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Analysis and Simulation the Effect of the Pierce Parameter on the Output of the Free Electron Laser system

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HIGHLIGHTS

- The effect of the Pierce parameter on the performance of the free electron laser system was shown using MATLAB simulation.
- Pierce parameter values ranged between (0.01 - 0.03) for the laser beam with long wavelengths.
- The Pierce parameter represents the ratio between the saturation power and electrons power.

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ABSTRACT

The free electron laser is considered one of these important and advanced inventions because it provides a laser beam within various wavelengths of the electromagnetic spectrum. In any laser system device, several parameters must be available on which the efficiency and performance of the laser system are dependent. The Pierce parameter is one of the important parameters in measuring the performance quality of the free electron laser. In this paper, the simulation results were obtained using the MATLAB program to show the important effect of the Pierce parameter on the performance of the free electron laser system. The simulation results showed that the values of the Pierce parameter ranged between (0.01 - 0.03) for the laser beam with long wavelengths, while it ranged between (0.001-0.0001) for the short wavelengths. The results showed an increase in the efficiency values of the free electron laser system with increasing the values of the Pierce parameter, which represents the ratio between the saturation power and the power of the electron; therefore, it can be used as a specific threshold to measure the efficiency of the laser system. Additionally, the simulation of the Pierce parameter has a direct relation to the small signal gain per unit length.

1. Introduction

In 1960, American engineer and physicist Theodore Maiman who operated the first laser at the Hughes Research Laboratory in California invented the laser [1]. The wavelength of the laser is a very important factor in determining the way by which lasers are utilized in various applications. These applications depend on the wavelength and power of the laser used, thus adjusting the wavelength of the output laser according to the target material became a necessity, because of that lasers vary in types [1,2,3].

Ten years later, in the seventies of the last century, the physicist at Stanford University, John Madey, invented the free electron laser FEL and obtained a patent for it [4]. The researchers competed in developing the free electron laser to maximize the best characteristics of the resulting laser beam, in a way that suits its various civil and military applications [5,6].

The Pierce parameters ρ is a tool for measuring FEL efficiency and bandwidth, as well as determining the undulator length to reach saturation The values of the ρ are very important for any FEL system design. According to the previous literature, the typical values of ρ range from (0.001-0.0001) at very short wavelengths and vary at longer wavelengths [7,8,9,10].

In this paper, the effect of the Pierce parameter on the output of the FEL system was analyzed and simulated to obtain the best values needed to attain the highest quality of the output laser beam.

2. Technique and Implementation of the Work

The free electron laser differs from traditional lasers in the active medium, as it uses a beam of free electrons generated by an electron bomber, hence the name "free electron laser".

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The electron in the atom is bound by the attractive forces of the nucleus and its participation with the rest of the electrons in different bonds [3,11].

Figure 1 shows the four basic components of a free electron laser represented by an electron gun, linear accelerator, undulator, and optical resonator [3,5,12,13,14].

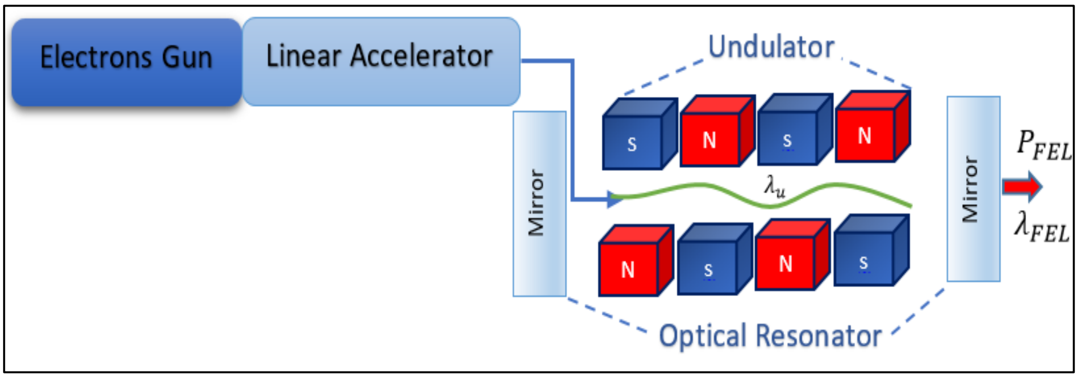

Figure 1: The four basic components of a free electron laser

In the beginning, the electron beam will be launched from the electron gun with an initial velocity, and then it is accelerated by the linear accelerator to a final velocity and a certain energy. After that, the electron beam enters the magnetic undulator part, which leads to its oscillation by the action of the Lorentz force in the form of a transverse sine wave around its axis of propagation. As a result of this oscillation, coherent photons are emitted forming synchrotron radiation. The photons keep oscillating between the two mirrors of the optical resonator to produce the laser beam at a certain threshold [5,6,11,15].

The most important characteristic of the resulting free electron laser is the possibility of tuning the wavelength λ_{FEL} and the power P_{FEL} of the output laser to any required value according to the following equations[1,6,12,13,16,17].

$$\lambda_{FEL} = (3.35 \times 10^{-27}) \left(\frac{\rho\ \lambda_{und}}{{N_{PH}}^2\ {E_{PH}}^2}\right) \left(1 + 4354.7\ B^2\ {\lambda_{und}}^2\right) \quad (1)$$

Where: ρ is the pierce parameter, λ_{und} is the undulator wavelength, N_{PH} is the coherent photons, E_{PH} is the photon energy and B is the magnetic field.

$$P_{FEL} = (3.33 \times 10^{+7}) \frac{N_{PH}\ \rho^2\ \ E_e\ R_1 R_2}{\lambda_{FEL}}\ \ \exp\left[\left(\frac{L_u}{G_L}\right) + 2L_R\,(G - \alpha)\right] \quad (2)$$

Where: E_e is the electrons energy, $R_1\ R_2$ is reflectivity of resonator mirrors, L_u is the undulator length, L_R is the undulator length, G_L is the Gain-Length, G is the gain coefficient and α is the loss coefficient.

The magnetic field B is given by Equation:

$$B = \frac{e^{(-5.08\, D/\lambda_{und}) - (1.54\, D^2/{\lambda_{und}}^2)}}{0.236} \quad (3)$$

Where: D is the distance between the rows of magnetics.

According to equations 1 and 2, the wavelength and power are affected by several parameters, and one of those important parameters represents the pierce parameter ρ, which will be discussed in some detail later on in this work.

In the one-dimensional approximation, the parameter used that determines the efficiency of the free electron laser system and the beam laser bandwidth is the Pierce parameter ρ which is given by the equation [8,18]:

$$\rho = \left(\frac{m_e c^2}{2\, E_e}\right) \left(\frac{I_e}{I_A}\right)^{1/3} \left(14.86 \times \frac{B\ {\lambda_{und}}^2\ \ f_B}{\sigma}\right)^{2/3} \quad (4)$$

Where: m_e is the electron mass, c is the electron velocity, I_e is the peak current, σ is the root-mean-square transverse radius, f_B is the coupling factor of Bessel function (For a helical undulator $f_B = 1$) and I_A is the Alfven current which is given by Equation:

$$I_A = \frac{4\pi\epsilon_0\, m_e\, c^3}{e} = 17069.3\ Amp \quad (5)$$

Where: ϵ_0 the vacuum permittivity constant.

It is clear that the Pierce parameter depends on the data of both the linear accelerator and the undulator, where the best values are chosen for each of them to reach the best performance of the free electron laser system.

Moreover, the Pierce parameter is a sensitive tool for measuring both setup errors and degrading effects; as a result, it characterizes the bandwidth [8].

Increasing the value of gain length inevitably leads to Large electron energy spreads, which leads to a broadening of the radiation beam, and thus will inhibit micro bunching and hence radiation amplification. The variations of laser wavelength must be less than the gain bandwidth. The Pierce parameter at saturation is given by [7,9]:

$$\frac{\Delta\,\lambda_{FEL}}{\lambda_{FEL}} \approx 2\,\rho \tag{6}$$

The relation between the pierce parameter ρ and the number of coherent photons N_{PH} is given by the equation [5,15,17,19,20]:

$$N_{PH} = 5.03 \times 10^{24}\ (\rho\,\lambda_{FEL}\,E_e) \tag{7}$$

According to Equation 2, the power P_{FEL} of the laser beam grows exponentially, and the growth increases especially in the last few gain lengths. The growth rate continues to increase until it reaches saturation state. The gain length G_L and saturation power P_s represent the main parameters to determine the performance of the FEL system, both depend on the Pierce parameter according to the following Equations [5,8,15,17,18,19,21]:

$$G_L = (0.0459)\,\frac{\lambda_{und}}{\rho} \tag{8}$$

The Pierce parameter has a direct effect on the process of transferring the energy of the electron beam to the beam of the laser radiation, due to its association with the gain resulting from the amplification of the synchrotron beam.

$$P_s = 6.25\ \times 10^{18}\,(\,\rho\,E_e\,I_e) \tag{9}$$

It is important to note that increasing the length of the undulator does not mean an improvement in the performance of the free electron laser system, as the electrons begin to regain energy from the radiation field, which leads to a decrease in the radiation energy.

Finally, the Peirce parameter ρ has an important relation with the small signal gain coefficient g_0 according to the following Equation [22,24]:

$$\rho = \frac{\lambda_{und}\,(\pi\,g_0\,)^{1/3}}{4\,\pi\,L_u} \tag{10}$$

Where L_u is the length of the undulator.

The g_0 represents an important tool for controlling small signal high gain dynamics and thus obtaining a high intensity laser beam. The g_0 is given by the equation [25,28]:

$$g_0 = (3.58 \times 10^{-8}\ )\,\frac{B^2\,L_u\,I_e}{I_A\ E_e\left(\sqrt{1+\epsilon_s^2}\right)\left(1+(8709.5\,\lambda_{und}{}^2\,B^2)\right)} \tag{11}$$

Where ϵ_s is the energy spread of the electron beam.

3. Results and discussion of Simulation

Obtaining the results requires a series of simulations, so a program was designed using the MATLAB language as shown in Figure 2. All the following analyses and discussions will depend on the obtained data to show the effect of the Pierce parameter on the output of the free electron laser system.

Table 1 clearly shows the dependence of the Pierce parameter on the dimensions of the undulator. Figure 3-a and Figure 3-b shows the direct relations between the ρ versus the λ_{und} and the B according to Equation 4. While Figure 3-c shows the dependence of the Pierce parameter on the velocity and energy of the electrons, where the relation is inverse. On the other hand, it has been noted the dependence of the Pierce parameter on the small signal gain per unit length, where the relation is direct according to Equation 10, as shown in Figure 3-d.

The Pierce parameter has an important effect on several characteristics of the free electron laser device. Where it can be seen that the number of photons increased according to Equation 7 as shown in Figure 4-a. The performance of the FEL system is determined by the gain length G_L, Figure 4-b shows the inverse relation of this parameter according to equation 8. The Pierce values decrease at the short wavelengths and low power of the output laser according to equations 1,2 and as shown in Figures 4-c and 4-d respectively.

Table 2 clearly shows the dependence of the Pierce parameter on the linear accelerator for electrons, to accelerate and gain high energies E_e. Figure 5-a shows the direct relation between the ρ versus the N_{PH} according to equation 7. While Figure 5-b shows, the inverse relation between the ρ versus the G_L according to Equation 8.

Figure 5-c and Figure 5-d show an increase in the Pierce values at long wavelengths and high powers of the output laser according to Equations 1, 2 respectively.

Table 3 clearly shows the dependence of the Pierce parameter on the small signal high gain g_0 according to equation 10. According to the simulation results shown in this table, changing the wavelength of the undulator λ_{und} will lead to a change in the small signal high gain g_0, so the values of the Pierce parameter will change, where the relation is direct according to Equation 11. The previous behavior of the Pierce parameter remains the same with both the wavelength and the power of the output laser.

Finally, the efficiency of the free electron laser system can be measured by determining the value of the Pierce parameter. Whereas, Pierce represents the ratio between the power of the electrons entering the system and the power of the output laser beam. Where all power values of P_{FEL} must be less than a certain threshold limit called saturated power as shown in Figure 6 for three Tables when changing the values of (λ_{und} , E_e , g_0) respectively. Figure 6-a shows that the last value of the laser power ($P_{FEL} = 3854180000\ MW$) has exceeded the threshold limit ($P_s = 99655.2\ MW$ and must be excluded and avoid reaching the λ_{und} values more than 0.09 m. Hence the importance of the Pierce parameter in improving the efficiency of the free electron laser system.

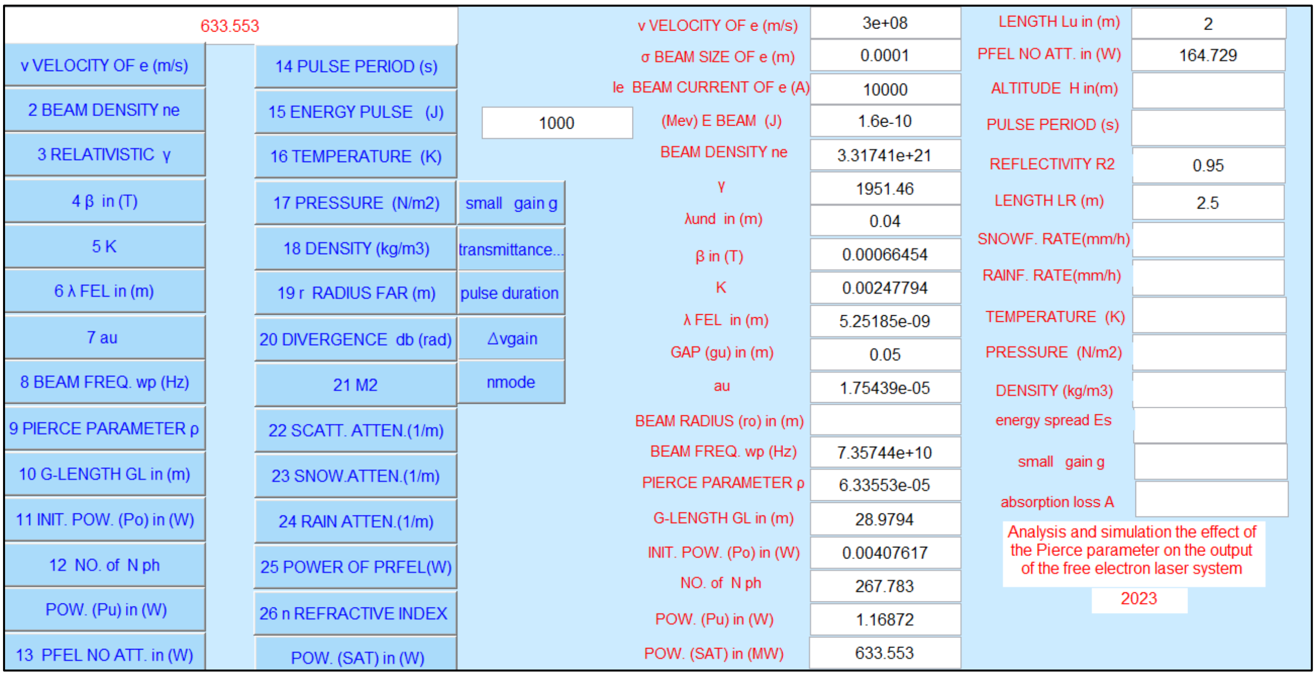

Figure 2: The simulation software interface

Table 1: The results of the simulation for λ_{und} versus $(B, N_{PH}\ \rho, \lambda_{FEL}, P_{FEL})$

$E_e = 300\ Mev$	$R_1 = 100\%$	$I_e = 10000\ A$		$I_A = 17069.3\ A$	D =0.05 m	
$(G-\alpha) = 1$	$R_2 = 95\%$	$B = (0.00066 - 0.2264)T$		$L_u = 2\ m$	$L_R = 2.5\ m$	$\sigma = 0.0001\ m$
λ_{und} (m)	N_{PH}	ρ	$G_L(m)$	λ_{FEL} (nm)	P_{FEL} (MW)	P_s (MW)
0.04	2975.37	0.000211184	8.69384	58.354	0.000645	633.552
0.05	20457.1	0.0011612	1.9764	72.967	0.234364	3483.6
0.06	74979.7	0.0035236	0.781587	88.134	30.7585	10570.8
0.07	202032	0.00776298	0.413887	107.791	3194.4	23288.9
0.08	500675	0.0141049	0.260335	147.02	331306	42314.7
0.09	1.33e+6	0.0226047	0.18275	243.797	35577000	67814.1
0.1	3.777e+06	0.0332184	0.138176	470.973	3854180000	99655.2

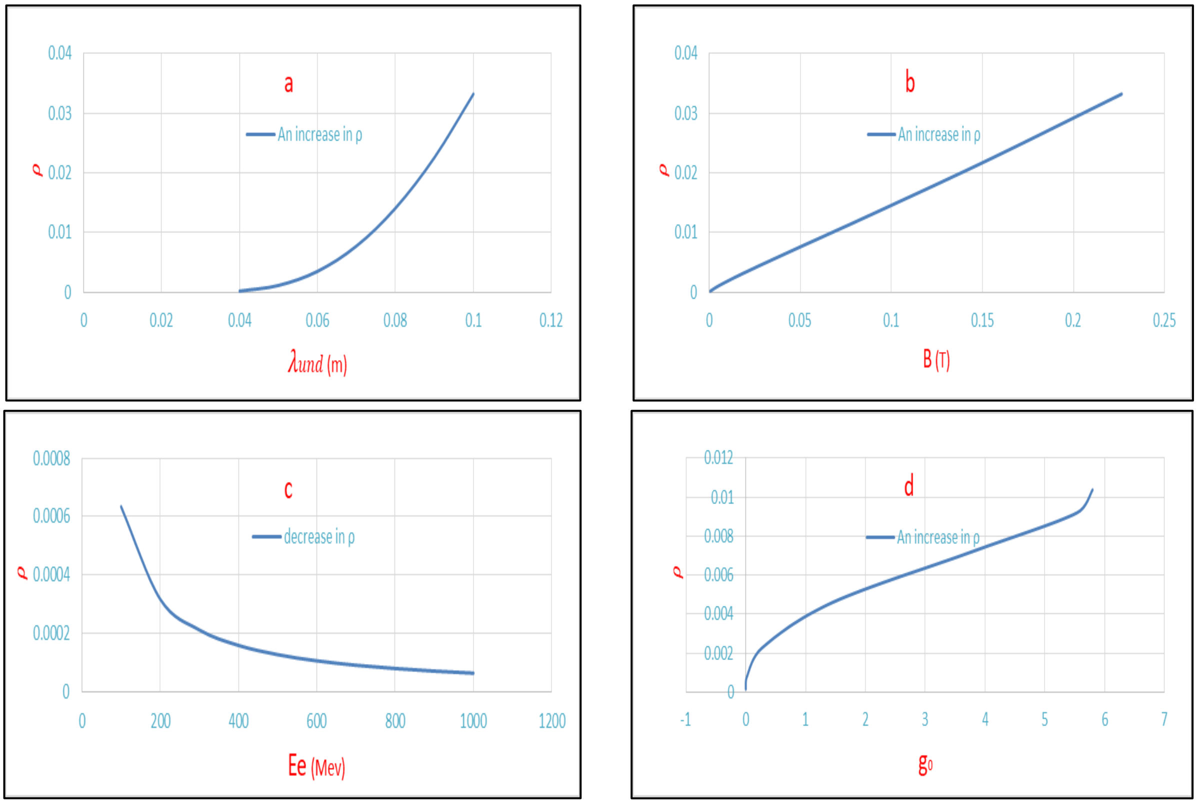

Figure 3: The relations between ρ versus (λ_{und} ,B, E_e, g_0)

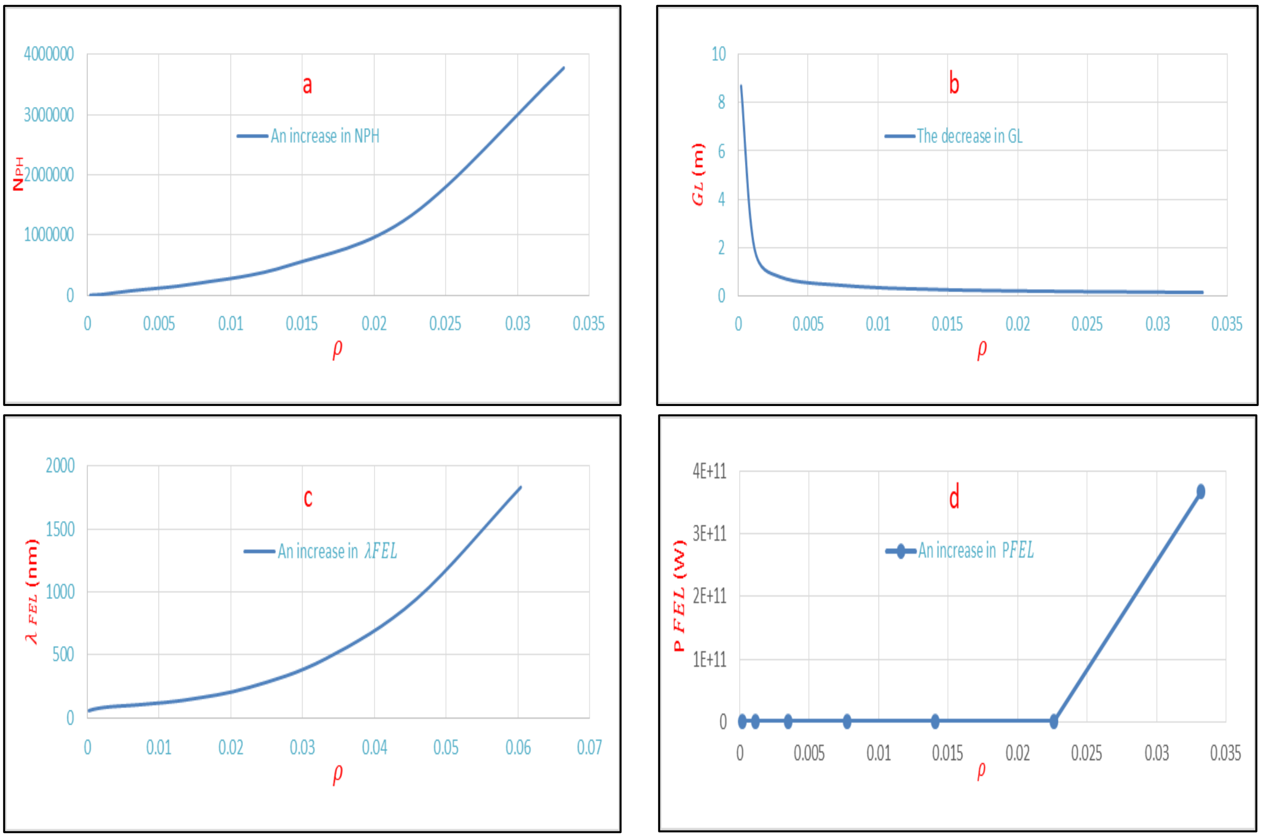

Figure 4: The relations between ρ versus $N_{PH}, G_L, \lambda_{FEL}, P_{FEL}$ when changing the λ_{und}

Table 2: The results of simulation for E_e versus $(B, N_{PH}\ \rho, \lambda_{FEL}, P_{FEL})$

$\lambda_{und} = 0.04$ m	$R_1 = 100\%$	$I_e = 10000\ A$		$I_A = 17069.3\ A$	D =0.05 m	$\epsilon_s = 1$
$(G-\alpha) = 1$	$R_2 = 95\%$	B= 0.0006645 T		$L_u = 2\ m$	$L_R = 2.5\ m$	$\sigma = 0.0001\ m$
$E_e\ Mev$	N_{PH}	ρ	G_L	$\lambda_{FEL}\ (nm)$	$P_{FEL} (MW)$	$P_s\ (MW)$
100	26778.3	0.000633553	2.89794	525.185	0.00306565	633.552
200	6694.61	0.000316776	5.79589	131.297	0.0010855	633.552
300	2975.37	0.00021118	8.69384	58.354	0.000645	633.552
400	1673.64	0.000158388	11.5918	32.8241	0.000456	633.552
500	1071.14	0.000126711	14.4897	21.0074	0.000353	633.552
600	743.847	0.000105592	17.3877	14.5886	0.000287	633.552
700	546.498	9.05076e-05	20.2856	10.7181	0.000242	633.552
800	418.41	7.91941e-05	23.1835	8.20599	0.000209	633.552
900	330.598	7.03948e-05	26.0815	6.48379	0.000184	633.552
1000	267.783	6.33553e-05	28.9794	5.25185	0.000164	633.552

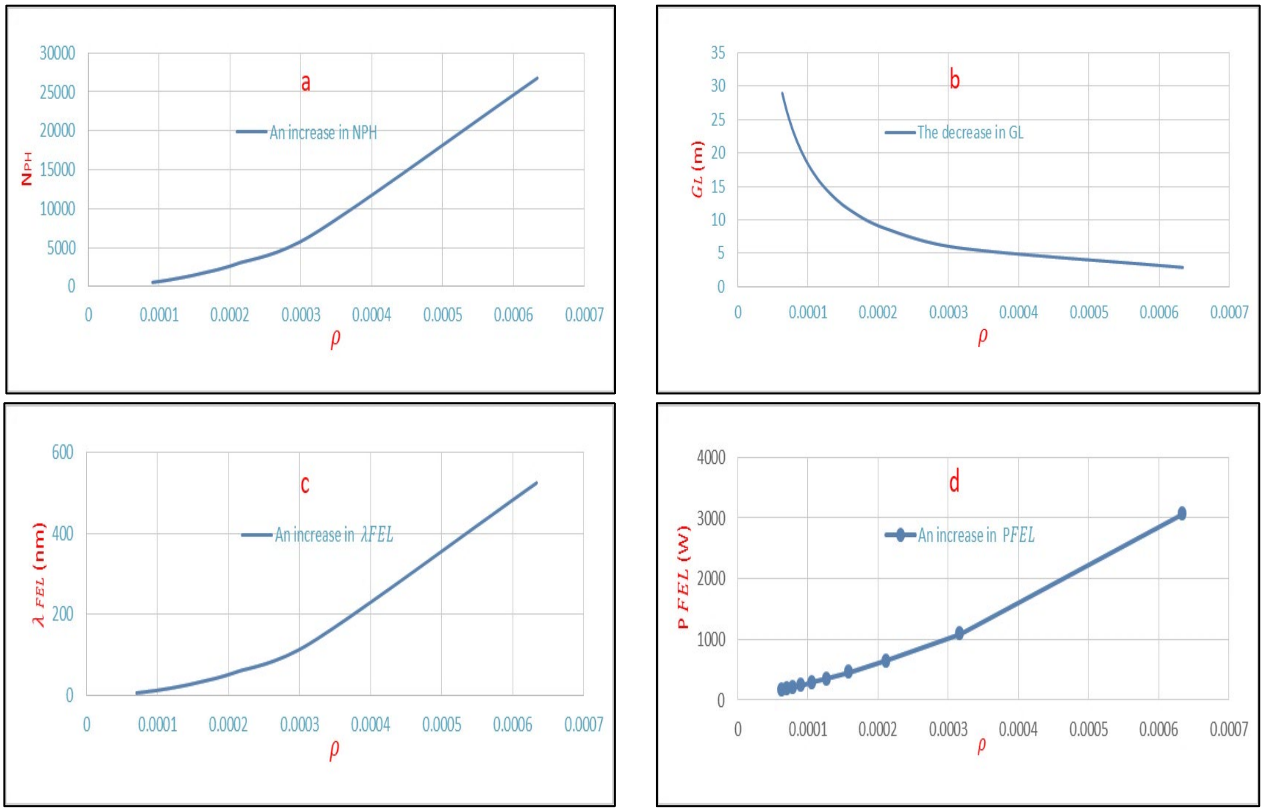

Figure 5: The relations between ρ versus $N_{PH}, G_L, \lambda_{FEL}, P_{FEL}$ when changing the E_e

Table 3: The results of simulation for g_0 versus $(B, N_{PH}\ \rho, \lambda_{FEL}, P_{FEL})$

$E_e = 300\ Mev$		$R_1 = 100\%$	$I_e = 10000\ A$	$I_A = 17069.3\ A$	D =0.05 m	$\epsilon_s = 1$
$(G-\alpha) = 1$		$R_2 = 95\%$	$\sigma = 0.0001\ m$	$L_u = 2\ m$	$L_R = 2.5\ m$	
$\lambda_{und}\ (m)$	$B\ (T)$	g_0	ρ	$\lambda_{FEL}\ (nm)$	$P_{FEL}\ (MW)$	$P_s\ (MW)$
0.04	0.00066454	0.000272886	0.000154846	58.354	0.000239	464.538
0.05	0.00562708	0.0195528	0.000792567	72.9674	0.0540	2377.7
0.06	0.0210061	0.268946	0.00225896	88.1349	3.2349	6776.88
0.07	0.0510755	1.45052	0.00459587	107.791	92.3045	13787.6
0.08	0.0966434	3.79547	0.00721464	147.02	1039.72	21643.9
0.09	0.156032	5.53596	0.00919313	243.797	3622.77	27579.4
0.1	0.226465	5.7971	0.0103711	470.973	5568.56	31113.3

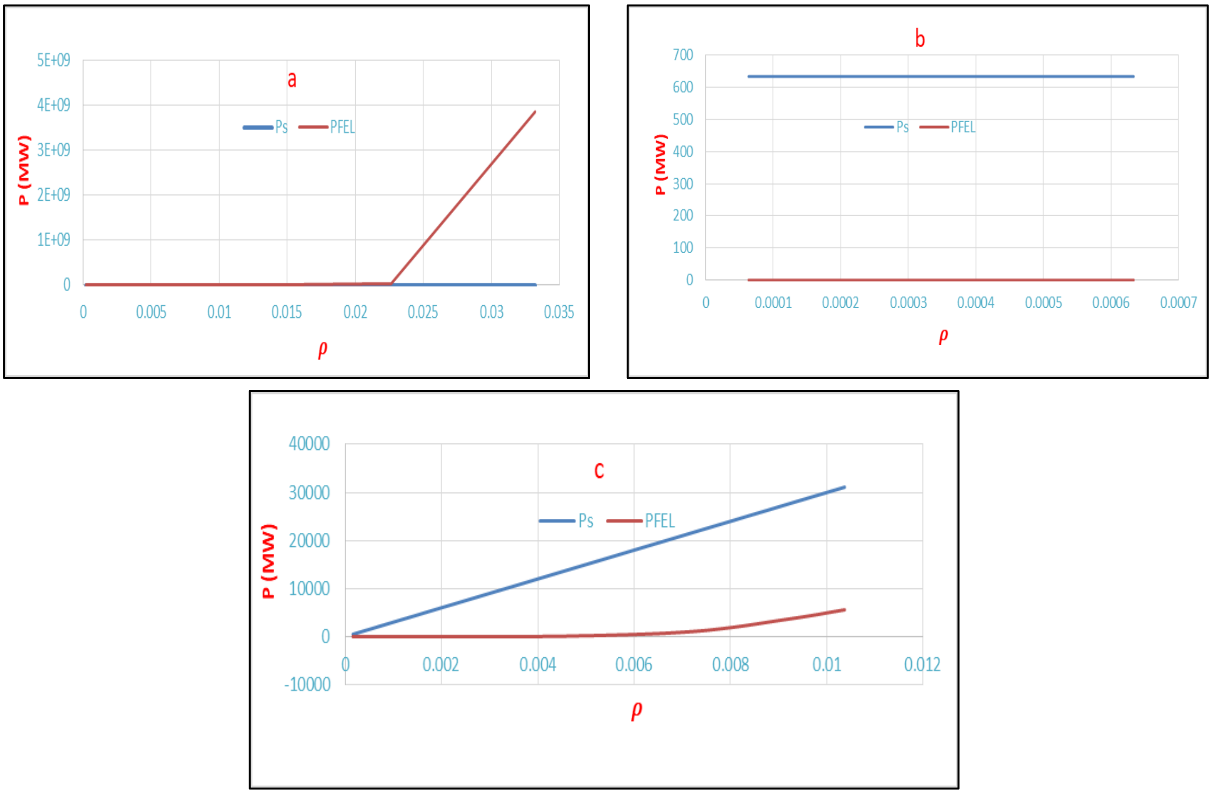

Figure 6: The powers for three Tables versus when changing the values of (λ_{und} , E_e , g_0) respectively

4. Conclusions

Through the simulation results, the importance of the Pierce parameter ρ in the free electron laser system became clear. Where it can be concluded that short wavelengths are suitable for small values of ρ ranging from (0.001-0.0001). While the larger wavelengths of the resulting laser within the visible region of the electromagnetic spectrum are suitable for values of the Pierce parameter ρ ranging from (0.01 - 0.03).

It can be concluded that the Pierce parameter represents a tool for measuring the efficiency of the free electron laser system, as it represents the ratio between the power of electrons ejected from the linear accelerator ($E_e\ I_e$) and the saturation power P_s. As the condition of high efficiency is that, the value of output laser power P_{FEL} does not exceed the saturated power value P_s. As the efficiency values of the free electron laser system increase with increasing the values of the Pierce parameter.

Author contribution

All authors contributed equally to this work.

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Data availability statement

Not applicable.

Conflicts of interest

The authors of the current work do not have a conflict of interest.

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