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A Comparison of Results among Waste Management Producers: A Case Study for the Process of Waste Management

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HIGHLIGHTS

- A mathematical model was applied to analyze cooperation among producers in waste management.
- The Shapely value was used to show the potential cost to the waste producers in the case of cooperation.
- A case study for the process of waste management was investigated.

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ABSTRACT

In this paper, we apply a mathematical model based on cooperative game theory to model cooperation among producers in waste management. Then, we use the Shapely value as a solution concept in a cooperative game to evaluate the cooperation existing among producers, in order to show the potential cost to the waste producers in the case of cooperation and reduce the overall costs of processing non-recyclable waste during the cooperation. Thus, this study concludes that all producers divide sanitary landfills or incineration of non-recyclable waste, meaning that for each producer who earns more, there will be a greater contribution to the disposal or treatment of non-recyclable waste.

1. Introduction

In most developed countries, modern industrialization systems are established based on reducing the involvement of new input factors and recycling waste. Since not all waste is recycled in waste recycling plants, there must be cooperation among waste producers to reduce the overall costs of processing non-recyclable waste. Many researchers have published several papers on this issue. The following is some of the previous research in the context of waste management: Cheng et al. [1] integrated multi-criteria decision analysis and linear programming to optimize the selection of a landfill site and the waste flows costs considered in the waste management problem; Karmperis et al. [2] reviewed multi-criteria decision analysis model and introduced the waste management bargaining game to support decision-makers; He et al. [3] proposed an agent-based waste treatment model as a complex adaptive system, and private operators set gate fees to vie for waste in low-information competition; Nguyen-Trong et al. [4] modeled the optimization of collection services on transportation of municipal solid waste; Feyzi et al. [5] provided a new MCDM framework and evaluated sustainability-based criteria to select the most appropriate site for solid waste incineration power plants.

Game theory is the analysis of a situation involving conflicting interests (as in business or military strategy) in terms of gains and losses among opposing players. The intention of game theory is to produce optimal decision-making of independent and competing actors in a strategic setting. The cooperative game has been applied to all situations in which product actions maximize utility and has shown to be a powerful tool for creating practical collaboration actions in a wide range of applications. According to the definition of cooperative game theory, this theory concentrates on mathematical models of interaction indices among rational participants (the players) of the modeled dispute (the game) [6]. In the garbage producers' conflict, the coalitional game structure was introduced [7- 8]. In addition, types of directed cooperative gaming applications have been built [9- 10] for waste management challenges. There are many important and interaction indices (see, e.g. [11- 17]), one of them is the Shapley important index which is used to demonstrate the possible cost to waste producers in the event of cooperation, as well as to minimize the overall costs of processing non-recyclable trash during the collaboration between the among producers. The interaction phenomena among a set of players, which can be seen as an extension of the notion of value, has been applied to multi-criteria decision-making in the framework of aggregation by the Choquet integral [18- 21]. The

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purpose of this paper is to describe waste management producer cooperation using a mathematical model based on cooperative game theory and use the Shapley important index to emphasize the potential cost to reduce the overall costs of non-recyclable rubbish processing during producer cooperation.

The structure of this paper is as follows. In section 2, we give the basic concepts and mathematical models for decision-making in waste management. The working procedure for the cooperative game problem is described in Section 3. In section 4, we introduce a study case for the processing of waste management producer cooperation, and then we give a compression of results in Section 5. Finally, some conclusions are presented.

2. Basic Concepts and Mathematical Model

Throughout the paper $N = \{1,\dots,n\}$ denotes the set of producers and the power set of N given by $G(N)$. Each non-empty subset of N is called a coalition. The set N is referred to as the grand coalition. For each coalition $S \subseteq N$, a worth function $v(S)$ gives the payoffs that any coalition of players can get. This function $v(S)$ is called the characteristic function of the game.

The usual conflicts among waste producers in waste management can be formalized as a game with the set of producers is $N = \{1, \dots, n\}$ and the characteristic function $v(S)$, where $S \subseteq N$ given by the following. Let $M = \{1, \dots, m\}$ be a finite set of waste recycling plants; $w_1^c, \dots, w_m^c$ denote their capacities and $c_1^g, \dots, c_m^g$ denote their gate fees. For the set of producers $N = \{1, \dots, n\}$, we denote their waste productions by $w_1^p, \dots, w_n^p$. The matrix specifies the transportation costs $[c_{i,j}^t]$, where $c_{i,j}^t$ indicates the cost of transportation from the point of origin to the point of destination (i.e. from the producer $i \in N$ to the plant $j \in M$). The quantity of garbage sent by the manufacturer $i \in N$ to the waste recycle plant $j \in M$ in tonnes is denoted by $x_{i,j}$. The distinguishing feature of the characteristic function $v(S)$, $S \subseteq N$ is given by following Equations (1) to (8) as an optimization problem. Note that, the characteristic function for the empty coalition equals zero by definition.

$$v(S) = \min_{x_{i,j}:\, i \in S, j \in M} \sum_{j \in M} \sum_{i \in S} \left(c_{i,j}^t + c_j^g \right) x_{i,j} \tag{1}$$

Subject to

$$\sum_{i \in S} x_{i,j} \leq w_j^c - \sum_{i \in N \setminus S} x'_{i,j}, \ \forall j \in M, \tag{2}$$

$$\sum_{j \in M} x_{i,j} = w_i^p, \ \forall i \in S, \tag{3}$$

$$x_{i,j} \geq 0, \ \forall i \in S, \ \forall j \in M, \tag{4}$$

$$\left\{ x'_{i,j} : \ i \in N \setminus S, \forall j \in M \right\} = \min_{x_{i,j}:\, i \in N \setminus S, j \in M} \sum_{j \in M} \sum_{i \in N \setminus S} \left(c_{i,j}^t + c_j^g \right) x_{i,j} \tag{5}$$

Subject to

$$\sum_{i \in N \setminus S} x_{i,j} \leq w_j^c, \ \forall j \in M, \tag{6}$$

$$\sum_{j \in M} x_{i,j} = w_i^p, \ \forall i \in N \setminus S, \tag{7}$$

$$x_{i,j} \geq 0, \ \forall i \in N \setminus S, \ \forall j \in M. \tag{8}$$

Waste handling expenses are calculated by multiplying transportation and entrance fees by the amount of processed waste. The entire strategy is predicated on the premise that all garbage generated can be processed by waste recycling plants. After a coalition of all outsiders has decreased the entire cost of the operation, the value of a characteristic function $v(S)$ corresponds to the minimum of the total costs of coalition members who have made the proper option. In disputes between waste producers, it reflects general principles of decision-maker cooperation. As a result, the previously specified characteristic function will be used to evaluate all game classes. The generated costs are compared utilizing the cooperative game problem for waste producers in order to arrive at a fair allocation.

The Shapley value presents a sharing structure that poises cooperation power in cooperative game problems, and this value is usually called the power index.

Definition 1 [6]: For the player $\boldsymbol{i} \in \boldsymbol{N}$, the Shapley value $\boldsymbol{\varphi_i(N, v)}$, is a value on cooperative game $(\boldsymbol{N}, \boldsymbol{v})$ defined by

$$\boldsymbol{\varphi_i(N, v) = \sum_{S \subseteq N, i \in S} \frac{(s-1)!(n-s)!}{n!} \left(v(S) - v(S - i) \right)} \tag{9}$$

where, $\boldsymbol{n = |N|}$, $\boldsymbol{s = |S|}$ and when $\boldsymbol{i \notin N}$, implies $\boldsymbol{\varphi_i(N, v) := 0}$.

Consider three-person players game with $N = \{1, 2, 3\}$, to calculate the Shapley value for this example of these players we need characteristic values as in the following:

$$v(\emptyset) = 0\,,\ \ v(\{1\}) = 8\,,\ v(\{2\}) = 7,\quad v(\{3\})\ = 8$$

$$v(\{1,\ 2\}) = \ 14,\quad v(\{1,\ 3\}) = 15\,,\ \ v(\{2,\ 3\}) = 13\,,\ \ v\ (\{1,\ 2,\ 3\}) = 20$$

Now, for example calculate the shapely value for the first player, $\varphi_1(N, v)$ by using Eq. (9):

$$\varphi_1(N,v) = \frac{0!\,2!}{3!}(v(\{1\}) - v(\emptyset)) + \frac{1!\,1!}{3!}(v(\{1,2\}) - v(\{2\})) + \frac{1!\,1!}{3!}(v(\{1,3\}) - v(\{3\})) + \frac{2!\,0!}{3!}(v(\{1,2,3\}) - v(\{2,3\})),$$

$$\varphi_1(N,v) = \tfrac{2}{6}(8-0) + \tfrac{1}{6}(14-7) + \tfrac{1}{6}(15-8) + \tfrac{2}{6}(20-13),$$

$$\varphi_1(N,v) = \tfrac{44}{6} \cong 7.33\,.$$

In the same technique we can find the Shapley value for the other players (2 and 3). Therefore, the Shapley vector $\varphi_i(N, v)$ for players have been found as

$$\varphi_i(N,v) = (\varphi_1,\ \varphi_2, \varphi_3) = (7.33,\ 5.83, 6.83).$$

3. Working Procedure

In order to fair allocation for the cooperative game problem for the waste producers, the resulting costs are compared using the Shapely value with a writing algorithm and computer program (MatLab program) as a working procedure. The following algorithm is the way to compute the values of the characteristic function and the values of Shapley $\boldsymbol{\varphi}$.

Algorithm: Computations the characteristic function and Shapley value $\boldsymbol{\varphi}$

Step 1: Let $\boldsymbol{N} = \{\mathbf{1}, \dots, \boldsymbol{n}\}$, $\boldsymbol{M} = \{\mathbf{1}, \dots, \boldsymbol{m}\}$,
Input: Gate fees $(\boldsymbol{c}_1^g, \dots, \boldsymbol{c}_m^g)$, and transportation costs $(\boldsymbol{c}_{1,1}^t, \dots, \boldsymbol{c}_{n,m}^t)$,
Step 2:
$for\ \ i = 1$ to number of players n, $i \in S$,
$for\ all\ \ j = 1\ to\ m$ do
Input: $x(i, j)$;
Step 3: if

$$x'_{i,j} = \underset{x(i,j)}{\mathrm{Min}} \left(\sum_{i\in n} \sum_{j\in m} c_{i,j}^t + c_{i,j}^g \right) x(i,j);$$

end if
$\boldsymbol{end\ i}$
$\boldsymbol{end\ j}$
Step 4: Return to Step 2
Step 5: $for\ i \in \boldsymbol{s}$,
Calculate the Shapley value $\varphi(i)$ from Eq. (9)
end for
Step 7: Return to **Step 6**

4. Case Study

This paper was utilized to investigate a mathematical model of the waste management process at the Mahmoudia solid waste management factory (Al-Yusufiyah district) shown in Appendix A (Figure 1), which is the only trash sorting and recycling facility in the Baghdad governorate. In the Mahmoudia solid waste management factory, there are five waste producers whose monthly imports and production values are arranged according to the strength of their cooperation in terms of the largest contribution to the producing company. The waste producers are the AL Central Bank of Iraq piston (CBI), the Dora piston (DP), the Directorate of Water and Sewage piston (DWS), and the Asia dyes piston (AD), and the fifth is Al-Rawi Water Company piston (AWC).

The information about waste sorting and recycling plant capacity, gate fees, waste production, and transportation costs from the place of origin to the planned destination was obtained from Mahmoudia solid waste management factory as per its one-month imports, as shown in Table 1.

Table 1: The data of waste producers and Mahmoudia solid waste management plant

W_i^P	$W_1^P = 300kt$	$W_2^P = 600kt$	$W_3^P = 550kt$	$W_4^P = 550kt$	$W_5^P = 500kt$
w_j^c	$W_1^c = 300$	$W_1^c = 300$	$W_1^c = 300$	$W_1^c = 300$	$W_1^c = 300$
c_{ij}^t	$c_{11}^t = 20$	$c_{21}^t = 40$	$c_{31}^t = 30$	$c_{41}^t = 35$	$c_{51}^t = 25$
c_j^g	$c_1^g = 55$	$c_1^g = 55$	$c_1^g = 55$	$c_1^g = 55$	$c_1^g = 55$

The characteristic functions for all coalitions of waste producers $\boldsymbol{v(S)}$, $\boldsymbol{S \subseteq N = \{1,2,3,4,5\}}$are computed by Equations (1) to (8) as an optimization problem through the algorithm presented in the previous section, and as shown in Table 2.

Table 2: The characteristic function values

$\boldsymbol{S}$	$\{\emptyset\}$	{1}	{2}	{3}	{4}	{5}	{1,2}	{1,3}
$\boldsymbol{v(S)}$	0	22.5	57	46.75	49.5	40	79.5	69.25
$\boldsymbol{S}$	{1,4}	{1,5}	{2,3}	{2,4}	{2,5}	{3,4}	{3,5}	{4,5}
$\boldsymbol{v(S)}$	72	62.25	103.75	106.5	97	96.25	86.75	`89.5
$\boldsymbol{S}$	{1,2,3}	{1,2,4}	{1,2,5}	{1,3,4}	{1,3,5}	{1,4,5}	{2,3,4}	{2,3,5}
$\boldsymbol{v(S)}$	126.25	129	119.5	118.75	109.25	112	153.25	143.75
$\boldsymbol{S}$	{2,4,5}	{3,4,5}	{1,2,3,4}	{1,2,4,5}	{1,3,4,5}	{1,2,3,5}	{2,3,4,5}	{1,2,3,4,5}
$\boldsymbol{v(S)}$	146.5	136.25	175.75	169	178.75	166.25	193.25	215.75

5. Compression of Results

The preference relation of the Decision Maker (DM) over alternatives is denoted by $\prec$. For any two waste producers i_1, $i_2 \in \boldsymbol{N}$, $i_1 \prec i_2$ means that the DM prefers alternative i_2 to i_1.

Using Equation (2), the Shapley value for each producer is presented in Table 3.

Table 3: The Shapley values

Waste producer	i	The Shapley values
CBI	i = 1	22.5
DP	i =2	57
CBI	i = 1	22.5
AD	i =4	49.5
AWC	i =5	40

Thus, the Shapley values vector φ, for waste producers is (22.5, 57, 46.75, 49.5, 40). Evidently, the highest value is the AL Dora piston (DP)= 57, therefore waste producer 2 has the maximum capacity to influence the outcome of the game, and it will be a greater contribution to the disposal or treatment of non-recyclable waste. That is, it will be a greater contribution than the rest of the waste producers. On the other hand waste producer 1 (CBI) has the minimum capacity to influence the outcome of the game, and it will be the least contribution than the rest of the waste producers.

As a result, the preference relation of the Decision Maker over the five producing companies that will be ranked according to their contribution to sorting and recycling waste is $\boldsymbol{1} \prec 5 \prec \boldsymbol{3} \prec \boldsymbol{4} \prec \boldsymbol{2}$. Therefore, after comparing the preference relation of our results with the data of Mahmoudia Solid Waste Management Factory, it was found to be mostly identical.

6. Conclusion

According to our present data, which is shown in Table 1 and Table 2, we have examined the results through the proposed framework to display the potential cost to waste producers in case of cooperation. Then, as shown in Table 3, we have concluded that waste producer 2 (AL Dora piston) has the maximum capacity to influence the outcome of the game, and it will be a greater contribution to the disposal or treatment of non-recyclable waste. That is, it will be a greater contribution than the rest of the waste producers.

Although the conclusion of the results from this study is sufficient to formulate well-structured strategies for the waste management system in the future, there is room for improvement of the proposed framework. For a problem of large scale, cooperation may become useful when the capacity of the local waste processor (Mahmoudia solid waste management factory) is insufficient and producers are forced to send their waste to further regions. Therefore, we recommend the establishment of more waste processors in Baghdad and other administrative units in Iraq.

Appendix A: Visit to the Mahmoudia solid waste management factory

Our visit to the Mahmoudia solid waste management factory was in order to the discussion of the mechanism of the factory's work in terms of waste management and recycling from the first step to production. Figure 1 shows our visit to the Mahmoudia solid waste management factory.

Figure 1: Mahmoudia solid waste management factory

[*] The geographic location of El Mahmoudeya city is south of Baghdad, which is known as the "Gateway to Baghdad".

Appendix B: Table of nomenclatures and abbreviations

N	Set of players (Set of producers)
$\boldsymbol{G}(\boldsymbol{N})$	The power set (probabilistic values)
S	A non-empty subset of N is called a coalition
$v(S)$	The characteristic function
φ	Shapley value
M	be a finite set of waste recycling plants
$[c_{i,j}^t]$	The transportation costs
c_i^g	Gate fees
DM	Decision Maker
CBI	AL Central Bank of Iraq piston
DP	the Dora piston
DWS	the Directorate of Water and Sewage piston
AD	and the Asia dyes piston
AWC	and the fifth is Al-Rawi Water Company piston
W_i^c	Capacities of waste recycling plants
$\prec$	The preference relation
$x_{i,j}$	The quantity of garbage sent by the manufacturer $i \in N$ to the waste recycle plant $j \in M$

Author Contribution

The contribution to the current work was equally made by all authors.

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Data Availability Statement

Upon any demand made by the corresponding author, the data which supports the conclusions of the current work can be made available.

Conflicts of Interest

There is no conflict of interest in the current work.

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