Engineering and Technology Journal 41 (11) (2023) 1403-1413

Engineering and Technology Journal
Journal homepage: https://etj.uotechnology.edu.iq

Single Variable Thick Plate Buckling Problems Using Double Finite Sine Transform Method

Charles Chinwuba Ike

Civil Engineering Dept., Enugu State University of Science and Technology, Agbani, Enugu State, Nigeria.
*Corresponding author Email: charles.ike@esut.edu.ng

HIGHLIGHTS

- The double finite sine transform method was utilized to obtain exact buckling solutions for thick plate problems
- The method simplified the problem to algebraic ones since the sinusoidal function satisfies the boundary conditions
- The buckling solutions aligned with exact solutions for thick plates

ARTICLE INFO

Handling editor: Mahmoud S. Al-Khafaji
Keywords:
Double finite sine transform method; Single variable thick plate; Bending problem; Biaxial buckling; Critical buckling load

ABSTRACT

This paper derives buckling solutions for single variable thick plate buckling problems using the double finite sine transform method (DFSTM). The problem governing partial differential equation (GPDE), originally formulated by Shimpi and others, uses a refined plate theory (RPT) and accounts for transverse shear deformations, rendering it applicable to thick plates. The thick plate is simply supported and subjected to (i) uniform uniaxial compressive load in the x direction. (ii) uniform biaxial compressive load in the x and y axes. The DFSTM was applied to the GPDE, and the problem transformed into an algebraic equation, which was simpler in this case due to the Dirichlet boundary conditions satisfied by the sinusoidal kernel of the DFSTM. Analytical buckling solutions were determined for the two considered cases of uniaxial and biaxial compressive loads in terms of the buckling modes, Poisson's ratio (μ), and thickness (h) to least dimension (a) ratio. Critical buckling loads (P_{cr}) determined at the first buckling modes agreed with previously obtained Navier solutions for Mindlin, Reddy, and Refined plates. P_{cr} calculated for ratios of h/a equal to 0.01 converged to the solutions obtained using Kirchhoff-Love plate theory (KLPT), illustrating the applicability of the GPDE to thin and thin plate buckling.

1. Introduction

Determining least loads causing buckling failures in plates under in-plane compressive forces is an important research theme vital to effective plate analysis and design. The buckling problem has been very well discussed in the context of several theories of plates, such as the Kirchhoff-Love plate theory (KLPT), shear deformation theories, and refined plate theories (RPTs). KLPT neglects shear deformation and thus does not apply to thick plates where shear deformation plays a significant role [1–3]. KLPT overestimates the buckling load capacities of thick plates and is thus unsafe for such plates. The limitations of KLPT led to the development of shear deformation theories by Mindlin [4], Reddy [5-6] and refined theories by Shimpi [7] and Shimpi et al. [8] to account for shear deformation and render the formulations applicable to thick plates. Mindlin's first-order shear deformation plate problems were studied by Nwoji et al. [9-10], Ike [11-12]; Norouzzadeh et al. [13] and Bao et al. [14].

Composite and functionally graded material (FGM) plates have been studied by Do et al., [15], Soltani et al. [16], Kumar and Singh [17], Hadji and Arcar [18], and Mohseni and Naderi [19]. Mindlin's theory is a first-order shear deformation theory needing shear correction factors. Reddy's theory is a third-order shear deformation theory that does not need shear correction factors as shear stress-free boundary conditions are satisfied in the Reddy formulation. Reddy's theory, however is a set of five coupled PDEs with five unknown displacement variables, rendering it very complex for solutions. Shimpi's RPT is a displacement-based theory that is variationally consistent and has been formulated in terms of two displacement variables in Shimpi [7] and in terms of a single variable in Shimpi et al., [8]. Onyeka and Mama [20] used the direct variational energy approach for the bending analysis of plates. In another research, Onyeka and Okeke [21] used the polynomial shear deformation theory to analyze thick rectangular plates with clamped and free edges. Onyeka and Okeke [22] studied thick plates using refined shear deformation theory and energy method. Onyeka et al., [23-25] have used energy methods for thick plate problems under buckling and bending. Recent studies by Onyeka et al., [26-28] considered thick rectangular plates with various edge support conditions.

http://doi.org/10.30684/etj.2023.142504.1537
Received 10 August 2023; Received in revised form 13 October 2023; Accepted 14 October 2023; Available online 19 November 2023
20232412-0758/University of Technology-Iraq, Baghdad, Iraq

Onah et al., [29] used the stress function method to derive elastostatic solutions to thick circular plate bending problems. Singh and Kumar [30] studied the buckling response of functionally graded plates (FGP) modeled using a refined hyperbolic shear deformation theory (RHSDT). Their formulated equations considered higher-order effects of shear, normal stress deformations, and thickness stretching. They used both polynomial and hyperbolic functions to obtain a parabolic distribution of shear stresses across the thickness, thus ensuring that the formulated boundary value problem satisfied the shear stress-free boundary conditions at the top and bottom surfaces of the plate. Thus, the shear modification factor was eliminated. The finite element method and MATLAB code were used to solve the resulting BVP. Convergence studies were used to illustrate the effectiveness of the solutions and validation of the results.

Nareen and Shimpi [31] have also developed solutions for thick plates using refined hyperbolic shear deformation theory. Ferreira and Roque [32] used the radial basis function method for solving thick plate problems but did not consider rectangular thick plate buckling problems. Gomaa et al., [33] used finite element techniques to solve thick isotropic plates but did not consider buckling analysis. Zargaripoor et al., [34] used an exact wave propagation approach to solve the eigen vibration and eigen buckling problems of thick plates within the framework of the third-order shear deformation plate theory. They obtained exact solutions using the method. Malikan and Nguyen [35] used a single variable first-order shear deformation theory to solve biaxial stability problems of plates based on Eringen's nonlocal elasticity approach. Zhong and Qu [36] analyzed clamped rectangular thick plate bending problems but did not consider buckling. Literature review shows several methods used to solve the plate buckling problems include Navier, Levy, Generalized integral transform, and approximate or numerical methods like the finite element, finite difference, and variational methods.

Reddy and Phan [37], Deepak et al., [38], Timoshenko and Gere [39], Thai and Choi [40], Srinivas and Rao [41], and Hashemi et al., [42] have used Navier method for buckling analysis of simply supported thick plates. For simply supported plates, the Navier method and occasionally the Levy method have been used by Timoshenko and Gere [39]. Galerkin methods have been used for thin plate buckling problems by Onyia et al., [43-44] and by Oguaghamba and Ike [45]. Their studies did not extend to thick plates. Single finite integral transform methods have been used by Onah et al. [46] and Onyia et al. [47] for thin plate buckling solutions for CCSS and SSCF and SSSS plates, respectively. They obtained analytical solutions that are exact within the framework of the thin plate buckling theory adopted. Ike et al., [48] used the generalized integral transform method to solve the thin SSCC plate buckling problem and obtained exact solutions. Nwoji et al., [49] obtained exact buckling solutions for simply supported thin plates using DFSTM. They did not study thick plate buckling. Oguaghamba and Ike [50] used the single finite Fourier sine transform method (SFFSTM) for the natural frequency analysis of flexural vibrations of Kirchhoff plates.

Mama et al., [51] used the SFFSTM for the exact bending solutions of rectangular Kirchhoff plates submitted to triangular load distribution. However, the double finite sine transform method (DFSTM) has not been used to solve the single variable plate buckling problems (SVPBPs) presented in this paper. This paper thus presents DFSTM for solving SVPBPs with simply supported edges and for uniaxial and biaxial compressive loads.

2. Governing Equations

The displacement field components of the single variable plate buckling theory (SVPBT) are formulated using only one unknown u_z as given by Equations (1,2) and (3) [7,52]:

$$u_x = -z\frac{\partial u_z}{\partial x} - \frac{D}{G}\left(\frac{3}{10}\left(\frac{z}{h}\right) - 2\left(\frac{z}{h}\right)^3\right)\frac{\partial}{\partial x}\nabla^2 u_z \quad (1)$$

$$u_y = -z\frac{\partial u_z}{\partial y} - \frac{D}{G}\left(\frac{3}{10}\left(\frac{z}{h}\right) - 2\left(\frac{z}{h}\right)^3\right)\frac{\partial}{\partial y}\nabla^2 u_z \quad (2)$$

$$u_z = u_{zb} - \frac{D}{Gkh}\nabla^2 u_{zb} \quad (3)$$

where u_x, u_y, and u_z are displacements in the x, y, and z directions, respectively. u_{zb} is the bending component of u_z. G is the shear modulus, D is the modulus of flexural rigidity, and h is the plate thickness. The material forces are [7]:

$$M_{xx} = -D\left(\frac{\partial^2 u_z}{\partial x^2} + \mu\frac{\partial^2 u_z}{\partial y^2}\right) \quad (4a)$$

$$M_{yy} = -D\left(\frac{\partial^2 u_z}{\partial y^2} + \mu\frac{\partial^2 u_z}{\partial x^2}\right) \quad (4b)$$

$$M_{xy} = -D(1-\mu)\frac{\partial^2 u_z}{\partial x \partial y} \quad (5)$$

$$Q_x = -D\frac{\partial}{\partial x}\nabla^2 u_z \quad (6)$$

$$Q_y = -D\frac{\partial}{\partial y}\nabla^2 u_z \tag{7}$$

where M_{xx}, M_{yy}, are the bending moments, M_{xy} is the twisting moment, and Q_x, Q_y are shear force distributions μ is Poisson's ratio and∇^2 is the two-dimensional Laplacian.

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \tag{8}$$

2.1 Governing Partial Differential Equations of Equilibrium (GPDE)

For the plate shown in Figure 1, which is subjected to a combination of in-plane forces P_x (acting in the x direction), P_y (acting in the y direction), and P_{xy} (acting in the xy direction), and a transverse distributed load of intensity $q(x,y)$, the equations of equilibrium are [7]:

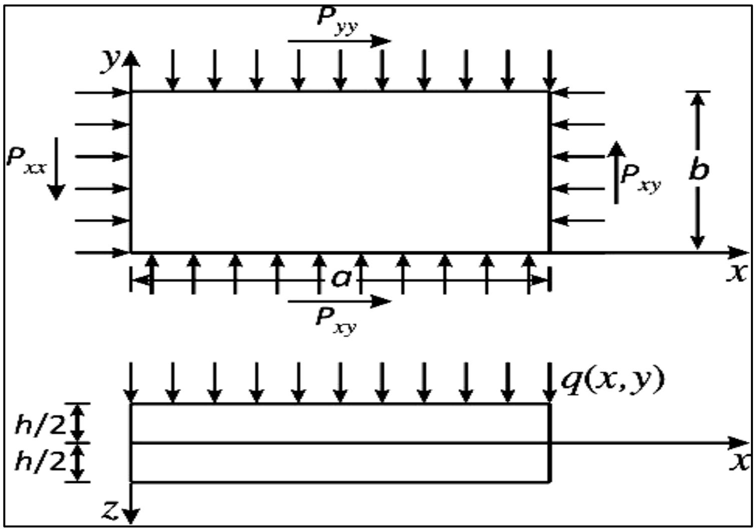

Figure 1: Thick plate under combined in-plane forces and transverse forces

$$\frac{\partial P_{xx}}{\partial x} + \frac{\partial P_{xy}}{\partial y} = 0 \tag{9}$$

$$\frac{\partial P_{xy}}{\partial x} + \frac{\partial P_{yy}}{\partial y} = 0 \tag{10}$$

$$\frac{\partial^2 M_{xx}}{\partial x^2} - 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_{yy}}{\partial y^2} = -\left(q(x,y) + P_{xx}\frac{\partial^2 u_z}{\partial x^2} + 2P_{xy}\frac{\partial^2 u_z}{\partial x \partial y} + P_{yy}\frac{\partial^2 u_z}{\partial y^2}\right) \tag{11}$$

Substituting Equations (3) – (5) into Equation (11) gives, after simplification:

$$D\nabla^2 u_{zb} = q(x,y) + P_{xx}\frac{\partial^2 u_z}{\partial x^2} + 2P_{xy}\frac{\partial^2 u_z}{\partial x \partial y} + P_{yy}\frac{\partial^2 u_z}{\partial y^2} \tag{12}$$

$$\nabla^4 = \nabla^2\nabla^2 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4} \tag{13}$$

where ∇4 is the biharmonic operator.
From Equation (13), the governing equation of equilibrium of the SVPBT is:

$$D\nabla^4 u_{zb} = q(x,y) + \frac{P_{xx}}{D}\frac{\partial^2}{\partial x^2}\left(u_{zb} - \frac{D}{Gkh}\nabla^2 u_{zb}\right) + \frac{P_{yy}}{D}\frac{\partial^2}{\partial y^2}\left(u_{zb} - \frac{D}{Gkh}\nabla^2 u_{zb}\right) + \frac{2P_{xy}}{D}\frac{\partial^2}{\partial x \partial y}\left(u_{zb} - \frac{D}{Gkh}\nabla^2 u_{zb}\right) \tag{14}$$

where k is the shear force distribution.

3. Formulation of the Double Finite Sine Transform (DFST) of the GPDE

The problem considered in this paper is shown in Figure 2, which is a simply supported thick plate under in-plane forces P_{xx} and P_{yy} only.

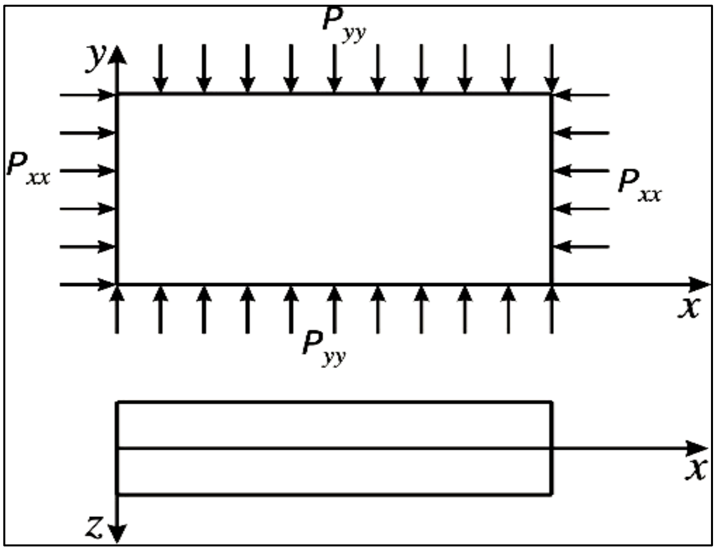

Figure 2: Simply supported thick plate under biaxial loading

Thus, $P_{xy} = 0,\ q(x, y) = 0$. The GPDE is then:

$$\nabla^4 u_{zb} - \frac{P_{xx}}{D}\frac{\partial^2}{\partial x^2}\left(u_{zb} - \frac{D}{Gkh}\nabla^2 u_{zb}\right) - \frac{P_{yy}}{D}\frac{\partial^2}{\partial y^2}\left(u_{zb} - \frac{D}{Gkh}\nabla^2 u_{zb}\right) = 0 \tag{15}$$

Applying the DFST method [18] gives:

$$\int_0^a\int_0^b\left(\nabla^2 u_{zb} - \frac{P_{xx}}{D}\frac{\partial^2}{\partial x^2}u_{zb} + \frac{P_{xx}}{Gkh}\frac{\partial^2}{\partial x^2}\nabla^2 u_{zb} - \frac{P_{yy}}{D}\frac{\partial^2}{\partial y^2}u_{zb} + \frac{P_{yy}}{Gkh}\frac{\partial^2}{\partial y^2}\nabla^2 u_{zb}\right)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\,dxdy = 0 \tag{16}$$

By the linearity properties of the DFSTM, [18] the equation simplifies as follows:

$$\begin{aligned}&\int_0^a\int_0^b \nabla^4 u_{zb}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}dxdy - \frac{P_{xx}}{D}\int_0^a\int_0^b\frac{\partial^2}{\partial x^2}u_{zb}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}dxdy + \\ &\frac{P_{xx}}{Gkh}\int_0^a\int_0^b\frac{\partial^2}{\partial x^2}\nabla^2 u_{zb}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}dxdy - \frac{P_{yy}}{D}\int_0^a\int_0^b\frac{\partial^2}{\partial y^2}u_{zb}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}dxdy + \\ &\frac{P_{yy}}{Gkh}\int_0^a\int_0^b\frac{\partial^2}{\partial y^2}\nabla^2 u_{zb}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}dxdy = 0\end{aligned} \tag{17}$$

Each integral in Equation (17) can be further simplified. Thus,

$$\begin{aligned}\int_0^a\int_0^b \nabla^4 u_{zb}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}dxdy &= \int_0^a\int_0^b\left(\frac{\partial^4 u_{zb}}{\partial x^4} + 2\frac{\partial^4 u_{zb}}{\partial x^2\partial y^2} + \frac{\partial^4 u_{zb}}{\partial y^4}\right)\times\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}dxdy \\ &= \left(\left(\frac{m\pi}{a}\right)^4 + 2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 + \left(\frac{n\pi}{b}\right)^4\right)\times\int_0^a\int_0^b u_{zb}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}dxdy = \\ &\left(\left(\frac{m\pi}{a}\right)^4 + 2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 + \left(\frac{n\pi}{b}\right)^4\right)\bar{U}_{zb}\end{aligned} \tag{18}$$

$$\bar{U}_{zb} = \int_0^a\int_0^b u_{zb}\sin\frac{m\pi x}{a}\sin\frac{n\pi x}{b}dxdy \tag{19}$$

Where, $\bar{U}_{zb}$ is the DFST of u_{zb}

Similarly,

$$\int_0^a\int_0^b \frac{\partial^2 u_{zb}}{\partial x^2}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}dxdy = -\left(\frac{m\pi}{a}\right)^2 \bar{U}_{zb}$$

$$\int_0^a\int_0^b \frac{\partial^2 u_{zb}}{\partial y^2}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}dxdy = -\left(\frac{n\pi}{b}\right)^2 \bar{U}_{zb} \tag{20}$$

$$\int_0^a\int_0^b \frac{\partial^2}{\partial x^2}\nabla^2 u_{zb}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}dxdy = \tag{21}$$

$$\left(\left(\frac{m\pi}{a}\right)^4 + \left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2\right)\bar{U}_{zb} \tag{22}$$

$$\int_0^a\int_0^b \frac{\partial^2}{\partial y^2}\nabla^2 u_{zb}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}dxdy = \left(\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 + \left(\frac{n\pi}{b}\right)^4\right)\bar{U}_{zb} \tag{23}$$

Then, by substitution, the transformed equation is:

$$\left(\left(\frac{m\pi}{a}\right)^4 + 2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 + \left(\frac{n\pi}{b}\right)^4\right)\bar{U}_{zb} + \frac{P_{xx}}{D}\left(\frac{m\pi}{a}\right)^2\bar{U}_{zb} +$$
$$\frac{P_{xx}}{Gkh}\left(\left(\frac{m\pi}{a}\right)^4 + \left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2\right)\bar{U}_{zb} + \frac{P_{yy}}{D}\left(\frac{n\pi}{b}\right)^2\bar{U}_{zb} +$$
$$\frac{P_{yy}}{Gkh}\left(\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 + \left(\frac{n\pi}{b}\right)^4\right)\bar{U}_{zb} = 0 \tag{24}$$

For nontrivial solutions, $\bar{U}_{zb} \neq 0$

Hence,

$$\left(\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right)^2 + \frac{P_{xx}}{D}\left(\left(\frac{m\pi}{a}\right)^2 + \frac{D}{Gkh}\left(\left(\frac{m\pi}{a}\right)^4 + \left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2\right)\right) + \frac{P_{yy}}{D}\left(\left(\frac{n\pi}{b}\right)^2 +\right.$$
$$\left.\frac{D}{Gkh}\left(\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 + \left(\frac{n\pi}{b}\right)^4\right)\right) = 0 \tag{25}$$

4. Results and Discussion

Case 1: $P_{yy} = 0,\ P_{xx} = -P_0$

$$\left(\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right)^2 - \frac{P_0}{D}\left(\left(\frac{m\pi}{a}\right)^2 + \frac{D}{Gkh}\left(\left(\frac{m\pi}{a}\right)^4 + \left(\frac{n\pi}{b}\right)^2\left(\frac{m\pi}{a}\right)^2\right)\right) = 0 \tag{26}$$

$$\frac{P_0}{D} = \frac{\left(\frac{m\pi}{a}\right)^4 + 2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 + \left(\frac{n\pi}{b}\right)^4}{\left(\frac{m\pi}{a}\right)^2 + \frac{D}{Gkh}\left(\left(\frac{m\pi}{a}\right)^4 + \left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2\right)} \tag{27}$$

$$\frac{P_0 a^2}{\pi^2 D} = \frac{m^4 + 2m^2n^2\frac{a^2}{h^2} + \frac{n^4a^4}{b^4}}{m^2 + \frac{D}{Gka^2h}\left(m^4\pi^2 + \frac{\pi^2a^2}{b^2}\right)} \tag{28}$$

$$\frac{D}{Gkh} = \frac{Eh^3}{12(1-\mu^2)}\frac{2(1+\mu)}{Eh}\frac{1}{5/6} = \frac{h^2}{5(1-\mu)} \tag{29}$$

$$\frac{D}{Gkha^2} = \frac{1}{5(1-\mu)}\left(\frac{h}{a}\right)^2 \tag{30}$$

$$\frac{P_0 a^2}{\pi^2 D} = \frac{m^4 + 2m^2n^2\left(\frac{a}{b}\right)^2 + n^4\left(\frac{a}{b}\right)^4}{m^2 + \frac{1}{5(1-\mu)}\left(\frac{h}{a}\right)^2\left(m^4\pi^2 + \pi^2\left(\frac{a}{b}\right)^2\right)} \tag{31}$$

P_0 is critical when $n = 1,\ m = 1$.
Hence,

$$\frac{P_0 a^2}{\pi^2 D}(m=1, n=1) = \frac{P_{cr} a^2}{\pi^2 D} = \frac{1 + 2\left(\frac{a}{b}\right)^2 + \left(\frac{a}{b}\right)^4}{1 + \frac{1}{5(1-\mu)}\left(\frac{h}{a}\right)^2\left(\pi^2 + \pi^2\left(\frac{a}{b}\right)^2\right)} \tag{32}$$

For square plates, $(a/b) = 1$

$$\frac{P_{cr} a^2}{\pi^2 D} = \frac{4}{1 + \frac{1}{5(1-\mu)}\left(\frac{h}{a}\right)^2 (2\pi^2)} \tag{33}$$

Hence, $P_{cr}a^2/\pi^2 D$ depends upon (h/a) and can be computed if (h/a) is known. Values of $P_{cr}a^2/\pi^2 D$ for $\mu = 0.30$ and $h/a = 0.01$, $h/a = 0.05$, $h/a = 0.1$, and $h/a = 0.2$ are calculated using Equation (33) and presented in Table 1 together with values from previous researchers. Table 1 shows excellent agreement between the present SVPBT results and the results of previous researchers who used Mindlin, Reddy, and Refined plate theories.

Case 2: $P_{xx} = P_{yy} = -P_0$

Then Equation (25) becomes:

$$\left(\frac{m\pi}{a}\right)^4 + 2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 + \left(\frac{n\pi}{b}\right)^4 - \frac{P_0}{D}\left(\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right) + \frac{D}{Gkh}\left(\left(\frac{m\pi}{a}\right)^4 + 2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 + \left(\frac{n\pi}{b}\right)^4\right) = 0 \tag{34}$$

Rearranging,

$$\frac{P_0}{D} = \frac{\left(\frac{m\pi}{a}\right)^4 + 2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 + \left(\frac{n\pi}{b}\right)^4}{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \frac{D}{Gkh}\left(\left(\frac{m\pi}{a}\right)^4 + 2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 + \left(\frac{n\pi}{b}\right)^4\right)} \tag{35}$$

$$\frac{P_0 a^2}{\pi^2 D} = \frac{m^4 + 2m^2 n^2\left(\frac{a}{b}\right)^2 + n^4\left(\frac{a}{b}\right)^4}{m^2 + n^2\left(\frac{a}{b}\right)^2 + \frac{D}{Gkha^2}\left(m^2\pi^2 + 2n^2\pi^2\left(\frac{a}{b}\right)^2 + n^4\pi^2\left(\frac{a}{b}\right)^2\right)} \tag{36}$$

$$\frac{P_0 a^2}{\pi^2 D} = \frac{m^4 + 2m^2 n^2\left(\frac{a}{b}\right)^2 + n^4\left(\frac{a}{b}\right)^4}{m^2 + n^2\left(\frac{a}{b}\right)^2 + \frac{1}{5(1-\mu)}\left(\frac{h}{a}\right)^2\left(m^2\pi^2 + 2n^2\pi^2\left(\frac{a}{b}\right)^2 + n^4\left(\frac{a}{b}\right)^4\pi^2\right)} \tag{37}$$

The least buckling load occurs when $m = 1$, $n = 1$, and is found as:

$$\frac{P_0 a^2}{\pi^2 D} = \frac{1 + 2\left(\frac{a}{b}\right)^2 + \left(\frac{a}{b}\right)^4}{1 + \left(\frac{a}{b}\right)^2 + \frac{1}{5(1-\mu)}\left(\frac{h}{a}\right)^2\left(\pi^2 + 2\pi^2\left(\frac{a}{b}\right)^2 + \pi^2\left(\frac{a}{b}\right)^4\right)} \tag{38}$$

For square plates $(a/b) = 1$, and

$$\frac{P_0 a^2}{\pi^2 D} = \frac{4}{2 + \frac{1}{5(1-\mu)}\left(\frac{h}{a}\right)^2 (4\pi^2)} \tag{39}$$

For $\mu = 0.30$, the critical buckling load, P_{cr} is

$$\frac{P_{cr} a^2}{\pi^2 D} = \frac{4}{2 + \frac{1}{3.5}\left(\frac{h}{a}\right)^2 4\pi^2} = \frac{4}{2 + \frac{4\pi^2}{3.5}\left(\frac{h}{a}\right)^2} \tag{40}$$

Thus, in this case $P_{cr}a^2/\pi^2 D$ depends upon (h/a) and can be computed if (h/a) is given. Values of $P_{cr}a^2/\pi^2 D$ for $h/a = 0.01$, $h/a = 0.05$, $h/a = 0.10$, and $h/a = 0.20$ are determined using Equation (40) and presented in Table 2 with the values from previous researchers using other plate theories and Navier's method. Table 2 illustrates the excellent agreement between the present SVPBT solution and the results of previous researchers who used Mindlin, Reddy, and Refined plate theories.

This paper uses the double finite sine transform method (DFSTM) to derive the solution for the single variable plate buckling problems (SVPBPs) under uniform uniaxial and uniform biaxial compressive loads. SVPBPs formulated by Shimpi et al., [5] is a refined plate theory derived from a a single unknown displacement variable. The governing partial differential equation (GPDE) takes account of shear deformation and is thus applicable to thick plates. The DFSTM adopted in this work transforms the GPDE into an algebraic problem for simply supported plates, which is simpler to solve.

Closed-form solutions were found for two cases: uniform uniaxial compression load in the *x* direction and uniform biaxial compression load in the *x* and *y* directions. In both cases, analytical expressions were derived for the eigenvalues in terms of *m*, *n*, the kernel function parameters of the integral transform, and the plate geometrical features viz thickness, *h,* and *a*, plate in-plane dimension. It is found that critical buckling load corresponds to the least value when $m = n = 1$, and are found in terms of h/a. Non-dimensional critical buckling loads were computed for Poisson's ratio of 0.30 and varying h/a values and tabulated in Tables 1 and 2, respectively, for the two cases considered. Tables 1 and 2 were also used to compare the present results with previous results from other theories.

Tables 1 and 2 illustrate the agreement of present results with previous results of Hashemi et al., [42] using Mindlin theory, Reddy and Phan [37] using Reddy theory, and Thai and Choi [40] using Refined plate theory (RPT). Tables 1 and 2 further show the convergence of the present and previous results to the KLPT results when $h/a = 0.01$ illustrating the applicability of the SVPBT to thin plates and thick plate buckling analysis. Tables 1 and 2 are represented graphically as Figures 3 and 4, where Figure 3 is the graph of the buckling coefficient K_{cr} plotted against a/h for the uniaxial buckling case, and Figure 4 is the graph of K_{cr} plotted against a/h for the biaxial buckling case. Figures 3 and 4 illustrate the very close agreement between present and previously obtained thick plate.

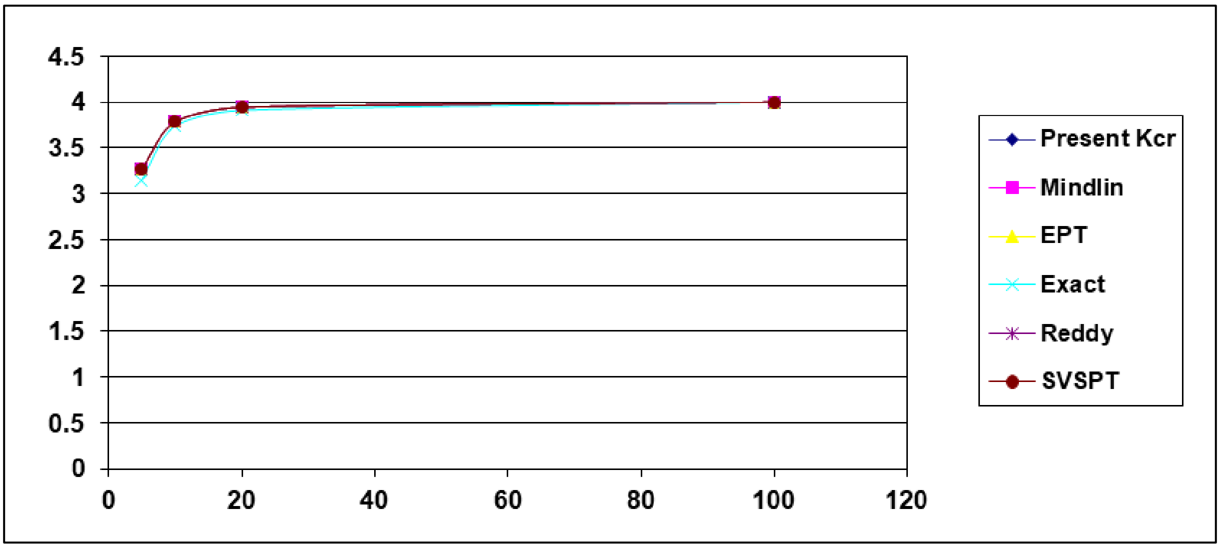

Figure 3: Graph of variation of K_{cr} with a/h for uniaxial buckling of thick plate where $K_{cr} = P_{cr}a^2/\pi^2 D$

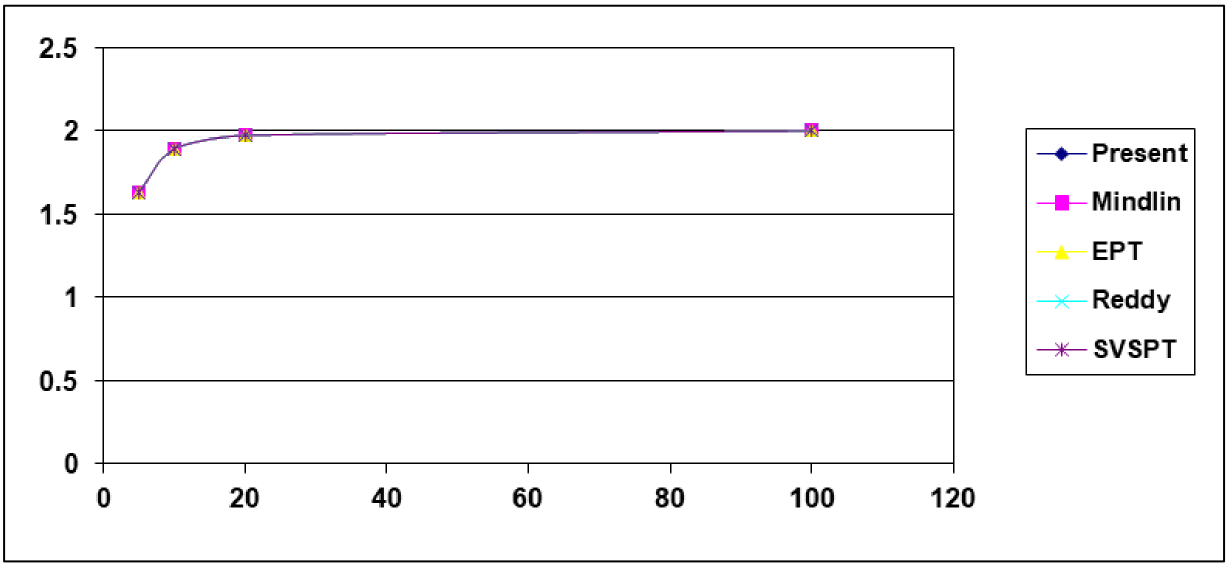

Figure 4: Graph of variation of K_{cr} with a/h for biaxial buckling of thick plate where $K_{cr} = P_{cr}a^2/\pi^2 D$

Table 1: Critical buckling load coefficients of a simply supported square plate under uniform uniaxial compression load (for $\mu = 0.30$)

	$P_{xx} = -P_0$, $P_{yy} = 0$, $P_{xy} = 0$			
Reference/Theory	$h/a = 0.01$	$h/a = 0.05$	$h/a = 0.10$	$h/a = 0.20$
Present study SVPBT	4.0000	3.9444	3.7864	3.2637
Mindlin (Hashemi et al., [42])	4.0000	3.9444	3.7864	3.2637
RPT (Thai and Choi., [40])	4.0000	3.9443	3.7865	3.2653
Exact (Srivinas and Rao., [41])	4.0000	3.9110	3.7410	3.1500
Reddy (Reddy and Phan, [37])		3.9443	3.7865	3.2653
SVSPT (Deepak et al., [38])	4.0000	3.9444	3.7864	3.2637
KLPT (Timoshenko and Gere., [39])	4.000 0	4.0000	4.0000	4.0000

Table 2: Critical buckling load coefficients of a simply supported square plate under uniform biaxial compression load (for $\mu = 0.30$)

	$P_{yy} = P_{xx} = -P_0$, $P_{xy} = 0$			
Reference /Theory	$h/a = 0.01$	$h/a = 0.05$	$h/a = 0.10$	$h/a = 0.20$
Present study SVPBT	2.0000	1.9722	1.8932	1.6319
Mindlin (Hashemi et al., [42])	2.0000	1.9722	1.8932	1.6319
RPT (Thai and Choi., [40])	2.0000	1.9722	1.8932	1.6327
Reddy (Reddy and Phan, [37])	2.0000	1.9722	1.8933	1.6327
SVSPT (Deepak et al., [38])	2.0000	1.9722	1.8932	1.6312
Exact (Srivinas and Rao., [41])	–	–	–	–
KLPT (Timoshenko and Gere., [39])	2.0000	2.0000	2.0000	2.0000

5. Conclusion

In conclusion, the DFSTM has been used to derive analytical buckling load solutions to simply supported plates using single variable refined plate buckling theory formulated by Shimpi et al., [5]. The DFSTM transforms the GPDE from simply supported boundaries into algebraic problems, which are easier to solve. For both cases of uniform uniaxial compressive loading in the *x* direction and uniform biaxial compressive loads, analytical expressions were determined for the buckling loads for any buckling mode *m, n*. Critical values of the buckling loads found at $m = 1$, $n = 1$, showed agreement with previously obtained values that used Mindlin theory, Reddy theory and RPT. The solution showed convergence to thin plate buckling solutions when $h/a = 0.01$.

Notation / Nomenclature

x, y	in-plane Cartesian coordinates
z	transverse Cartesian coordinate
u_x	displacement component in *x* direction
u_y	displacement component in *x* direction
u_z	displacement component in *z* direction
G	shear modulus
D	modulus of flexural rigidity
M_{xx}, M_{yy}	bending moment
M_{xy}	twisting moment
Q_x, Q_y	shear force distributions
μ	Poisson's ratio
∇^2	two-dimensional Laplacian
P_{xx}	in-plane force acting in the *x*-direction
P_{yy}	in-plane force acting in the *y*-direction
P_{xy}	in-plane twisting force
$q(x, y)$	transverse distributed load intensity
∇^4	biharmonic differential operator
u_{z_b}	bending component of u_z
h	thickness of plate
a, b	in-plane dimensions of plate
U_{z_b}	double finite sine transform of u_{z_b}
$\frac{\partial}{\partial x}$	partial derivative with respect to *x*
$\frac{\partial^2}{\partial x^2}$	second partial derivative with respect to *x*

$\frac{\partial}{\partial y}$	partial derivative with respect to *y*
$\frac{\partial^2}{\partial y^2}$	second partial derivative with respect to *y*
$\frac{\partial^2}{\partial x \partial y}$	mixed partial derivative with respect to *x* and *y*
DFST	double finite sine transform
DFSTM	double finite sine transform method
GDPE	governing partial differential equation
RPT	refined plate theory
KLPT	Kirchhoff-Love plate theory
RPT	refined plate theories
CCSS	rectangular plate with two clamped edges and two simply supported edges
SSCF	rectangular plate with two simply supported edges, one clamped edge and one free edge
SSSS	rectangular plate with four simply supported edges
SVPBPs	single variable plate buckling problems
SVPBT	single variable plate buckling theory.

Funding

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

Data availability statement

The data that support the findings of this study are available on request from the corresponding author.

Conflicts of interest

The authors declare that there is no conflict of interest.

References

[1] C.C. Ike, Exact analytical solutions to bending problems of SFrSFr plates using variational Kantorovich-Vlasov method, J. Compu. Appl. Mech., 54 (2023) 186 – 203. https://doi.org/10.22059/jcamech.2023.351563.776

[2] C.C. Ike, Variational Ritz-Kantorovich-Euler Lagrange method for the elastic buckling analysis of fully clamped Kirchhoff thin plate, ARPN J. Eng. Appl. Sci., 16 (2021) 224 – 241.

[3] C.C. Ike, Generalized integral transform method for bending analysis of clamped rectangular thin plate, J. Comput. Appl. Mech., 53 (2022) 599 – 635. https://doi.org/10.22059/jcamech.2022.350620.768

[4] R. D. Mindlin, Influence of rotary inertia and shear on flexural motions of isotropic elastic plates, ASME J. Appl. Mech., 18 (1951) 31 – 38. https://doi.org/10.1115/1.4010217

[5] J. N. Reddy, A simple higher order theory for laminated composite plates, ASME J. Appl. Mech., 51(1981) 745 – 752.

[6] J. N. Reddy, A refined non-linear theory of plates with transverse shear deformation., Int. J. Solids Struct., 20 (1984) 881 – 896. https://doi.org/10.1016/0020-7683(84)90056-8

[7] R.P. Shimpi, Refined plate theory and its variants, AIAA J., 40 (2002) 137 – 146. https://doi.org/10.2514/2.1622

[8] R. P. Shimpi, R.A. Shetty, A. Guha, A single variable refined theory for free vibrations of a plate using inertia related terms in displacements, Eur. J. Mech. A. Solids, 65 (2017) 136 – 148.https://doi.org/10.1016/j.euromechsol.2017.03.005

[9] C. U. Nwoji, B.O. Mama, H.N. Onah, C.C. Ike, Flexural analysis of simply supported rectangular Mindlin plates under bisinusoidal transverse load, ARPN J. Eng. Appl. Sci., 13 (2018) 4480 – 4488.

[10] C.U. Nwoji, H.N. Onah, B.O. Mama, C.C. Ike, Theory of elasticity formulation of the Mindlin plate equations, Int. J. Eng. Technol., 9 (2018) 4344 – 4352.

[11] C.C. Ike, Equilibrium approach in the derivation of differential equations of homogeneous isotropic Mindlin plates, Niger. J. Technol., 36 (2017) 346 – 350. https://doi.org/10.4314/njt.v36i2.4

[12] C.C. Ike, Mathematical solutions for the flexural analysis of Mindlin's first order shear deformable circular plates, J. Math. Models Eng., 4 (2018) 50 – 72. https://doi.org/10.21595/mme.2018.19825

[13] A. Norouzzadeh, R. Ansari, H. Rouhi, Isogeometric analysis of Mindlin nanoplates based on the integral formulation of nonlocal elasticity Multidiscip, Model. Mater. Struct., 14 (2018) 810 – 827. https://doi.org/10.1108/mmms-09-2017-0109

[14] Y. Bao, S. Cen, C. Li, Distortion-resistant and locking free eight node elements effectively capturing the edge effects of Mindlin-Reissner plates, Eng. Comput., 34 (2017) 548 – 586. https://doi.org/10.1108/EC-04-2016-0143

[15] Do V.T., Pham V.V., Nguyen H.N., On the development of refined plate theory for static bending behaviour of functionally graded plates, Math. Model. Eng. Probl., 2836763, 2020. https//doi.org/10.1155/2020/2836763

[16] K. Soltani, A. Bassaim, M.S.A. Houari, A. Kaci, M. Benguediab, A. Tounsi, M.Sh. Alhodaly, A novel hyperbolic shear deformation theory for the mechanical buckling analysis of advanced composite plates resting on elastic foundations, Steel Compos. Struct., 30 (2019) 13 – 29. https://doi.org/10.12989/scs.2019.30.1.013

[17] R. Kumar. J. Singh, Assessment of higher order transverse shear deformation theories for modelling and buckling analysis of FGM plates using RBF based meshless approach, Multidiscip Model. Mater. Struct., 14 (2018) 891 – 907. https://doi.org/10.1108/mmms-07-2017-0069.

[18] L. Hadji, M. Avcar, Free vibration analysis of FG porous sandwich plates under various boundary conditions, J. Appl. Comput. Mech., 7 (2021) 505 – 519. https://doi.org/10.22055/jacm.2020.35328.2628

[19] E. Mohseni, A. Naderi, Effect of higher order shear and normal deformations theory in buckling analysis of thick functionally graded plates, J. Comput. Appl. Mech., 54 (2023) 347 – 364. https://doi.org/10.22059/jcamech.2023.361677.852

[20] F.C. Onyeka, B.O. Mama, Analytical study of bending characteristics of an elastic rectangular plate using direct variational energy approach with trigonometric function, Emerg. Sci. J., 5 (2021) 916-928. https://doi.org/10.28991/esj-2021-01320

[21] F.C. Onyeka, E.T. Okeke, Analytical solution of thick rectangular plate with clamped and free support boundary condition using polynomial shear deformation theory, Adv. Sci. Technol. Eng. Syst. J., 6 (2021) 1427 – 1439. https://dx.doi.org/10.25046/aj0601162

[22] F.C. Onyeka, T.E. Okeke, New refined shear deformation theory effect on non-linear analysis of a thick plate using energy method, Arid Zone. J. Eng. Technol. Environ., 17 (2021) 121 – 140.

[23] F.C. Onyeka, B.O. Mama, T.E. Okeke, Exact three-dimensional stability analysis of plate using a direct variational energy method, Civ. Eng. J., 8 (2022) 60 – 80. https://doi.org/10.28991/CEJ-2022-08-01-05

[24] F.C. Onyeka, C.D. Nwa-David, T.E. Edozie, Analytical solution for the static bending elastic analysis of thick rectangular plate structures, Eng. Technol. J., 40 (2022)1548 – 1559. https://doi.org/10.30684/etj.2022.134687.1244

[25] F.C. Onyeka, T.E. Okeke, B.O. Mama, Static elastic bending analysis of a three-dimensional clamped thick rectangular plate using energy method, High Tech Innov. J., 3 (2022) 267 – 281. https://doi.org/10.28991/HIJ-2022-03-03-03

[26] F.C. Onyeka, E.D. Nwa-David, B.O. Mama, Static bending solutions for an isotropic rectangular clamped/simply supported plate using 3-D plate theory, J. Comput. Appl. Mech., 54 (2023) 1–18. https://doi.org/10.22059/JCAMECH.2022.349835.764

[27] F.C. Onyeka, T.E. Okeke, C. Nwa-David, B.O. Mama, Analytical elasticity solution for accurate prediction of stresses in a rectangular plate using exact 3-D theory, J. Comput. Appl. Mech., 54 (2023) 167-185. https://doi.org/10.22059/jcamech.2022.351892.781

[28] F.C. Onyeka, B.O. Mama, C.D. Nwa-David Stress analysis of transversely loaded isotropic three-dimensional plates using a polynomial shear deformation theory, Eng. Technol. J., 41 (2023) 603 – 618. https://doi.org/10.30684/etj.2023.137410.1345

[29] H.N. Onah, M.E. Onyia, B.O. Mama, C.U. Nwoji, C.C. Ike, First principles derivation of displacement and stress function for three-dimensional elasto static problems, and application to the flexural analysis of thick circular plates, J. Comput. Appl. Mech., 51 (2020) 184 – 198. https://doi.org/10.22059/JCAMECH.2020.295989.471

[30] J. Singh, A. Kumar, Vibration and the buckling response of functionally graded plates, according to a refined hyperbolic shear deformation theory, Mech. Compos. Mater., 59 (2023) 725 – 742. https://doi.org/10.1007/s11029-023-10127-5

[31] K. Nareen, R.P. Shimpi, Refined hyperbolic shear deformation plate theory, Proc. Inst. Mech. Eng. C., J. Mech. Eng. Sci. 229 (2015) 2675 – 2686. http://dx.doi.org/10.1177/0954406214563739

[32] A.J.M. Ferreira, C.M.C. Roque, Analysis of thick plates by radial basis functions, Acta Mech., 217 (2011) 177 – 190. https://doi.org/10.1007/s00707-010-0395-5

[33] S.T. Gomaa, M.H. Baluch, H.H. Abdel-Rahman, A.K. Mohammed, Finite element modelling of thick isotropic plates, Eng. Comput., 8 (1991) 361 – 378. https://doi.org/10.1108/eb023845

[34] A. Zargaripoor, A. Bahrami, M. Nikkah Bahrami, Free vibration and buckling analysis of third-order shear deformation plate theory using exact wave propagation approach, J. Comput. Appl. Mech., 49 (2018) 102 – 124. https://doi.org/10.22059/JCAMECH.2018.249468.227

[35] M. Malikan, V.C. Nguyen, A novel one-variable first order shear deformation theory for biaxial buckling of a size-dependent plate based on Eringen's nonlocal differential law, World J. Eng., 15 (2018) 633 – 645. https://doi.org/10.1108/WJE-11-2017-0357

[36] Zhong Y., Xu Q., Analysis bending solutions of clamped rectangular thick plate, Math. Probl. Eng., 7539276, 2017. https://doi.org/10.1155/2017/7539276

[37] J.N. Reddy, N.D. Phan, Stability and vibration of isotropic, orthotropic and laminated plates according to a higher-order shear deformation theory, J. Sound Vib., 98 (1985) 158 – 170. https://doi.org/10.1016/0022-460X(85)90383-9

[38] S.A. Deepak, R.A. Shetty, K.K. Sudheer, G.L. Dushyanthkumar, Buckling analysis of thick plates using a single variable simple plate theory, J. Mines. Metals Fuels, 2021 (2021) 67 – 72.

[39] Timoshenko, S.P., Gere, G.M., Theory of Elastic Stability 2nd Edition, McGraw Hill Book Company, New York, 1961.

[40] H.T. Thai, D.H. Choi, Analytical solutions of refined plate theory for bending, buckling and vibration analyses of thick plates, Appl. Math. Modell., 37 (2013) 8310 – 8323.https://doi.org/10.1016/j.apm.2013.03.038

[41] S. Srinivas, A.K. Rao, Buckling of thick rectangular plates, AIAA J., 7 (1969) 1645 – 1646.

[42] S.H. Hashemi, K. Khorshidi, M. Amabili, Exact solutions for linear buckling of rectangular Mindlin plates, J. Sound Vib., 315 (2008) 318 – 342. http://dx.doi.org/10.1016/j.jsv.2008.01.059

[43] M.E. Onyia, E.O. Rowland-Lato, C.C. Ike, Galerkin-Kantorovich method for the elastic buckling analysis of thin rectangular SCSC plates, Int. J. Eng. Res. Technol., 13 (2020) 613 – 619. https://dx.doi.org/10.37624/IJERT/13.4.2020.613-619

[44] M.E. Onyia, E.O. Rowland-Late, C.C. Ike, Galerkin-Vlasov variational method for the elastic buckling analysis of SSCF and SSSS rectangular plates, Int. J. Eng. Res. Technol.,13 (2020) 1137 – 1146.http://dx.doi.org/10.37624/IJERT/13.6.2020.1137-1146

[45] O.A. Oguaghamba, C.C. Ike, Galerkin-Vlasov method for the elastic buckling analysis of Kirchhoff plate with one free edge and three simply supported edges under uniform uniaxial compression, ARPN J. Eng. Appl. Sci., 15 (2020) 1574 – 1581.

[46] H.N. Onah, C.U. Nwoji, C.C. Ike, B.O. Mama, Elastic buckling analysis of uniaxially compressed CCSS Kirchhoff plate using single finite Fourier sine transform method, Modell. Meas. Control, B, 87 (2018) 107 – 111. https://doi.org/10.18280/mmc_b.870208

[47] M.E. Onyia, E.O. Rowland-Late, C.C. Ike, Elastic buckling analysis of SSCF and SSSS rectangular thin plates using the single finite Fourier sine integral transform method, Int. J. Eng. Res. Technol., 13 (2020) 1147 – 1158. https://dx.doi.org/10.37624/IJERT/13.6.2020.1147-1158

[48] C.C. Ike, M.E. Onyia, E.O. Rowland-Lato, Generalized integral transform method for bending and buckling analysis of rectangular thin plate with two opposite edges simply supported and other edges clamped, Adv. Sci. Technol. Eng. Sys. J., 6 (2021) 283 – 296. http://dx.doi.org/10.25046/aj060133

[49] C.U. Nwoji, H.N. Onah, B.O. Mama, C.C. Ike, E.U. Ikwueze, Elastic buckling analysis of simply supported thin plates using the double finite sine transform method, Explorematics J. Innovative Eng. Technol., 1 (2017) 37 –47.

[50] O.A. Oguaghamba, C.C. Ike, Single finite Fourier sine integral transform method for the determination of natural frequencies of flexural vibration of Kirchhoff plates, Int. J. Eng. Res. Technol., 13 (2020) 470 – 476. https://dx.doi.org/10.37624/IJERT/13.3.2020.470-476

[51] B.O. Mama, O.A. Oguaghamba, C.C. Ike, Single finite Fourier sine integral transform method for the flexural analysis of rectangular Kirchhoff plate with opposite edges simply supported, other edges clamped for the case of triangular load distribution, Int. J. Eng. Res. Technol., 13 (2020) 1802 – 1813. https://dx.doi.org/10.37624/IJERT/13.7.2020.1802-1813

[52] R.P. Shimpi, P.J. Guruprasad, K.S. Pakhare, Single variable new first-order shear deformation theory for isotropic plates, Lat. Am. J. Solids Struct., 15 (2018) e124, 1-25. https://doi.org/10.1590/1679-78254842