Interaction Diagrams for Reinforced Concrete T-Columns

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Abstract
The present investigation gives a comprehensive theoretical study for the analysis of reinforced concrete T-columns under the simultaneous action of axial compressive load and uniaxial bending. A computer program is written which enables calculating the compressive and tensile forces together with their moments, and eventually gives the results in the form of design charts which can be used directly for design or analysis.
In order to illustrate the use of the proposed design charts, four examples are given, two of which are design examples while the other two are analytsis.

Keywords: Computer program; interaction diagram; reinforced concrete; T-columns; uniaxial bending.

منحنيات التصميم للأعمدة الخرسانية المسلحة ذات مقطع على شكل حرف *T*

الخلاصة
يقدم البحث الحالي دراسة نظرية موسعة لتحليل الأعمدة الكونكريتيــة المــسلحة علــى شــكل حرف *T* تحت تأثير حمل أنضغاط محوري و عزم انحناء أحادي المحور تم كتابة برنامج لإدخال متطلبات التحليل وحساب قوى الأنضغاط و الشد وعز ومهما باســتخدام الحاســوب واســتنباط النتائج على شكل منحنيات للتصميم يمكن استخدامها بشكل مباشر.
ولغرض توضيح عملية استخدام منحنيات التصميم المقترحة والمستحدثة في هذا البحث تــم وضع أربعة أمثلة ، اثنان منها تصميمية و الاثنان الباقية تحليلية.

Notation

b_f flange width,
b_w web width,
c depth of compression zone,
C_c calculated force for compression region,
d_{si} distance from the extreme compression fibre to the centroid of any arbitrary reinforcing bar,
f_c' specified compressive strength concrete of cylinder,
f_y specified yield strength of reinforcement,
g ratio of center-to-center distance between exterior layers of longitudinal reinforcement to overall depth of section,
h overall thickness of the cross-section in plane of bending,
h_f overall thickness of flange,
h_w overall thickness of web,
m $= \dfrac{f_y}{0.85\, f_c'}$
M_n nominal flexural strength about the axis of bending,

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$N.A$	neutral axis
P_n	nominal allowable load in uniaxial bending with eccentricity e,
RC	reinforced concrete,
α_1	ratio of the stress in the equivalent stress block to the specified compressive strength of concrete =0.85 ;
β_1	ratio of the depth of the equivalent stress block to neutral axis depth,
ε_{cu}	specified ultimate compressive strain of concrete =0.003, and
ρ_t	Gross ratio of reinforcement.

Introduction

Arbitrarily shaped reinforced concrete (RC) members subjected to uniaxial or biaxial bending and axial compression are frequently used in multistory tall buildings and bridge piers.

In recent years some methods have been presented for the ultimate strength analysis of various concrete sections, such as L-, and channel-shaped, under symmetrical bending or combined biaxial bending and axial compression [1-4]. These methods compute the ultimate flexural capacity of section. For design purposes they require trial and error procedures.

The present research also aims at obtaining direct relationships between the compression load and the uniaxial bending capacities which can be used as ready design charts for T-columns.

Research significance

This search deals with reinforced concrete T-columns which are used for the external building edges supposing that neighbouring spaces in the buildings are balanced in loads. These columns are subjected to axial compressive load plus uniaxial bending. This uniaxial bending apply tension on flange and compression on web.

The principal aims of this work is to furnish a method for analysing tied columns under the combined action of axial compressive load and uniaxial bending that is simple in concept and can be beneficially used in providing easy way to deal with charts for the design of such columns.

Description of the procedure

For columns subjected to uniaxial bending, the neutral axis (N.A) always remains parallel to the axis about which the moment is being applied. Since the position of the neutral axis depends on the value of the eccentricity (e), therefore the variation of the neutral axis position may in general leads to the two possible cases of compression zone shown in Fig.(1).

Estimating concrete compression force

Depending on an equivalent rectangular compression block for concrete,[5] the compressive force of the concrete is

$$C_c = \text{area of compression zone} * (\alpha_1 f_c') \qquad ...(1)$$

Where

α_1 = ratio of the stress in the equivalent stress block to the specified compressive strength of concrete, and

f_c' = specified cylinder compressive strength of concrete.

Estimating strain in steel reinforcement

Based on the chosen value of ultimate usable strain at extreme concrete compression fibre (ε_{cu}) which is equal to 0.003[5] and the linear strain distribution across the depth of the cross section; (Fig. 2), a correlation between the strain (ε_{si}) in any arbitrary reinforcing bar and the depth of the compression zone (c) can be obtained. Let (d_{si}) denotes the distance from the extreme compression fibre to the centroid of any arbitrary reinforcing bar.

Referring to Fig. (2), the strain in any steel bar can therefore be obtained

$$\varepsilon_{si} = \frac{\varepsilon_{cu}}{c}\left| c - d_{si} \right| \qquad ...(2)$$

since steel can be idealized as elastic-perfectly plastic material with maximum value of stress (f_y), therefore the stress in any steel bar is simply

$$f_{si} = \varepsilon_{si} E_s \leq f_y \qquad ...(3)$$

where

f_{si} = stress in reinforcement,

E_s = modulus of elasticity of reinforcement, and

f_y = specified yield strength of reinforcement.

Equilibrium criteria

For a given eccentricity (e), the value of the compressive load (P) can be estimated from the following simple equilibrium equation

$$P = C_c + \sum C_{si} - \sum T_{si} \qquad ...(4)$$

The associated uniaxial bending moment (*M*) can also be estimated by summing up the moment of the resulting forces on the cross section around the plastic centroid (*PC*) of the section,

$$M = C_c \text{ * its lever arm to the PC } + \sum C_{si} \text{ * its lever arm to the PC } + \sum T_{si} \text{ * its lever arm to the PC} \qquad ...(5)$$

where C_{si} and T_{si} represent the compressive and tensile force in the i^{th} reinforcing bar respectively, see Fig. (2). The subscript (*i*) refers to the reinforcing steel layer position.

Program description

The computer program is developed in Microsoft Quick-Basic Version 4.5. It is capable of producing points that describe the axial load versus moment interaction diagram for any short T-column under uniaxial bending.

Input data for program include: the material and section properties, and the area and coordinates of each longitudinal bar. The output of the program consists of a series of data points (P and M vales) that could be used in drawing the interaction diagram for the column.

The program assumed a linear variation of strain over the depth of the section. Strain hardening of steel, tensile strength of concrete, and slenderness effects are ignored. In addition, the output does not include the capacity reduction factor (ϕ).

Design charts

In order to make direct use of the present method in the design of RC T-columns subjected to axial load and uniaxial bending moment, charts are constructed in a manner analogous to those given in text books for the case of rectangular columns with uniaxial bending. Charts 1 through 8 have been prepared for the case of uniaxial bending of T-columns. These charts are designed to cover a wide range of the cross sectional parameters.

Fig. (3) shows a cross section of a typical RC T-columns. For convenience, these design charts are presented in this study which cover the following cases

$\frac{b_f}{b_w} = 2$ and 4

g = 0.6, 0.7, 0.8 and 0.9

No. of bars = 8

$f_c' \leq 30$ MPa and $f_y = 414$ MPa

For values of $(\frac{b_f}{b_w})$ and (g) other than those listed above, linear interpolation between neighbouring values can be used.

Examples

Example (1) – Design problem

A short RC T-column subjected to nominal compressive load of 2016 *kN* acting at a position with eccentricity e = 232 *mm*. Use $f_c' = 28$ MPa, $f_y = 414$ MPa, No.of bars = 8, $b_f = 525$ *mm*, $b_w = 175$ *mm*, $h_f = 200$ *mm*, $h_w = 380$ *mm* and cover = 58 *mm*. It is required to determine the steel ratio by using the proposed charts.

Solution:-

$$m = \frac{fy}{0.85 * f_c'} = \frac{414}{0.85 * 28} = 17.39;$$

$$\frac{b_f}{b_w} = \frac{525}{175} = 3;$$

$$h = h_f + h_w = 200 + 380 = 580\ mm;$$

$$g = \frac{580 - 2 * 58}{580} = 0.8;$$

$$\frac{e}{h} = \frac{232}{580} = 0.4;$$

$$\frac{Pn}{f_c' b_w h} = \frac{2016 * 10^3}{28 * 175 * 580} = 0.71$$

Enter chart 3 (g = 0.8 and $\frac{b_f}{b_w} = 2$)with $\frac{e}{h} = 0.4$ and $\frac{Pn}{f_c' b_w h} = 0.71$, read $\rho_t m = 0.85$

Enter chart 7 (g = 0.8 and $\frac{b_f}{b_w} = 4$)with $\frac{e}{h} = 0.4$ and $\frac{Pn}{f_c' b_w h} = 0.71$, read $\rho_t m = 0.34$

Interpolating (g = 0.8 and $\frac{b_f}{b_w} = 3$):

$\rho_t m = 0.595$;

$$\therefore \rho_t = \frac{0.595}{17.39} = 0.034$$

Example (2) – Design problem

A short RC T-column subjected to nominal compressive load of 2650 *kN* acting at a position with eccentricity e = 200 *mm*. Use $f_c' = 25$ MPa, $f_y = 414$ MPa, No.of bars = 8, $h_f = 200$ *mm*, $h_w = 380$ *mm*, cover = 58 *mm* and steel ratio ($\rho_t = 0.0426$). It is required to determine values of dimensions (b_f and b_w) by using the proposed charts.

Solution :-

$$m = \frac{414}{0.85 * 25} = 19.48;$$

$$\rho_t m = 0.0426 * 19.48 = 0.83;$$

$$h = 200 + 380 = 580\ mm;$$

$$g = \frac{580 - 2 * 58}{580} = 0.8;$$

$$\frac{e}{h} = \frac{200}{580} = 0.34$$

assume $\frac{b_f}{b_w} = 2$ and $b_w = 250\ mm$

Enter chart 3 (g = 0.8 and $\frac{b_f}{b_w} = 2$)with $\frac{e}{h} = 0.34$ and $\rho_t m = 0.83$; read $\frac{Pn}{f_c^{'} b_w h} = 0.76$, therefore

$Pn = 0.76 * 25 * 250 * 580 * 10^{-3} = 2755$ kN

Since this value agrees closely with actual nominal load, therefore b_f =500 *mm* and b_w = 250 *mm* are acceptable

Example (3) – Analysis problem

A short RC T-column subjected to nominal compressive load of (Pn) acting at a position with eccentricity e = 195 *mm*. Use $f_c^{'}$ = 30 MPa, f_y = 414 MPa, No.of bars = 8, b_f =600 *mm*, b_w = 300 *mm*, h_f=275 *mm*, h_w = 375 *mm* ; cover = 65 *mm* and steel ratio ($\rho_t = 0.0308$). It is required to determine the allowable nominal compressive load (P_n) by using the proposed charts.

Solution:-

$m = \frac{414}{0.85 * 30} = 16.235$;

$\rho_t m = 0.0308 * 16.235 = 0.5$;

$\frac{b_f}{b_w} = \frac{600}{300} = 2$;

$h = 275 + 375 = 650\, mm$;

$g = \frac{650 - 2 * 65}{650} = 0.8$;

$\frac{e}{h} = \frac{195}{650} = 0.3$

Enter chart 3 (g = 0.8 and $\frac{b_f}{b_w} = 2$) with $\frac{e}{h} = 0.3$ and $\rho_t m = 0.5$; read $\frac{Pn}{f_c^{'} b_w h} = 0.7$, therefore

$Pn = 0.7 * 30 * 300 * 650 * 10^{-3} = 4095\, kN$

Example (4) – Analysis problem

A short RC T-column subjected to nominal compressive load of 1330 *kN* acting at a position with eccentricity e . Use $f_c^{'}$ = 26 MPa, f_y = 414 MPa, No. of bars = 8, b_f =400 *mm*, b_w = 200 *mm*, h_f=200 *mm*, h_w = 380 *mm* ; cover = 58 *mm* and steel ratio ($\rho_t = 0.0427$). It is required to determine the allowable nominal moment by using the proposed charts.

Solution :-

$m = \frac{414}{0.85 * 26} = 18.733$;

$\rho_t m = 0.0427 * 18.733 = 0.8$;

$\frac{b_f}{b_w} = \frac{400}{200} = 2$;

$h = 200 + 380 = 580$ mm;

$g = \frac{580 - 2 * 58}{580} = 0.8$;

$\frac{Pn}{f_c^{'} b_w h} = \frac{1330 * 10^3}{26 * 200 * 580} = 0.44$

Enter chart 3 (g = 0.8 and $\frac{b_f}{b_w} = 2$)with $\frac{Pn}{f_c^{'} b_w h} = 0.44$ and $\rho_t m = 0.8$, read $\frac{e}{h} = 0.73$;

therefore

$e = 0.73 * 580 = 423.4\, mm$;

$M_{allowable} = 1330 * 423.4 * 10^{-3} = 563.1\, kN.m$

Conclusions

The analysis and design of reinforced concrete T-sections subjected to axial compression and uniaxial bending are very tedious and time consuming because

1. in the analysis, a trial and error procedure is required to find the depth of the neutral axis satisfying the equilibrium conditions.
2. in the design process, a trial and error procedure is required to find the steel ratio (ρ_t) satisfying the strength requirements.

On the other head, the simplicity of the present approach enabled the construction of new design charts which can be used directly in design.

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Future research

The following iterms are recommended for future studies

1. Applying the new approach to analyse high strength RC T-columns and to study the effect of some important parameters on their capacity such as the amount of reinforcement ratio, yield strength of reinforcement and ultimate compressive strain value of concrete.
2. Extending the new approach to analyse irregular shapes such as L-, and channel shaped and providing design charts.

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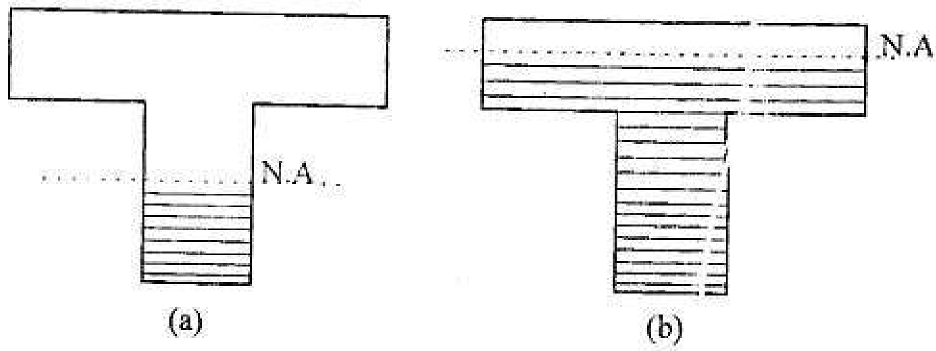

Fig. (1) The two possible cases of compression zone.

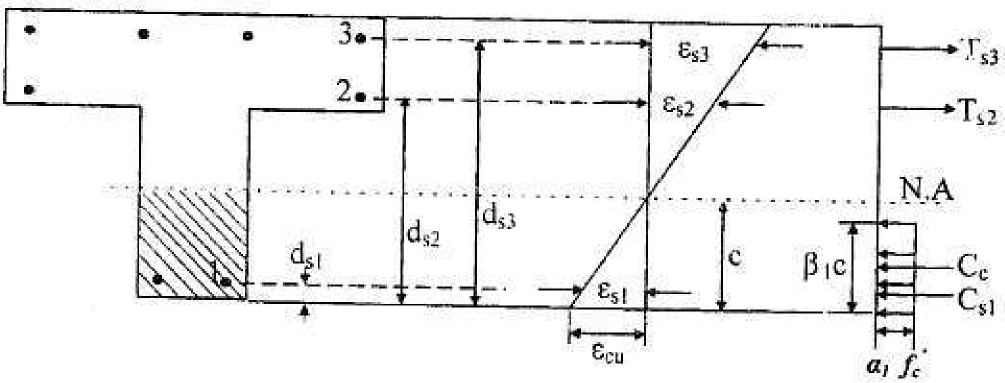

Fig. (2) Eccentrically loaded columns.

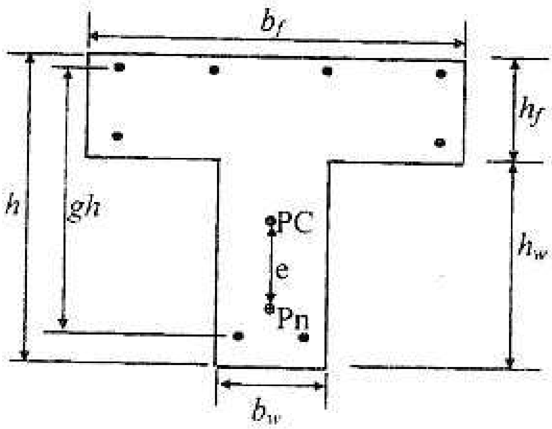

Fig. (3) Symbol details of RC T-column subjected to uniaxial bending.

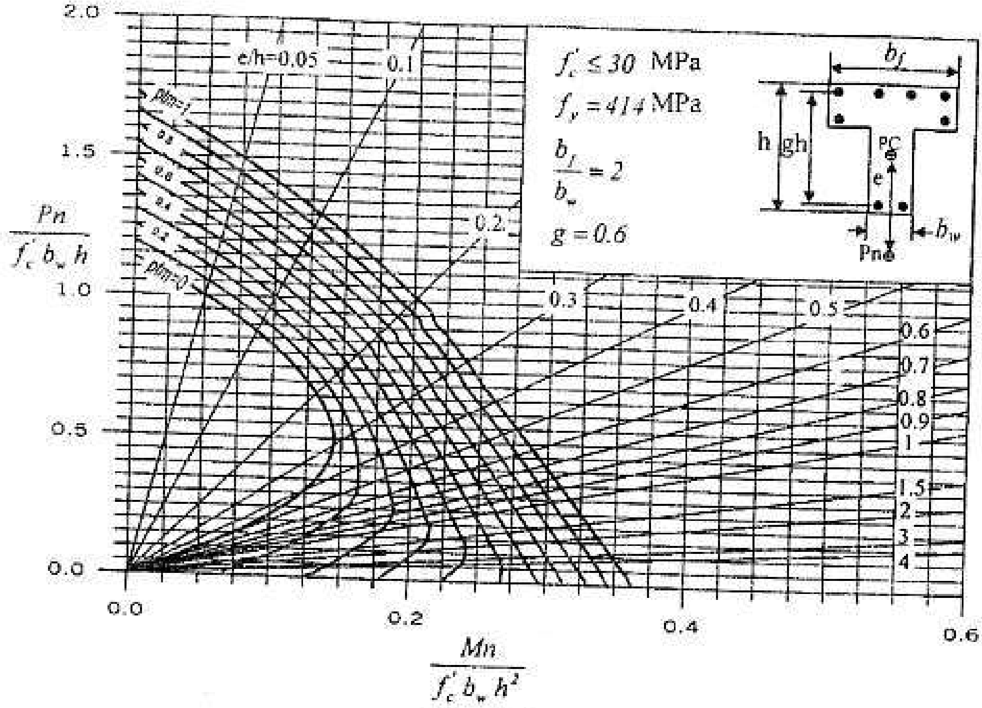

Chart 1

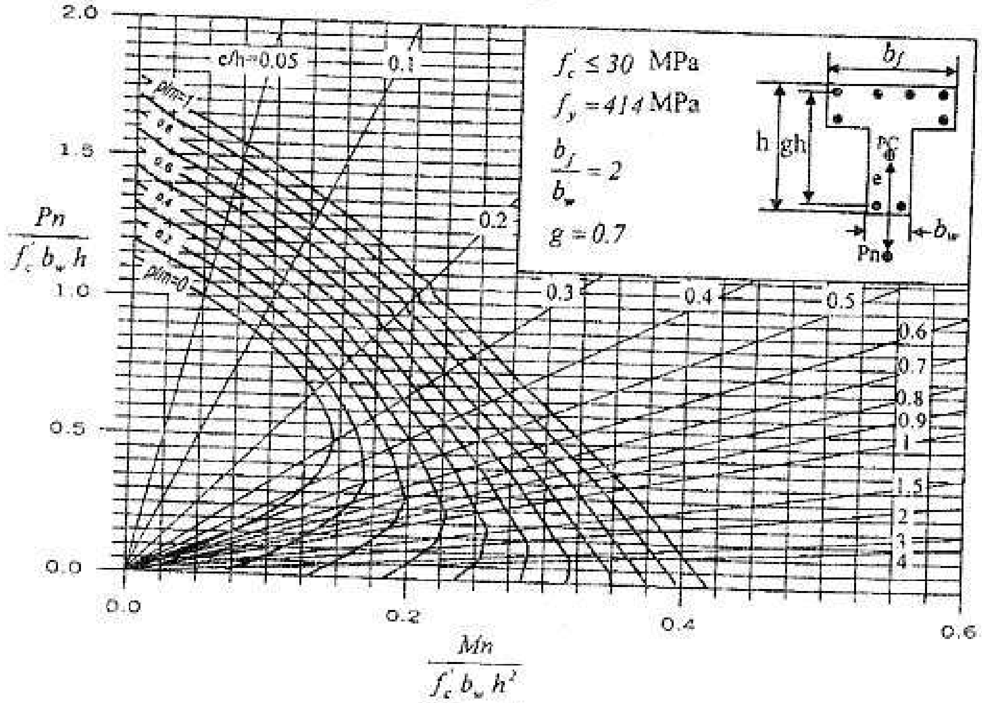

Chart 2

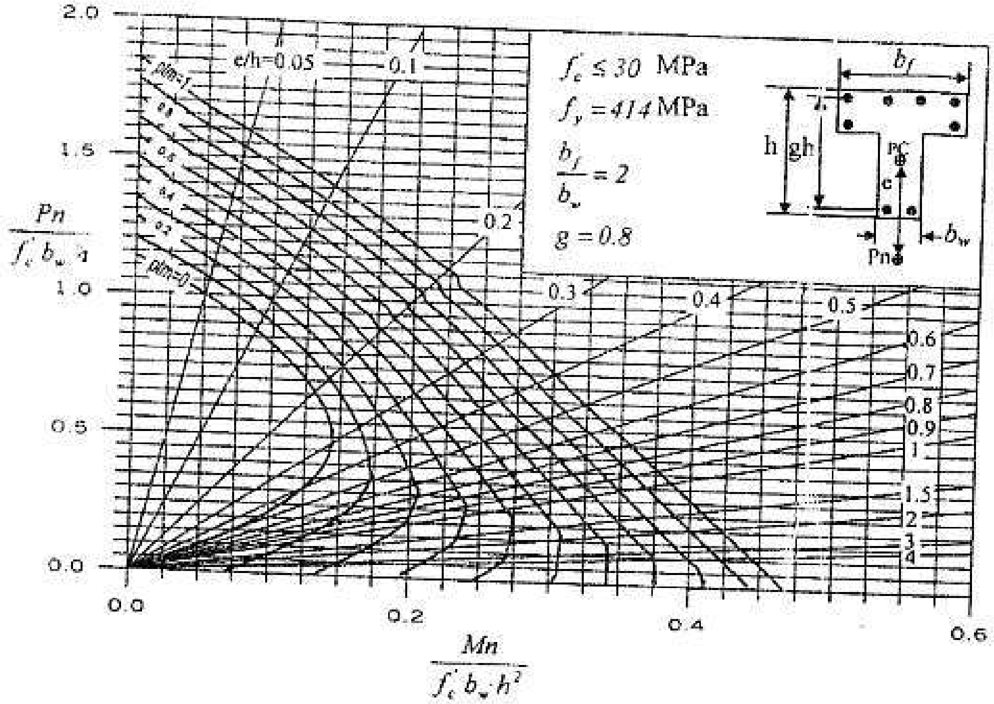

Chart 3

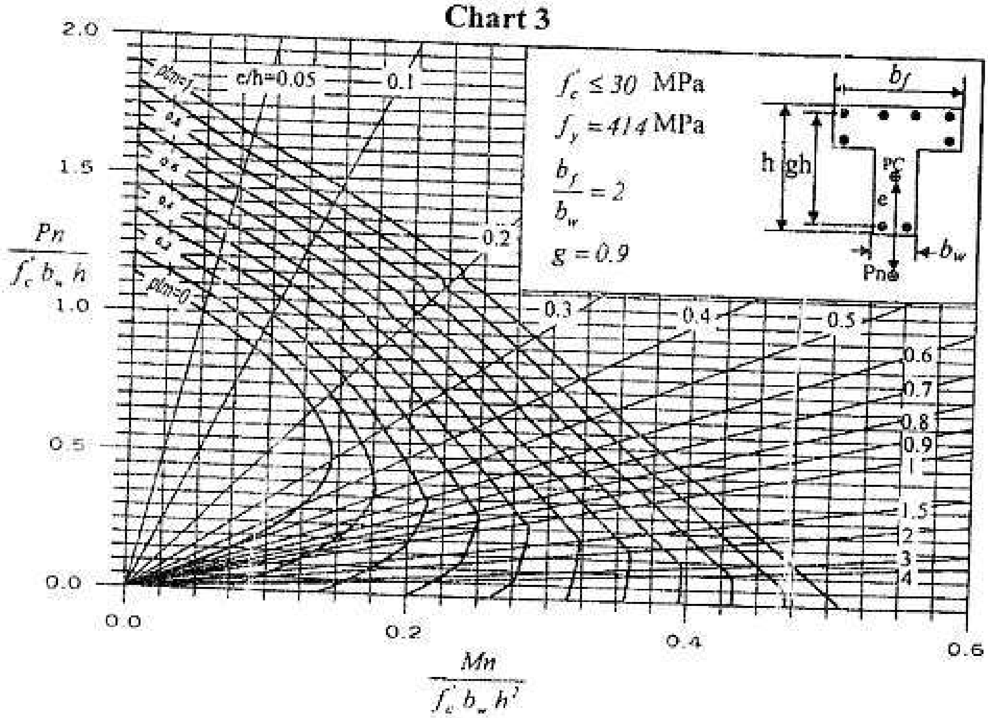

Chart 4

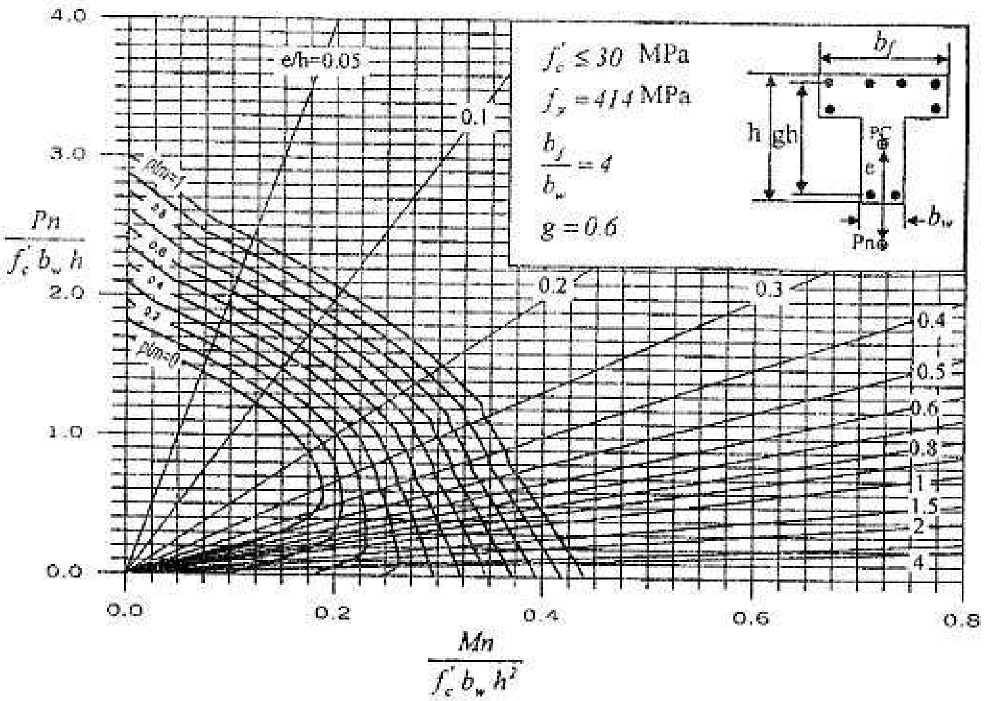

Chart 5

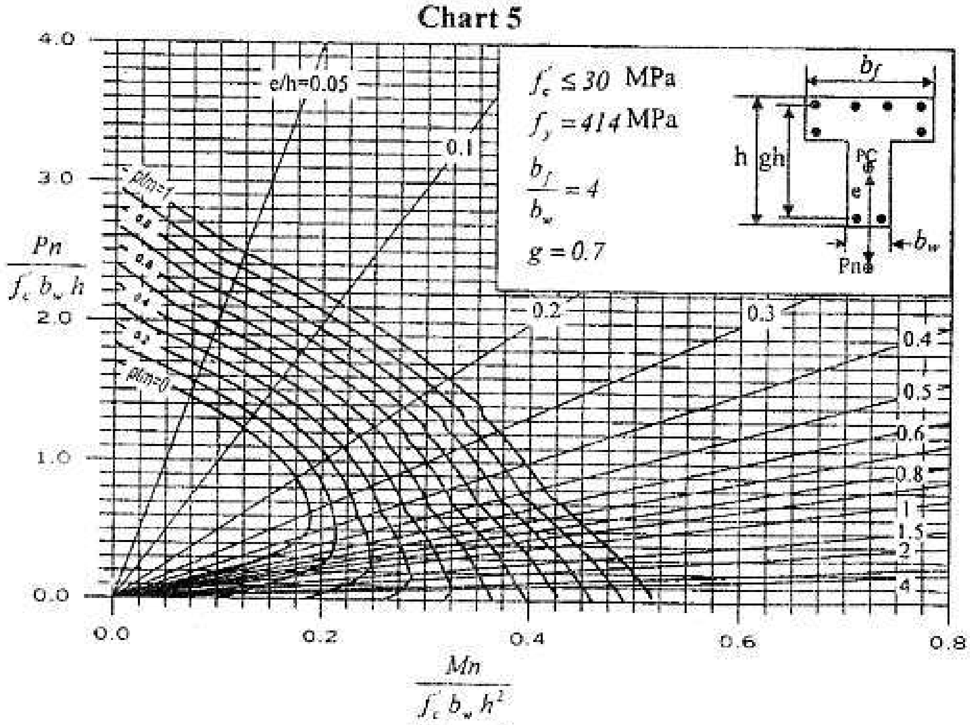

Chart 6

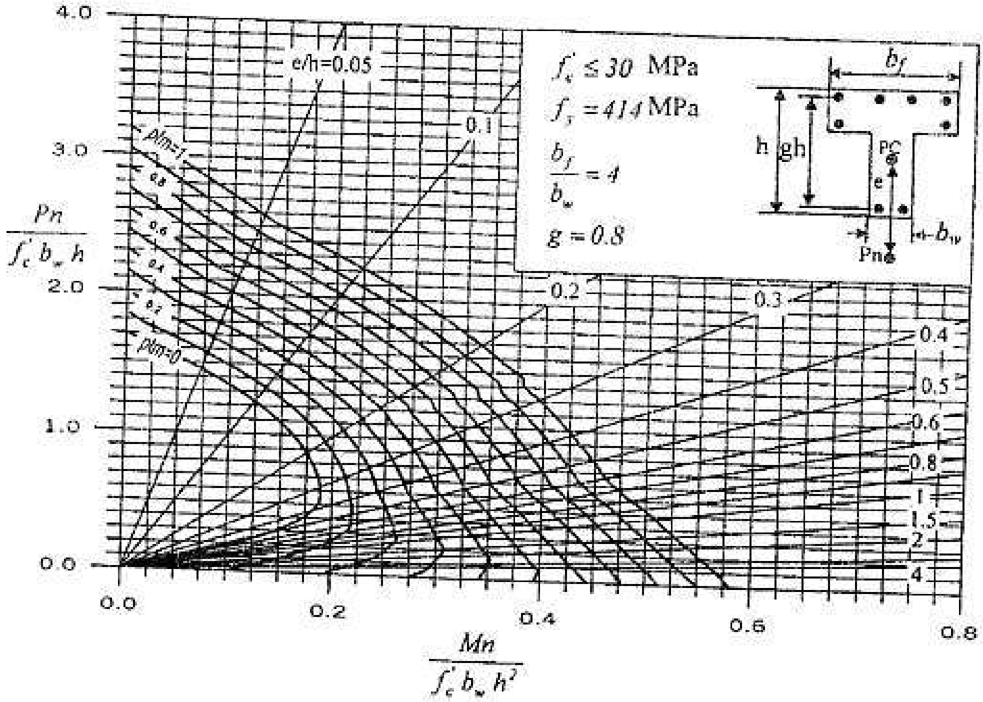

Chart 7

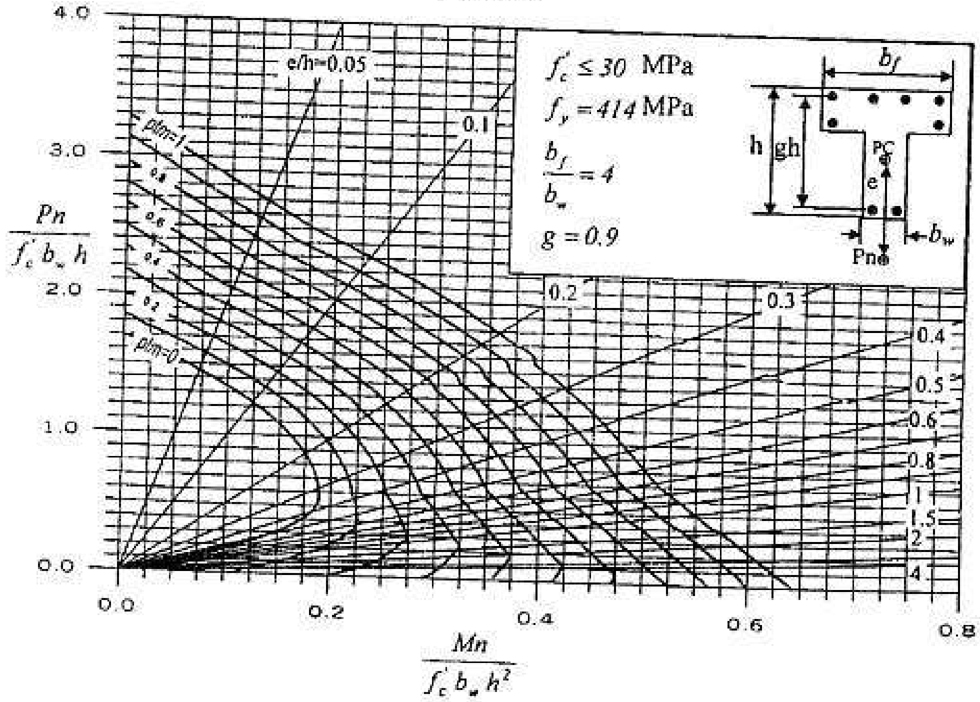

Chart 8