I + md^2)\ddot{\theta}(t) = md\dot{\theta}(t)v_x(t) + \frac{L}{2}\big(f_2(t) + f_1(t)\big) \quad (7)$$

Now, considering the kinematics of the wheels which are given by;

$$v_x(t) = \frac{1}{2}\left(V_1(t) + V_2(t)\right) = \frac{r}{2}\left(\dot{\emptyset}_1(t) + \dot{\emptyset}_2(t)\right) \quad (8)$$

$$\dot{\theta}(t) = \frac{1}{L}\left(V_2(t) - V_1(t)\right) = \frac{r}{L}\left(\dot{\emptyset}_2(t) + \dot{\emptyset}_1(t)\right) \quad (9)$$

Where;

$V_1(t), V_2(t)$: are the corresponding linear velocities for the right and left wheels, respectively;

$\emptyset_1(t), \emptyset_2(t)$: are the corresponding angular velocities for the right and left wheels; respectively; and

r : is the radius of wheels;

By differentiating Eq. (8) and (9) with respect to time, results;

$$\dot{v}_x(t) = \frac{r}{2}\left(\ddot{\emptyset}_1(t) + \ddot{\emptyset}_2(t)\right) \quad (10)$$

$$\ddot{\emptyset}(t) = \frac{r}{L}\left(\ddot{\emptyset}_2(t) + \ddot{\emptyset}_1(t)\right) \quad (11)$$

We have the dynamics of each wheel are given by;

$$J\ddot{\emptyset}_1(t) = \tau_1(t) - f_1(t)r \quad (12)$$

$$J\ddot{\emptyset}_2(t) = \tau_2(t) - f_2(t)r \quad (13)$$

Where;

$\tau_1(t), \tau_2(t)$: are corresponding right and left wheels actuators torques, respectively; and

J: is the rotational inertia of the wheels about their axis of rotation;

By adding and subtracting Eq. (12) and (13) together, and rearrange

$$f_1(t) + f_2(t) = \frac{1}{r}\left(\tau_1(t) + \tau_2(t)\right) - \frac{J}{r}(\ddot{\emptyset}_1(t) + \ddot{\emptyset}_2(t)) \quad (14)$$

$$f_2(t) + f_1(t) = \frac{1}{r}\left(\tau_2(t) + \tau_1(t)\right) - \frac{J}{r}(\ddot{\emptyset}_2(t) + \ddot{\emptyset}_1(t)) \quad (15)$$

The next step is to substitute Eq. (10) and (11) into Eq. (14) and (15), respectively;

$$f_1(t) + f_2(t) = \frac{1}{r}\left(\tau_1(t) + \tau_2(t)\right) - \frac{2J}{r^2}\dot{v}_x(t) \quad (16)$$

$$f_2(t) + f_1(t) = \frac{1}{r}\left(\tau_2(t) + \tau_1(t)\right) - \frac{LJ}{r^2}\ddot{\theta}(t) \quad (17)$$

By substituting Eq. (16) and (17) into Eq. (1) and (7), the equations of motion will be;

$$\left[m + \frac{2J}{r^2}\right]\dot{v}_x(t) = md\dot{\theta}^2(t) + \frac{1}{r}\left(\tau_1(t) + \tau_2(t)\right) \quad (18)$$

$$\left[I + md^2 + \frac{JL^2}{2r^2}\right]\ddot{\theta}(t) = md\dot{\theta}(t)v_x(t) + \frac{L}{2r}\left(\tau_2(t) + \tau_1(t)\right) \quad (19)$$

The projection of velocity of the center of mass of the robot onto the inertial reference frame can be obtained as;

$$\dot{x}(t) = v_x(t)\cos\left(\theta(t)\right) - v_y(t)\sin(\theta(t)) \quad (20)$$

$$\dot{y}(t) = v_x(t)\sin(\theta(t)) - v_y(t)\cos\left(\theta(t)\right) \quad (21)$$

By Substituting Eq. (5) into Eq. (20) and (21), and using Eq. (18) and (19), the complete set of equations of motion is;

$$\dot{x}(t) = v_x(t)\cos\left(\theta(t)\right) - d\omega(t)\sin\left(\theta(t)\right) \quad (22)$$

$$\dot{y}(t) = v_x(t)\sin\left(\theta(t)\right) - d\omega(t)\cos(\theta(t)) \quad (23)$$

$$\dot{\theta}(t) = \omega(t) \quad (24)$$

$$\dot{v}_x(t) = \frac{md}{\tilde{m}}\omega^2(t) + \frac{1}{\tilde{m}r}\left(\tau_1(t) + \tau_2(t)\right) + g_1(x, y, \theta, v_x, \omega) \quad (25)$$

$$\dot{\omega}(t) = -\frac{md}{\tilde{I}}\omega(t)v_x(t) + \frac{L}{2\tilde{I}r}\left(\tau_2(t) + \tau_1(t)\right) + g_2(x, y, \theta, v_x, \omega) \quad (26)$$

Where;

$$\tilde{m} = m + \frac{2J}{r^2};$$

$$\tilde{I} = I + md^2 + \left[\frac{L^2}{r^2}\right]J;$$

$\omega(t)$: Robot's angular velocity; and

g_1 and g_2: are uncertainty functions affecting robot dynamics.

3. Design of Discontinuous Sliding Mode Controller

The first step in designing the controller is to differentiate Eq. (22) and (23) with respect to time and rearrange in the following form;

$$\ddot{x}(t) = \tilde{f}_1(x, y, \theta, v_x, \omega) + u_1(t)\cos\left(\theta(t)\right) - du_2(t)\sin\left(\theta(t)\right) + \tilde{g}_1(x, y, \theta, v_x, \omega) \quad (27)$$

$$\ddot{y}(t) = \tilde{f}_2(x, y, \theta, v_x, \omega) + u_1(t)\sin\left(\theta(t)\right) - du_2(t)\cos\left(\theta(t)\right) + \tilde{g}_2(x, y, \theta, v_x, \omega) \quad (28)$$

Where;

$$u_1(t) = \frac{1}{\tilde{m}r}\left(\tau_1(t) + \tau_2(t)\right) \quad (29)$$

$$u_2(t) = \frac{1}{2\tilde{I}r}\left(\tau_2(t) - \tau_1(t)\right) \quad (30)$$

$$\tilde{f}_1 = \left(\frac{m}{\tilde{m}} - 1\right)d\omega^2\cos\left(\theta(t)\right) + \left(\frac{md^2}{\tilde{I}} - 1\right)\omega v_x\sin\left(\theta(t)\right) \quad (31)$$

$$\tilde{f}_2 = \left(\frac{m}{\tilde{m}} - 1\right)d\omega^2\sin\left(\theta(t)\right) + \left(\frac{md^2}{\tilde{I}} - 1\right)\omega v_x\cos\left(\theta(t)\right) \quad (32)$$

$$\tilde{g}_1 = g_1\cos(\theta(t)) - dg_2\sin(\theta(t)) \quad (33)$$

$$\tilde{g}_2 = g_1\sin(\theta(t)) - dg_2\cos(\theta(t)) \quad (34)$$

With this step has been done, it is needed to compensate the nominal dynamics in Eq. (27) and (28), this can be done by using the following feedback linearization control laws [11];

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} cos(\theta(t)) & -d\, sin(\theta(t)) \\ sin(\theta(t)) & d\, cos(\theta(t)) \end{bmatrix}^{-1} \begin{bmatrix} -\tilde{f}_1 + v_1 \\ -\tilde{f}_2 + v_2 \end{bmatrix} \quad (35)$$

Using the control law Eq. (35), the nominal terms $\tilde{f}_1$ and $\tilde{f}_2$ will be canceled, so that Eq. (27) and (28) became;

$$\ddot{x}(t) = v_x + \tilde{g}_1(x, y, \theta, v_x, \omega) \quad (36)$$

$$\ddot{y}(t) = v_y + \tilde{g}_2(x, y, \theta, v_x, \omega) \quad (37)$$

Where; v_x and v_y the new control inputs, which will counteract the effect of perturbations in the dynamics of Eq. (27) and (28), respectively. The objective is to design a controller that steers the system dynamics to the desired position and orientation which are defined by the fictitious leader robot dynamics, let's denote the leader robot position and orientation variables as $[x_L, y_L, \theta_L]^T$ [in this work it is assumed that the leader mobile is operated with the nonholonomic dynamics Eq. (22)-(26)]. On the same time, the formation of a group of robots is controlled with respect to leader mobile robot; therefore we define the following change of coordination;

$$e_x = x(t) - x_L(t) - l_x, \qquad t \geq 0 \quad (38)$$

$$e_y = y(t) - y_L(t) - l_y, \qquad t \geq 0 \quad (39)$$

Where; l_x and l_y are steady state differences of robot with respect to the leader's coordinate position. The proposed sliding mode functions are chosen as linear functions in the following forms;

$$s_x = c_x e_x + \dot{e}_x, \qquad c_x > 0 \quad (40)$$
$$s_y = c_y e_y + \dot{e}_y, \qquad c_y > 0 \quad (41)$$

Where the sliding manifolds for each coordinates dynamics are defined as the sets

$$\{s_x = c_x e_x + \dot{e}_x = 0\} \quad (42)$$
$$\{s_y = c_y e_y + \dot{e}_y = 0\} \quad (43)$$

Within these two sets each coordinate phase trajectory, the switching function (i.e.s_x or s_y) lines will be asymptotically stable, this can be easily proofed by using a Lyapunov function candidates V_x and V_y, on the following way;

For the x-axis coordinate dynamics;

$$V_x = \frac{1}{2}s_x^2 \quad (44)$$

The time derivative of this Lyapunov function should be negative for asymptotic stability condition is [14],

$$\dot{V}_x = s_x\dot{s}_x < 0$$
$$= s_x[c_x\dot{e}_x - \ddot{x}_L + v_x + \tilde{g}_1] < 0 \quad (45)$$

Where the first and second time derivatives of the x-axis coordinate error e_x are substituted to constitute $\dot{s}_x$. To achieve the above stability condition, the transformation $\eta$is introduced;

$$\eta = c_x\dot{e}_x - \ddot{x}_L + v_x + \tilde{g}_1 \quad (46)$$

To derive the control law we equate Eq. (47) to the following discontinuous form;

$$\eta = -k_x\, sign(s_x), \qquad k_x > 0 \quad (47)$$

Which will lead to the following control law;

$$v_x = -c_x\dot{e}_x + \ddot{x}_L - k_x - \tilde{g}_1 - sign(s_x) \quad (48)$$

Where;

$$k_x > c_x|\dot{e}_x| + |\ddot{x}_L| + |\tilde{g}_1| + max\left(\tilde{g}_1(e_x, e_y, e_\theta, v_x, \omega)\right) \quad (49)$$

For the y-axis coordinate dynamics;

Following the same above argument, we design the second control law v_y,

$$v_y = -c_y\dot{e}_y + \ddot{y}_L - \tilde{g}_2 - k_y\, sign(s_y) \quad (50)$$

Where;

$$k_y > c_y|\dot{e}_y| + |\ddot{y}_L| + |\tilde{g}_2| + max\left(\tilde{g}_2(e_x, e_y, e_\theta, v_x, \omega)\right) \quad (51)$$

Since the both control laws (Eq. (48), and Eq.(50)) has discontinuous term, this will cause a chattering

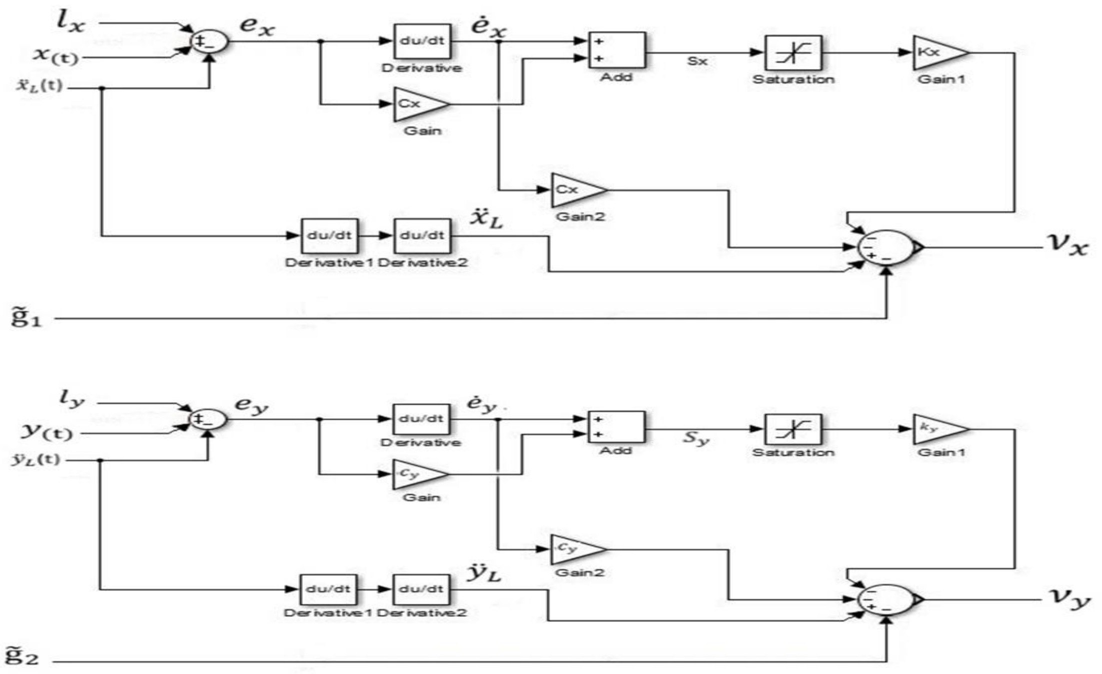

Figure 2: The block diagram for the x- and y- axes control laws.

appeared when the trajectory enters the manifold, to alleviate the chattering effect, one of the approaches used is to replace the signum function with a sigmoidal function like the saturation function [13]; therefore the control laws (Eq. (48), and Eq.(50) are modified in the following way;

$$\nu_x = -c_x\dot{e}_x + \ddot{x}_L - \tilde{g}_1 - k_x\, sat\left(^{S_x}/_{\mu}\right) \quad (52)$$

$$\nu_y = -c_y\dot{e}_y + \ddot{y}_L - \tilde{g}_2 - k_y\, sat\left(^{S_y}/_{\mu}\right) \quad (53)$$

Where the parameter $\mu > 0$ is chosen small enough. Here, in our design, it is assumed to have the boundary layer thickness to be the same for both controllers, but there is no problem of using different boundary layers thickness for each controller. In Figure (2), the schematic block diagram of control laws (52) and (53) are shown to illustrate the controller structure.

4. Design of Continuous Sliding Mode Controller Based on Conditional Servocompensator

In the previous section, we designed a discontinuous sliding mode control laws to steer each mobile robot agent. To overcome the problem of chattering behavior that arises due to the inclusion of discontinuous terms in control laws, we approximated the signum functions with a saturation functions. This approach is known as a boundary layer method [14]. This will remove the chattering, but this will come into the expense of scarifying the feedback performance, since an error steady state will not equal to zero in finite time, instead the trajectory will approach the manifold asymptotically. However, Seshagiri and Khalil [12] had proposed a solution to this dilemma by augmenting the feedback control system with a conditional integral servocompensator. Unlike the idea of reference [11], where the author has used a traditional integral servocompensator, which has the problem of performance degradation due to the effect of integral "windup". So replacing this traditional integral with a "conditional" one have shown its effect on recovering the performance of an ideal (discontinuous) sliding mode controller efficiently [12]. The conditional integrator from its name; provides integration action only inside he chosen boundary layer. In the following steps, we will show haw to modify the control laws (Eq. (52) and (Eq. (53)) by utilizing the idea of conditional integrator. The first step is to introduce the following two conditional servocompensators [12];

$$\dot{\sigma}_x = c_{xo}\sigma_x + \mu\, sat\left(^{S_x}/_{\mu}\right), \qquad c_{xo} > 0 \quad (54)$$

$$\dot{\sigma}_y = c_{yo}\sigma_y + \mu\, sat\left(^{S_y}/_{\mu}\right), \qquad c_{yo} > 0 \quad (55)$$

Where σ_x and σ_y are the servocompensator states for the x-and y-axes, respectively. The above two servocompensator integrator equations will be active only inside the boundary layers$\{s_x \leq 0\}$, and$\{s_y \leq 0\}$, respectively. The surface manifolds (Eq. (40) and (41)) have to be modified by the inclusion of the servocompensators states in the following way;

$$s_x = c_{xo}\sigma_x + c_x e_x + \dot{e}_x, \quad (56)$$

$$s_y = c_{yo}\sigma_y + c_y e_y + \dot{e}_y, \quad (57)$$

And the control laws (Eq. (52) and (53) will remain unchanged. Inside the layer the term $sat(^{s}/_{\mu}) = s,$ and the conditional servocompensator reduces to

$$\dot{\sigma}_x = c_{xo}\sigma_x + s_x \quad (58)$$

$$\dot{\sigma}_y = c_{yo}\sigma_y + s_y \quad (59)$$

Where, both Eq. (58) and (59) insures the existence of a zero-error invariant manifold for the corresponding trajectory. Each of the state of σ_x and σ_y is $O(\mu)$ (order of μ, i.e. $\sigma_x \leq k_{x\mu}s_x$, and $\sigma_y \leq k_{y\mu}s_y$, for some constants $k_{x\mu}, k_{x\mu} > 0$ [12]) for all $t \geq 0$. Therefore, the trajectories of the error functions e_x and $e_y$can be made arbitrarily close to the trajectories of ideal sliding mode controller by making μ smaller. In other words, the continuously implemented sliding mode controller ensures in finite time the trajectory will be in the neighborhood of the set $\{\sigma = 0, e = 0\}$and the error states tends to zero as t tends to infinity [please refer to Theorem 1 in [12]].

5. Simulation Results

In this section, the simulations using Matlab/Simulink package is conducted. For the sake of comparison, we achieved the simulations by considering the same system parameters (shown in Table (1)) and simulations conditions as in reference [11]. In the following simulations, three mobile robots in pursuit to a fictitious leader is considered. The leader robot is considered to move in the circular trajectory in counter-clockwise;

$$x_L = 3 + cos(^{\pi t}/_{30}) \quad (60)$$

$$y_L = 3 + sin(^{\pi t}/_{30}) \quad (61)$$

The aim of the simulation is that the three robots perform coordinated tracking to their respective trajectories in order to maintain a rectangular formation with respect to the leader robot position. The formation distance parameters, initial conditions, and controller gains are stated in Table (2) below, where all the parameters and gains of the controller **are tuned** after several trials to get the best performance. It can be seen in Fig. (3), that all the three robots trajectories are following the leader robot in circular trajectory as prescribed in (Eq. (60) and (61), also the three robots are gathered in the square formation as required. In Figures (4), (5), and (6), the errors time history for the x, y coordinates and the orientation angle $\theta$are shown. It can be noticed that compared to the results shown in ref. [11] [please refer to Figures (7), (8), and (9) therein], the results of this work are comparable and even show smoother behavior. In Figures (7) and (8), it is shown the time history for the switching manifolds for the corresponding robots in the x and y directions, respectively. The results show that all the surfaces converge to the origin in finite time less than 10 sec. Finally the control actions u in both x- and y-axis directions are demonstrated in Figures (9) and (10), respectively.

6. Conclusions

In this paper focuses on the problem of controlling a group of mobile robots to create specific formation or shape. The steps for solving this problem are starting by deriving the model of each mobile robot and make it suitable for the controller design process. The next step was to compensate for the nominal dynamics in the model, this was done using a feedback linearization control law. Which in turn, will inject the new control variables that will counteract the effect of uncertainty functions in the model. Using first the classical ideas of designing a discontinuous sliding mode controller, a component wise procedure is conducted to derive the discontinuous version of the controller. Then, we provide a way for modifying the control laws and sliding surfaces to achieve a continuous version of the controller by augmenting conditional servocompensator integral equations acting with the feedback system. Comparing the results with the one we have in ref. [11], the simulation results shown comparable results with smoother profiles with finite-time convergence for the tracking errors to zero.

Table 1: Robot parameters

Parameter	Value
m	$2.83\ kg$
I	$0.0226kg$
J	$5.1 \times 10^{-5}kg$
L	$0.315\ m$
d	$0.078\ m$
r	$0.045\ m$

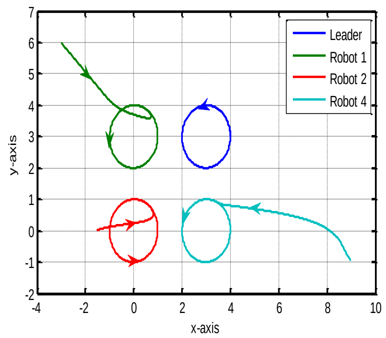

Figure 3: Phase Portraits for the three mobile robots agents with respect to the leader robot.

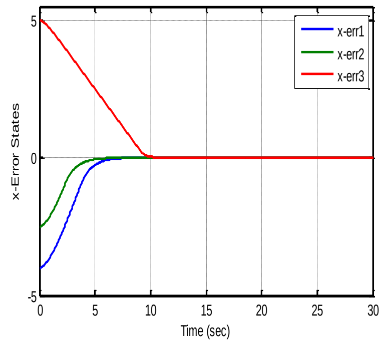

Figure 4: Simulation of error states in the x-axis direction e_x for the three robots.

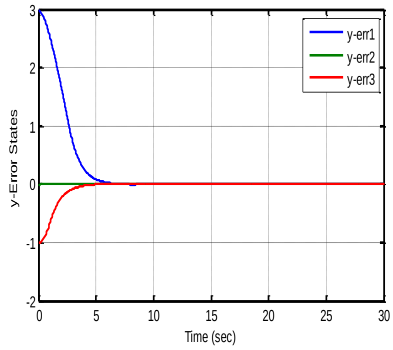

Figure 5: Simulation of error states in the y-axis direction e_y for the three robots.

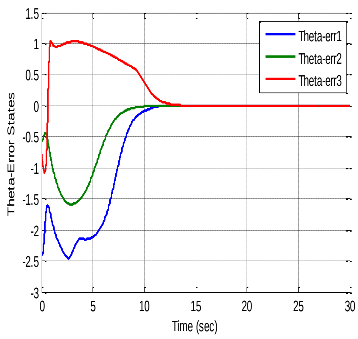

Figure 6: Simulation of error states of the orientation e_θ for the three robots.

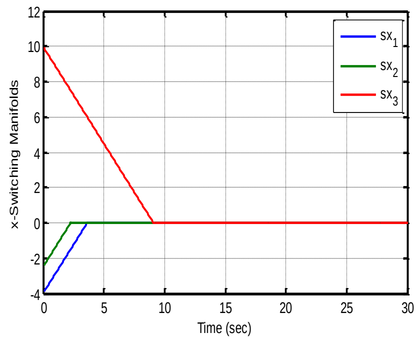

Figure 7: Simulation of switching manifolds in the x-axis direction s_x for the three robots.

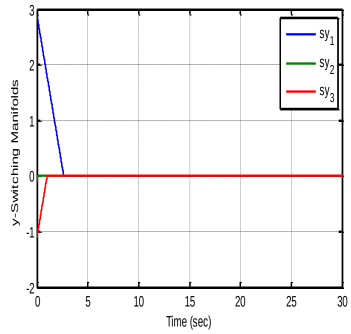

Figure 8: Simulation of switching manifolds in the x-axis direction s_y for the three robots.

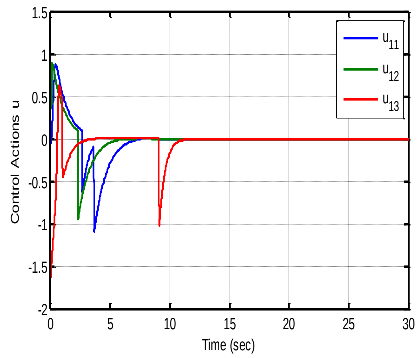

Figure 9: Simulation of control actions in the x-axis direction u_1 for the three robots.

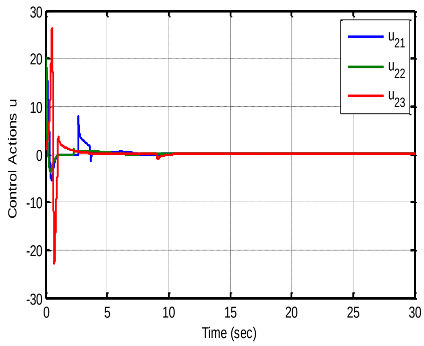

Figure 10: Simulation of control actions in the y-axis direction u_2 for the three robots.

Table 2: Formation distances, initial conditions, and controller parameters

Formation Distances	l_1	$[-3 \quad 0]^T$
	l_2	$[-3 \quad -3]^T$
	l_3	$[0 \quad -3]^T$
Initial Conditions	$[x_1(0) \quad y_1(0) \quad \theta_1(0)]^T$	$[-3 \quad 6 \quad {}^{-3\pi}/_{4}]^T$
	$[x_2(0) \quad y_2(0) \quad \theta_2(0)]^T$	$[-1.5 \quad 0 \quad {}^{-\pi}/_{6}]^T$
	$[x_3(0) \quad y_3(0) \quad \theta_3(0)]^T$	$[9 \quad -1 \quad {}^{-\pi}/_{4}]^T$
Controller Parameters	$c_{x1} = c_{x2}$	1
	c_{x3}	2
	$c_{y1} = c_{y2} = c_{y3}$	1
	$c_{xo1} = c_{xo2} = c_{xo3}$	1
	$c_{yo1} = c_{yo2} = c_{yo3}$	1
	μ	0.01
	$k_{x1} = k_{x2} = k_{x3}$	1.1
	$k_{y1} = k_{y2} = k_{y3}$	1.1

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