Parametric Study On The Effect Of The Fiber Reinforced Polymer(FRP) Pad On The Stress Intensity Factor Of A Cracked Pipe

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Abstract

In this paper the effect of using FRP bonded pad to reinforce damaged pipes subjected to internal pressure is investigated. Outer Longitudinal edge crack was simulated by using standard finite element method as a powerful tool for the numerical solution of a wide range of engineering problems. Cracks in the present study are investigated by using linear elastic fracture mechanics in order to calculate the stress intensity factor SIF. Mode I SIF was calculated by means of using standard FE program ANSYS5.4 and the element used was 8-node quadrilateral isoparametric element .A parametric study was done to investigate the effect of the pad width W, thickness H and the volume fraction V_f of the composite pad on the SIF of the cracked pipe .It was found that by using FRP pad to reinforce the cracked pipes , the SIF reduces in the vicinity of the crack tip, and the fiber volume fraction of the pad has a great influence on the SIF.

دراسة نظرية في تأثير استخدام وسادة من اللدائن المقواة بالألياف على معامل شدة الإجهاد للأنابيب التي تحتوي على شقوق

الخلاصة

في هذه الدراسة تم بحث استخدام وسادة من اللدائن المقواة بالألياف لتقوية الأنابيب المتضررة التي تتعرض لضغط داخلي.حيث تم دراسة الشقوق الطولية باستخدام طريقة العناصر المحددة كأداة فعالة في حل كثير من المشاكل الهندسية.الشقوق تم دراستها باستخدام علم ميكانيك الكسر المطيلي لحساب معامل شدة الإجهاد.شكل I لمعامل شدة الإجهاد تم حسابه باستخدام برنامج جاهز للعناصر المحددة ANSYS5.4 وكان العنصر المستخدم ثمانية عقد.تم دراسة عدة عوامل مثل عرض ،طول وكذلك النسبة الحجمية لألياف الوسادة المركبة على قيمة معامل شدة الإجهاد للأنابيب ذات الشقوق.لقد وجد بان استخدام الوسادة المقواة بالألياف تقلل من قيمة معامل شدة الإجهاد وكذلك زيادة النسبة الحجمية للألياف.

1.Introduction

Pipes usually subjected to external environment which led to damage the pipe ,such as externally corrosion and erosion and other factors which led to be uncomfortable for utilization and decrease their working life .The costly usual solutions of the rehabilitation process for damaged pipes fabricated from metals is very important mater. The rehabilitation technique for pipes is needed because of external erosion and corrosion for pipes or due to external damages .Usually such process requires using heavy machines and skill workers in

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addition to the time needed. By using FRP techniques to increase flexural strength for existing concrete and steel structural elements ,it is very important for the corroded and damaged pipes to increase the resistance to the surrounding environment .The traditional techniques for repairing damaged pipes needs operation of cutting ,welding and inspecting in addition that the system be stopped during the repair period .For many cases the repair process is uncomfortable because of the residual stress induced in the pipe during the welding process which tends to decrease the fatigue performance of the pipe material in addition to the weakness of the pipe wall due to pitting. The final rehabilitation cost of the new trends in repair is low compared with the usual trends because of the effectiveness of the new materials used. In the present work the study of the effect of the FRP strengthening pad on the SIF of a cracked pipe is based on the elastic fracture mechanics, which is based on neglecting the crack tip plasticity in order to calculate the SIF is carried out [1,2].

The present study focuses on the outer longitudinal edge crack ,Figure (1), were investigated rather than circumferential crack because it is subjected to girth stress twice than the longitudinal one [3]. Therefore for the same crack depth the longitudinal cracks are much dangerous than the circumferential one .In this study the rehabilitation techniques were modeled by using standard FE program ANSYS5.4 [4] .

2.Geometry, Material of the Pipe:

The investigated pipe, Figure(1), has the following material and geometric parameters, inner radius of R_i=50mm and outer radius of R_o =55mm.The pipe used was made of steel with Young modulus of $E=200*10^3$ N/mm^2 and Poisson's ratio of $\mu=0.3$.The crack depth varies from $0.2<a/t<0.8$ for all strengthen and unstrengthen pipes.

The cracked pipes were subjected to an internal pressure of $P=20\ N/mm^2$ A numerical analysis was used for the all pipe models ,Figures (2)&(3).

3.Finite Element modeling:

The numerical modeling for all cases was based on the finite element method FEM by using two dimensional plain strain 8-node quadrilateral isoparametric element (Plane 86)[4]. Because of symmetry, only on half of the pipe was modeled and all models were meshed with the same degree of refinement at the crack edge to obtain consistent result, Figure (4).The finite element mesh and the boundary condition of the cracked pipe is shown in Figure (2). Stress intensity factor K_I along the notch tip are computed by using absolutely standard eight node quadrilateral element with the mid-side node displaced from their normal position [5,6], by applying the extrapolating techniques for appropriate crack surface displacement .Figure (3) shows condition of the strengthed cracked pipe while Figure (4) shows the finite element mesh.

4.Pad Reinforcement Material:

Three types of fibers are used in the study to simulate the reinforcement pad. Table (1) and Figure (5) [7,8] show a comparison for the modulus of elasticity of the steel ,Glass, Aramid and Carbon fibers.

The bounding adhesive used is epoxy with traditional adhesive properties for young modulus and Poisson's ratio of E=3.5GPa μ=.25 respectively [9].

Because of the complexity of the problem investigated, the FRP pad was assumed to be isotropic for each of the materials.

The composite modulus of elasticity computed by using the relation [10]:

$$E_c=E_fV_f+E_mV_m$$

where

E_c, E_f, E_m =Modulus of Elasticity of Composite ,Fiber and Matrix respectively

V_f,V_m =Volume fraction of the Fiber and Matrix respectively

The volume fraction of the fibers used $0.25\% < V_f < 0.75\%$

Table(2) and Figure (6) shows the effect of V_f on the composite modulus of elasticity.

5. Linear Elastic Fracture Mechanics:

Linear elastic fracture mechanics technology is based on an analytical procedure that relates the stress field magnitude and distribution in the vicinity of a crack tip to the nominal stress applied to the structural member, size, shape and orientation of the crack and material properties [11]. The fundamental principle of the fracture mechanics is that the stress field a head of a sharp crack in structural member can be represented by a single parameter *K*, (the stress intensity factor), that has a unit of $MN.m^{-3/2}$. This parameter relates both the nominal stress level σ in the member and the size of the crack present a .Paris and Sih [11] showed that in order to establish methods of stress analysis for cracks in elastic solid, it is convenient to define three types of relative movement of two crack surfaces. These displacement modes represent the local deformation in an infinitesimal element containing a crack front as shown in Figure (13), which are the opening mode, *Mode I* .The sliding or shear mode, *Mode II*. The tearing mode, *Mode III* .It is clear that each of these modes of deformation corresponds to a basic type of stress field in the vicinity of crack tips. In any case the deformations at the crack tip can be treated as one or combined of these local displacement modes.

Moreover the stress field at the crack tip can be treated as one or a combination of the three basic types of stress fields. Most practical design situations and failures correspond to *Mode I*, which is the mode, considered in this study.

6.Elastic Stress Field Equations in the Vicinity of Crack Tip:

Westergaared and Irwin [11] developed a method to show that the stress and displacement field in the vicinity of crack tip for a linear, elastic, homogeneous, and isotropic solid subjected to the three modes of deformation are given by:

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}}\cos(\frac{\theta}{2})*$$
$$[1-\sin(\frac{\theta}{2})\sin(\frac{3\theta}{2})]...(1)$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}}\cos(\frac{\theta}{2})^* [1+\sin(\frac{\theta}{2})\sin(\frac{3\theta}{2})] \quad ...(2)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}}\cos(\frac{\theta}{2})^* \sin(\frac{\theta}{2})\cos(\frac{3\theta}{2}) \quad ....(3)$$

$$u = \frac{K_I}{2G}\sqrt{\frac{r}{2\pi}}\cos(\frac{\theta}{2})^* [k-1+2\sin^2(\frac{\theta}{2})] \quad ...(4)$$

$$v = \frac{K_I}{2G}\sqrt{\frac{r}{2\pi}}\sin(\frac{\theta}{2})^* [k+1-2\cos^2(\frac{\theta}{2})] \quad ...(5)$$

The stress component in the coordinates r, θ are shown in Figure (14).
where
K_I is stress intensity factor for mode *I*
.
G = Shear modulus = $E/2(1+\mu)$

$k = \frac{3-\mu}{1+\mu}$ For plane stress

$k = 3-4\mu$ For plane strain

μ =Poisson's ratio

$\sigma_z = 0$ For plane stress

$\sigma_z = \mu(\sigma_x + \sigma_y)$ For plane strain

It may be stated that most brittle fractures occur under conditions of *Mode I* loading [12], therefore the stress of primary intent in Figure (14) and in most practical applications is σ_y as shown in Figure (16) which shows for $r \to 0$, the stress becomes infinite and the stress intensity factor is then a measure of the stress singularity at the crack tip. For σ_y to be a maximum in equation (2), let $\theta = 0$, therefore:

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \Rightarrow K_I = \sigma_Y\sqrt{2\pi r} \quad ....(6)$$

It is clear that at location far from crack tip, σ_y decreases, and K_I remains constant and describes the intensity of the stress field just ahead of sharp crack.

7.Determination of the (SIF) by finite element method:

Fracture mechanics generally is based on two types of analysis [13], namely residual strength analysis to determine the maximum crack size that can be tolerated, and fatigue crack growth analysis to calculate the time for crack growth from a certain initial crack size until the maximum tolerable crack size in order to determine the safe life. These analyses are usually based on the stress intensity factor. Therefore, knowing the stress intensity factor is important for the fracture mechanics. Stress intensity factor can always be expressed as:

$$K = \sigma\sqrt{\pi a}\,Y \quad ....(7)$$

where

Y is a dimensionless factor which depends upon geometry of the body and crack size.The problem resides in the determination of *Y*. Solutions of *Y* for many configurations have been obtained. Compilations of these solutions known as stress intensity factor handbook [14] are available.
Components that contain cracks may be subjected to one or more different types of load, such as uniform tensile

loads, concentrated tensile loads, or bending loads.In such a case the total stress intensity factor can be found by algebraically adding the stress intensity factor that corresponds to each load if it available. This method is known as the superposition method [11].

It can be noted that superposition is the addition of stress intensity factor and not of *Y*. The value of *Y* of each case have to be obtained separately and stress intensity factor calculated separately and then added. This method is illustrated in Figure (17) [11]. Many structural configurations are so complicated, that a solution may not be available in handbooks. Consequently the finite element method was and still a powerful tool to determine crack tip stress fields. By finite element solution a complicated engineering geometry could be analyzed, also it has been extensively used in engineering crack problems, such as the determination of stress intensity factor.

Basically there are two different approaches to determine stress intensity factor by the finite element method [13]. One is the direct method in which stress intensity factor is calculated from stress field or from displacement field. The second is an indirect method in which stress intensity factor is calculated from other parameter, the line integral (*J*-Integral).

7.1 Direct Method:

The displacement method involves a correlation of the finite element nodal point displacements with the well-known crack tip displacement equation. For *Mode I*, the crack tip displacement equation (4), (5) can be written as:

$$U_i = \frac{K_I}{2G}\sqrt{\frac{r}{2\pi}}f_i(\theta) \ldots(8)$$

Where,

$U_1 = u, U_2 = v$

$$f_1 = \cos(\frac{\theta}{2})[k-1+2\sin^2(\frac{\theta}{2})]$$

$$f_2 = \sin(\frac{\theta}{2})[k+1-2\cos^2(\frac{\theta}{2})]$$

By substituting a nodal point displacement U_i at some point (r,θ) near the crack tip into equation (8) a quantity K_I can be calculated as:

$$K_I = \sqrt{\frac{2\pi}{r}}\frac{2GU_i}{f_i(\theta)} \ldots(9)$$

From plots of K_I as a function of *r* for fixed values of θ and a particular displacement component, estimates of K_I can be made. If the exact theoretical displacement is substituted in equation (9) then value of K_I obtained as *r* approaches zero would be the exact value of K_I. Since the finite element displacements are rather inaccurate at an infinitesimal distance from the crack tip, this process is not useable. Instead a tangent extrapolating of the K_I curve is used to estimate K_I [15]. With suitable refinement of element size the K_I curves obtained from element displacements rapidly approach a constant slope with increasing distance (r) from the crack tip.

The estimation of the stress intensity factor by the tangent extrapolating method could be made by plotting a tangent line to the constant slope portion of the curve and extending the line until an intersection with K_I is obtained at point, which is the estimated value of K_I as shown in Figure(15). The most accurate estimates are obtained from K_I curve corresponding to the normal displacement on the crack surface at $\theta = 180^o$ [5], hence equation (9) is therefore can be written as:

$$K_I = \frac{2Gv}{f_2(180^o)}\sqrt{\frac{2\pi}{r}} \text{ ...(10)}$$

The stress method for determining the crack tip stress intensity factor involves a correlation of the finite element nodal stresses with the well-known crack tip stress equations.

For *Mode I*, the crack tip stress, equations (1),(2),(3) can be written as:

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta) \text{.........(11)}$$

Where,

$\sigma_{11} = \sigma_{XX}$

$\sigma_{22} = \sigma_{YY}$ $\sigma_{12} = \tau_{XY}$

$f_{11} = f_{XX} =$

$$\cos(\frac{\theta}{2})[1 - \sin(\frac{\theta}{2})\sin(\frac{3\theta}{2})]$$

$f_{22} = f_{YY} =$

$$\cos(\frac{\theta}{2})[1 + \sin(\frac{\theta}{2})\sin(\frac{3\theta}{2})]$$

$f_{12} = f_{XY} =$

$$\cos(\frac{\theta}{2})\sin(\frac{\theta}{2})\cos(\frac{3\theta}{2})$$

Nodal point stresses σ_{ij} in the vicinity of the crack tip can be substituted into equation (11) and values of K_I can be calculated from:

$$K_I = \frac{\sqrt{2\pi r}}{f_{ij}(\theta)}\sigma_{ij} \text{............(12)}$$

The estimation of stress intensity factor could be made by plotting K_I as a function of r for fixed value of θ and a particular stress component. If the exact theoretical stresses were substituted into equation (12) then the intercept of the curve with the K_I axis at $r=o$ would be the exact value of K_I.

Due to the inability of the finite element method to represent the stress singularity conditions at the crack tip, the K_I curve for r greater than zero must again be extrapolated back to $r = 0$ The extrapolated value of K_I at $r = o$ is the estimated K_I.

Accurate evaluation of the stress intensity factor by stress method could be obtained from plotting K_I curve corresponding to the normal stress on the crack tip (σ_y) at angle $\theta = 0^o$ [15]. Hence equation (12) can be written

$$K_I = \sigma_Y \frac{\sqrt{2\pi r}}{f_Y(0^o)} \ldots\ldots(13)$$

In this study the direct method is considered.

Figure (7) shows a comparison between the closed form solution of the calculated SIF [6,14,16] for the unstrengthed cracked pipe and the estimated SIF from the FEM results in the present study .

7.2 Indirect Method:

Rice [17] stated that the value of the line integral

$$J = \int_{\Gamma}(W dy - T\frac{\partial u}{\partial x} ds) \ldots\ldots(14)$$

is proportional to the square of the crack tip stress intensity factor ,as shown in Figure (18).

$K_I = \sqrt{JE}$ For plane stress

$K_I = \sqrt{\frac{JE}{(1-\mu^2)}}$ For plane strain

$K_I = \sqrt{\frac{JE}{(1-\mu^2)}}$ For plane strain

where,

Γ : An arbitrary contour surrounding the crack tip.

W: Defined as the strain energy density.

T : The traction vector defined according to the outward normal along Γ.

U: The displacement vector.

ds: An element of arc length along Γ.

The line integral is evaluated in a counter clockwise sense starting from the lower crack surface and continuing along the path Γ to the upper surface. The strain energy densities were calculated from nodal point stresses, and the nodal point forces were used as a surface traction.

The indirect method has some advantages [13]. In the first place no extrapolations are required, and second, there is no need for a particularly fine mesh at the crack tip. On the other hand, there are several disadvantages, since only a single value of K_I is obtained, it is difficult to estimate the degree of error. Another complication is that at least two crack sizes have to be analyzed in order to carry out the differentiation. Also the loss of accuracy due to the differentiation procedure may occur.

8.Results and Conclusions:

1. The significant effect of using composite material reinforcing pad with fiber volume fraction(V_f=0.25%) on decreasing the value of the SIF of the unstrengthed pipe is shown in Figure(8).
2. It is shown from Figure (9) that the length of the reinforcement pad (H) has a little influence on decreasing the value of the SIF for t/H>1 ,while the influence of the pad thickness (W) has great effect on decreasing the SIF for a/t>0.4,as shown in Figure (10).
3. As the fiber volume fraction of the Pad increased (0.25%<V_f<0.75%), the value of SIF decreased ,as shown in Figure (11).
4. Figure(12) shows the effect of using different pad fiber material on lowering the value of SIF.

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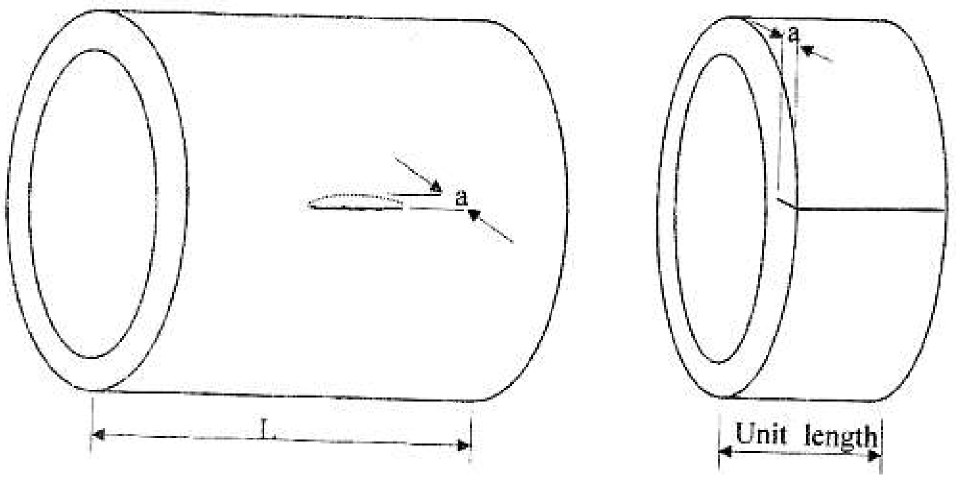


Figure 1.Cracked Pipe

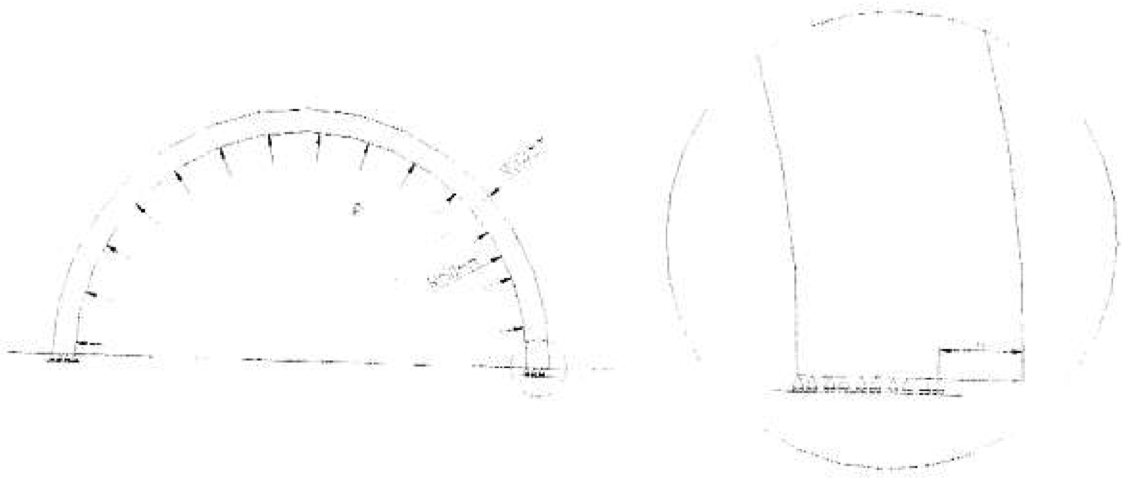


Figure 2.Geometry of the Cracked Pipe

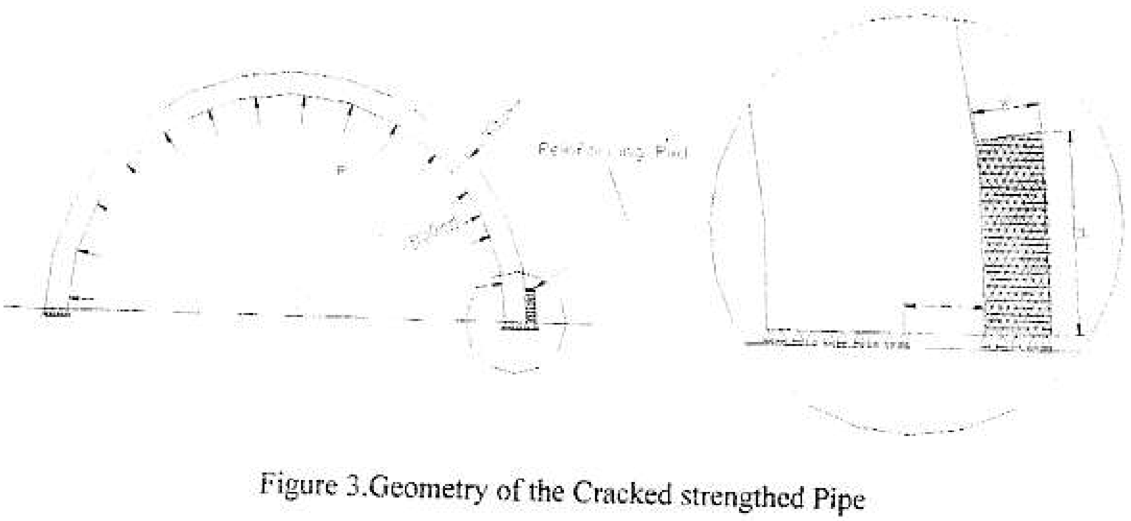


Figure 3.Geometry of the Cracked strengthed Pipe

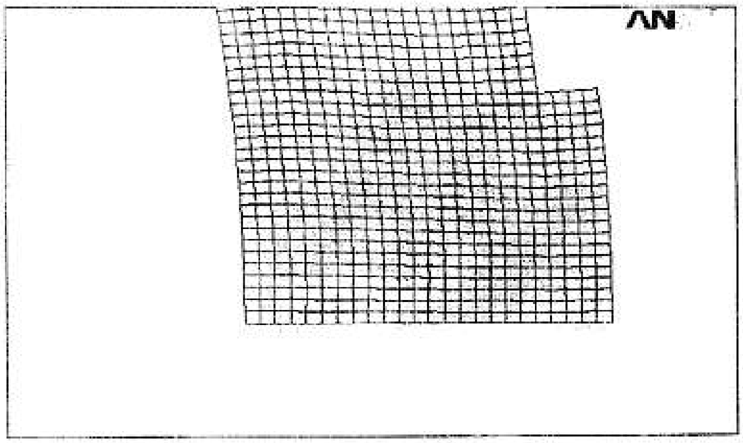


Figure 4.Part of the FE mesh for the Cracked strengthened Pipe

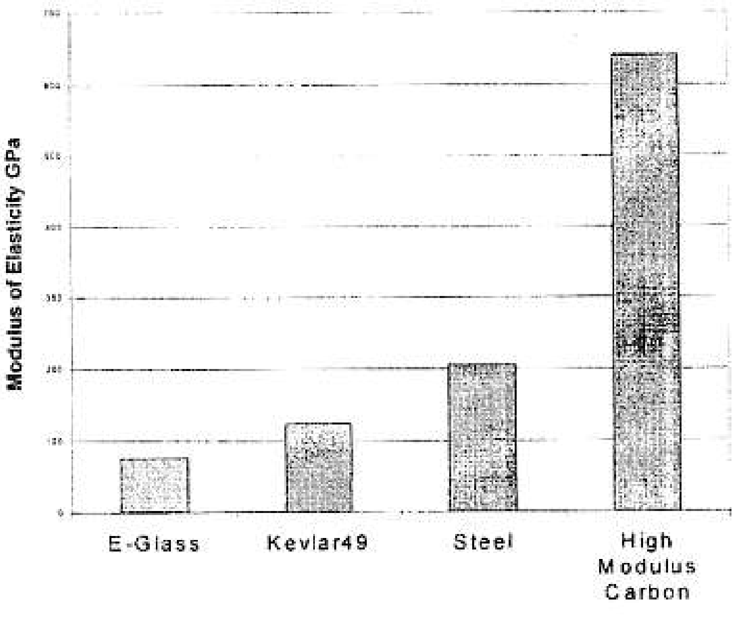
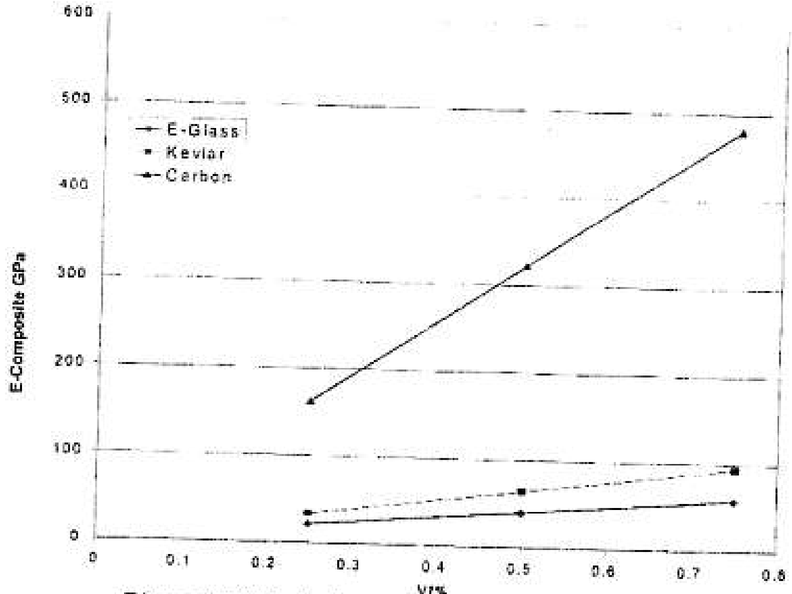
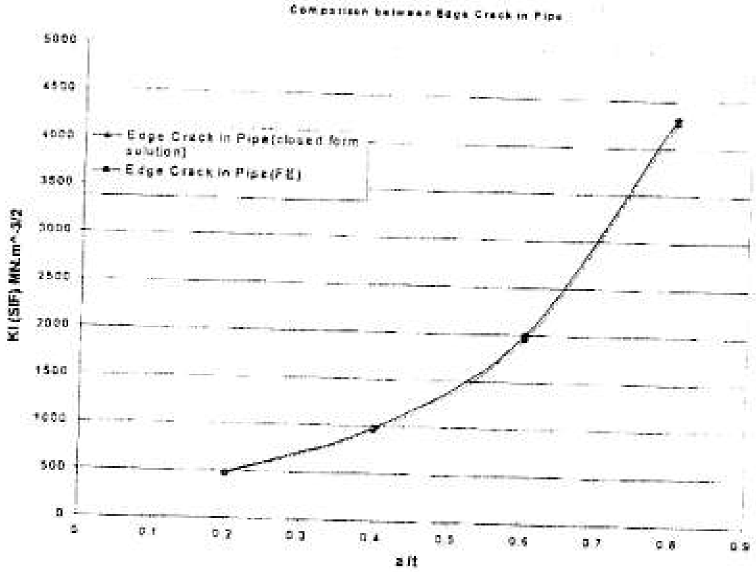


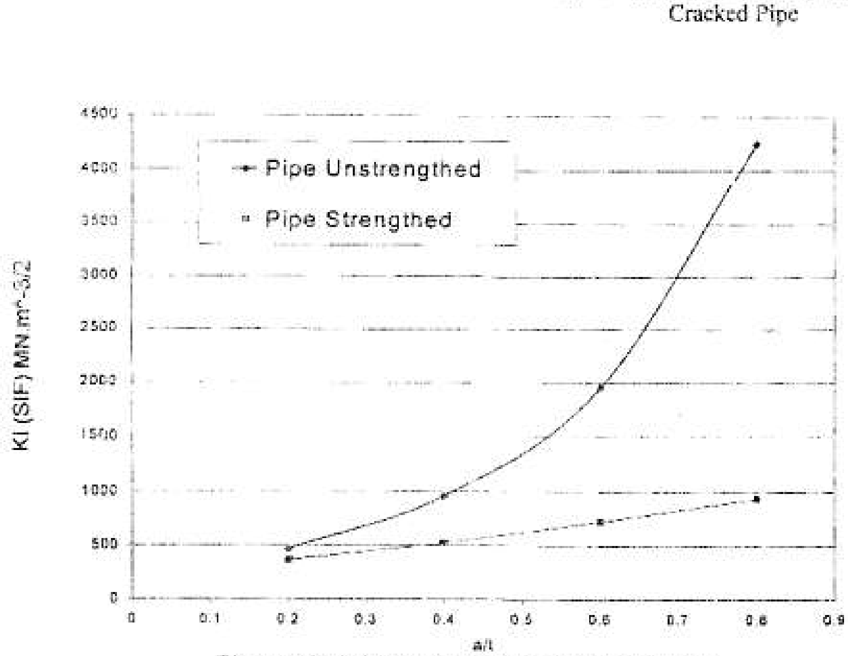
Figure 5 A comparison of modulus of elasticity



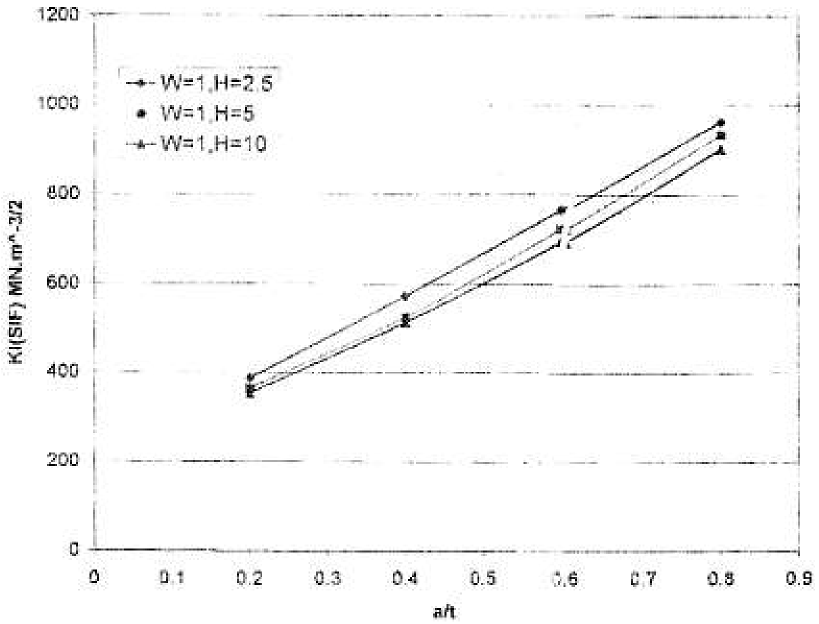
Figure(6) Variation of Vf on Modulus of Elasticity



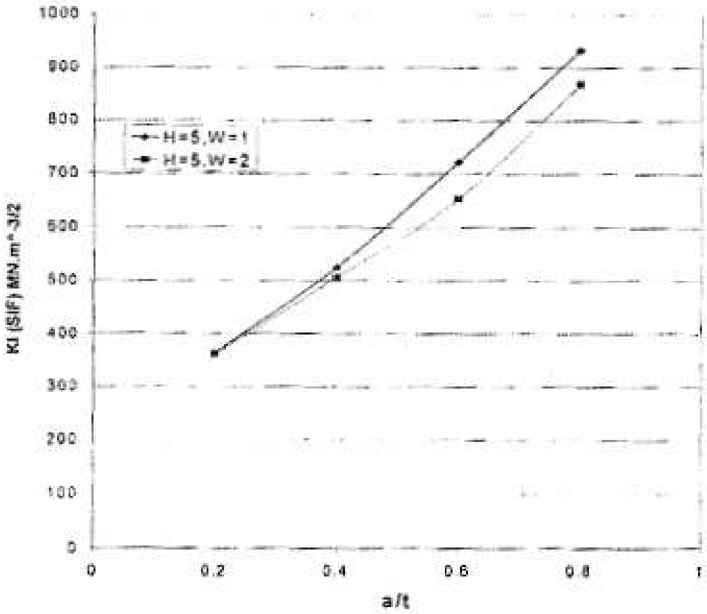
Figure(7) Comparison between Edge Crack in Pipes



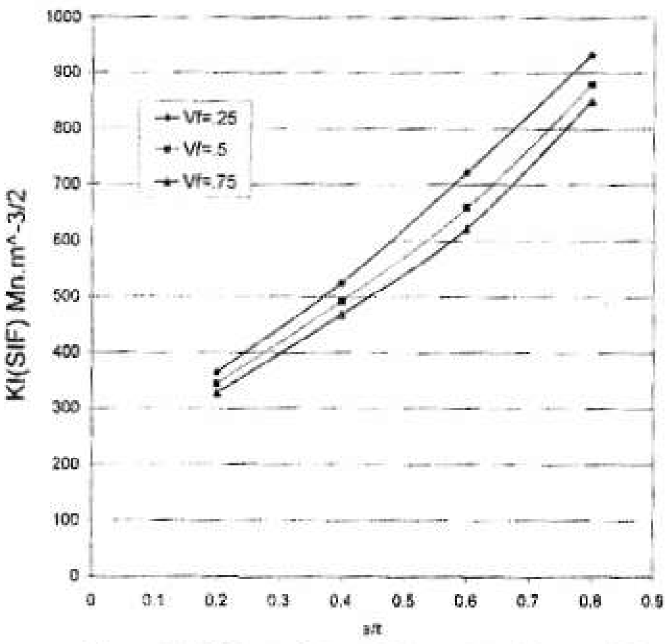
Figure(8) Effect of reinforcement on SIF
(E glass Vf=.25)



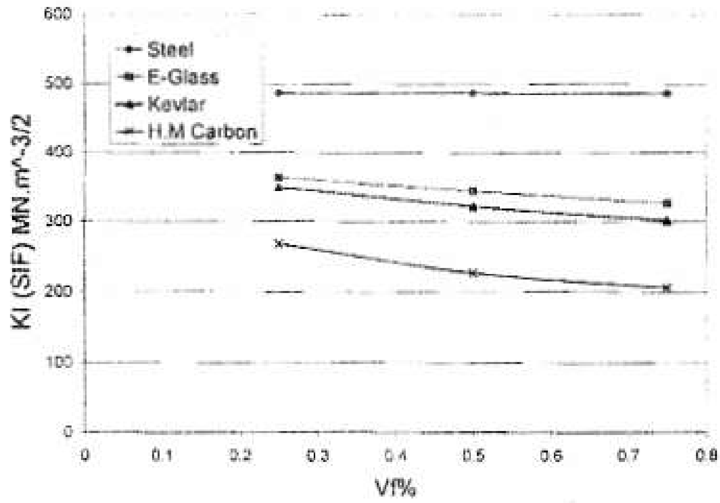
Figure(9) Effect of Pad Length(H) on SIF{E-Glass
Vf=.25 W=1mm }



Figure(10) Effect of Reinforcement Width(W) on SIF



Figure(11) Effect of Fiber Volume Fraction on SIF {E-Glass}



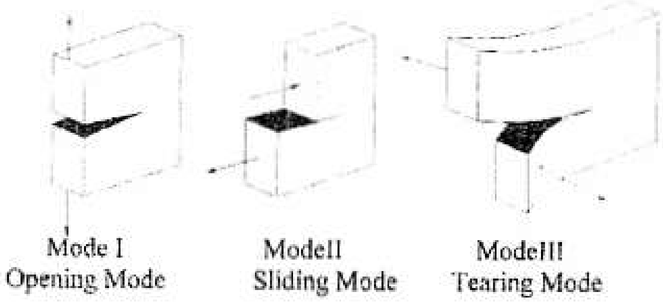
Figure(12) Effect of Fiber Volume Fraction on SIF

Table(1)

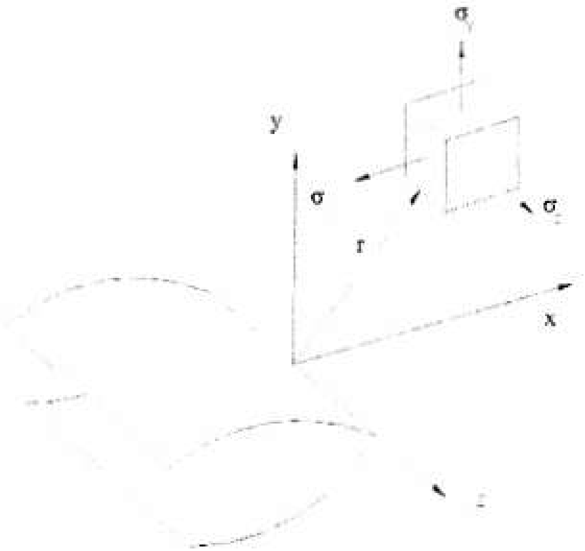
Modulus of Elasticity GPa				
Epoxy	E-Glass	Kevlar 49	Steel	High Modulus Carbon
3.5	76	124	205	640

Table(2)

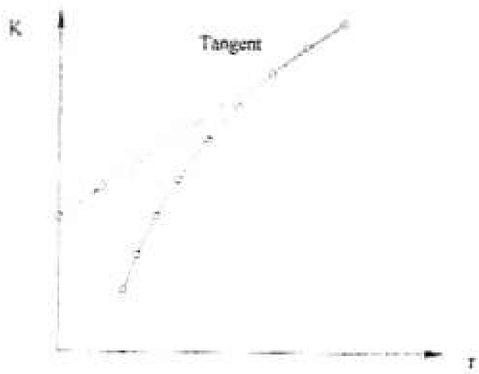
Modulus of Elasticity GPa			
Vf%	E-Glass	Kevlar 49	High Modulus Carbon
25	21.6	33.6	162.6
50	39.75	63.7	321.7
75	57.87	93.87	480



Figure(13) Crack Modes



Figure(14)- Stress components in polar coordinates r, θ at crack tip



Figure(15) Extrapolating method

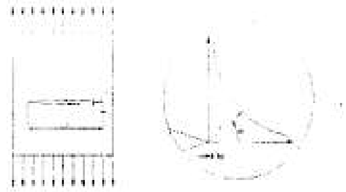


Figure (16) Stress singularity at crack tip

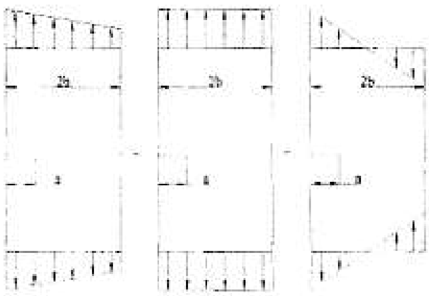


Figure (17) Super position Method

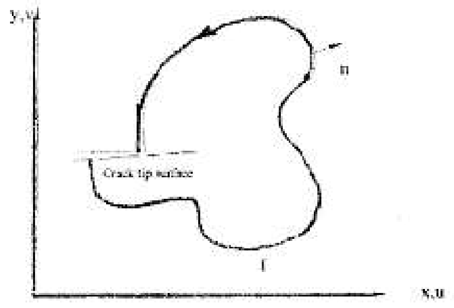


Figure (18) Indirect method (J-integral)