Effects Of WEB Openings On The Analysis Of Cellular Plate Structures Of Varying Depth By Grillage Method

Meethaq Sh. Al-Fatlawi*
Received on: 21/10/2004
Accepted on: 12/6/2005

Abstract
Effect of web openings on the elastic behavior of a tapered box girder is studied. The structures are analyzed by a grillage method including the effect of warping restraint implicitly through the torsional constant. The introduction of web openings results in a significant reduction in the transverse shear and torsional stiffness of the structure.
An appropriate reduction coefficient can be taken from empirical curves derived from a parametric finite element study; these coefficients are used in the grillage analysis.
Results obtained from the grillage method are compared with those gained from finite element solutions (plate/shell) elements by using the software program (MSC/ NASTRAN package), for different proportions of the structure and with different size of opening (12.9%, 25.5%, 36.3%).

Key Words: Cellular structure, Grillage method, plate/shell elements, tapered box beam, Torsion, Warping restraint, Web opening.

أجريت دراسة تأثير وجود الفتحات في الأغشية العمودية على السلوك المرن للمنشات اللوحية الخلوية متغيرة العمق. تم تحليل المنشأ بطريقة Warping المشابك (grillage method) مع إدخال تأثير تقييد الالتواء (torsional constant) من خلال ثابت الثني (transverse) لأظهر أن وجود الفتحات يؤدي إلى تقليل ملحوظ في صلادة القص العرضي وصلادة الثني (shear stiffness (empirical curves). وقد استخدمت منحنى تجريبي (finite elements) لدراسة تم تستخدم العناصر المحددة (finite elements) وهذه المعاملات استخدمت في التحليل بطريقة المشابك، تمت مقارنة النتائج

* Dept. of Civil Eng., University of Baghdad, Baghdad-IRAQ.

https://doi.org/10.30684/etj.24.5.4
University of Technology-Iraq, Baghdad, Iraq/2412-0758
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Introduction

Cellular plate structures are often used in bridges, double-wall storage tanks, dock gates and as double-layer bottoms in ship construction. To reduce the self-weight of such structures by adding accessories, large openings are cut in the web plates to provide access for inspection and maintenance and to allow any liquid being stored or carried to flow freely between the cells.

Many practical diaphragms installations contain large openings such as those required for roof lights.

In [1973] an approach to diaphragms analysis was developed by Bryan who derived simple expressions for both the strength and stiffness of regular diaphragms of rectangular shape. But Bryan’s method was not able to deal with irregular situations such as diaphragms with large openings for roof lights, and the need for a simplified analysis produced By Michael [1977].

Bakht and Jaeger [1981] showed that the grillage method was the most versatile and efficient on that can realistically model the transverse cell distortion of cellular and voided structures. It was noted that the grillage properties could be obtained by multiplying the appropriate section rigidity of the grillage beam by the relevant beam spacing.

Grillage Idealization Of A Cellular Structure

A grillage is composed of discrete one-dimensional members assembled into two dimensional arrangements, so that the interaction between longitudinal and transverse force system takes place at nodal points. Two problems are involved in establishing such a grillage idealization:

a) The representation of shear stiffness of web plates.

b) The representation of the torsional stiffness of the closed cells.

Evans and shanmugam [1979] showed that the introduction of web openings
has little influence on the shear lag effect. They applied a simplified grillage approach to the elastic analysis of cellular structure. The introduction of web openings results in a significant reduction in the bending and torsional stiffness of the structure. Such effect can be taken into account by the use of reduction coefficients. Empirical coefficients were derived from a parametric finite element study.

Husain and Meethaq\(^{(7)}\) [2004] had considered previously the application of grillage techniques to the elastic analysis of cellular structures of varying depth with inclusion of warping effects. The cellular plate structure is discretized into grillage beams running in two directions (at the same position of the webs). To each grillage beam of I-section of varying depth, flexural and rigidity, transverse shear rigidity and torsional rigidity are specified. These section properties are obtained from the properties of the cellular plate structure and from the spacing of the webs.

For structures without web openings, the effective shear area of I-beam element may be taken as the cross sectional area of the web plate. When a structure has openings in web plates, this leads to a significant reduction in the web shear stiffness and a resulting increase in the deflections of the girder. Secondary shearing effects arising from the Vierendeel action (by torsion) of a girder with large openings tend to increase further the deflections.

These effects are taken into account by introducing an effective shear area coefficient \((k_s)\) defined by

\[
\Lambda_{e} = k_s A_w
\]

(1)

To allow for the effects of openings on the torsional rigidity, an effective torsional constant coefficient \((k_T)\) is introduced such that

\[
J_{e} = k_T J_{\text{eff}}
\]

(2)

Where [5]

\[
J_{\text{eff}} = \frac{J}{1 - \frac{\mu \omega}{kL \tanh kL}}
\]

(3-a)

\[
J_{\text{eff}} = \frac{J}{\omega_1}
\]

(3-b)

\[
\omega_1 = \left[ 1 - \frac{\mu \omega}{kL \text{SH}} \right]^{-1}
\]

(4)

Where

\[
\text{CH} = \cosh (kL), \text{SH} = \sinh (kL)
\]
\[ \mu_\omega = 1 - \frac{J}{I_c} \]

\[ k = \left( \frac{\mu_\omega G J}{E I_\omega} \right)^{1/2} \]

\[ I_\omega = \int \omega(s)^2 \, dA \]

Here \( J_{\text{eff}} \) is the effective torsional constant for a single closed cell section under warping restraint. In this analysis, warping restraint effect is included implicitly through the use of the effective torsional constant.

Equation (3-a) is used for a member with one end restraint. Equation (3-b) is used for a member with two end restraints.

The calculation of the effective torsional constant is presented in details in previous paper by Husain and Meethaq\(^7\) [2004] and Al-Dussary\(^8\)

Evans and Shanmugam\(^6\) [1979] presented a parametric study of the effects of openings. They produced design curves for effective torsional constant coefficients \( k_t \), Fig.(4) and for shear area coefficient \( k_s \) as shown in Fig.(5) for girders containing opening.

The proportion of the girders (grillage member) may be defined by:

- Spacing of transverse webs = \( \frac{a}{d} \)
- Spacing of longitudinal webs = \( \frac{b}{d} \)
- Spacing of longitudinal webs = \( \frac{b}{t} \)

Typical girders (grillage member) in a cellular structure are shown in Fig. (1).

Fig. (1) Cellular structure of linearly varying depth
COMPARISON OF
RESULTS FOR
CELLULAR
STRUCTURES WITH
WEB OPENING

Results For A Cellular
Structure of Square Plan
Form

A cellular structure of
linearly varying depth
Fig.(2) was analyzed by the
grillage method in details in
a previous paper of Husain
and Meethaq. The results of
the analysis were compared
with those from the finite
elements (plate / shell
elements) from MSC/
NASTRAN Package.

The properties of the
square structure are
- b/d = 2 , a/d = 2
- E= 0.202 x 10^6 N/mm^2
- v = 0.3
- Three percentages of web
openings are considered.
These are 12.9 %, 25.5% and
36.3%. Rectangular openings
with different sizes are cut in
each case, between the
transverse diaphragms and
symmetrically about the web
as shown in Fig.(3). To avoid
an unrepresentative loss of
stiffness of the girder, the
depth of openings was not
allowed to exceed half the
depth of web.

- Two support conditions
are studied. In the first, all
four edges are simply
supported, and in the second
only two longitudinal edges
are simply supported.
Applied load of (300 kN) is
used at each internal web
intersection was considered.

- The values of k_T and k_S
are taken from Fig. (4) and
Fig.(5) for the structure with
different openings and they
are given also in Table (1).
Fig. (2) Details of square cellular structure of varying depth.

Table (1) Values of $k_s$ and $k_T$, taken from Fig. (4) and Fig. (5) for grillage idealization of structures.

<table>
<thead>
<tr>
<th>% web openings</th>
<th>Square structure $a/d=2$, $b/d=2$</th>
<th>Rectangular structure $a/d=2$, $b/d=4$</th>
<th>Rectangular structure $a/d=4$, $b/d=2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$k_s$</td>
<td>$k_T$</td>
<td>$k_s$</td>
</tr>
<tr>
<td>0.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>12.9</td>
<td>0.52</td>
<td>0.80</td>
<td>0.52</td>
</tr>
<tr>
<td>25.5</td>
<td>0.21</td>
<td>0.51</td>
<td>0.21</td>
</tr>
<tr>
<td>36.3</td>
<td>0.12</td>
<td>0.38</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Table (2) Results of deflections and normal stresses for cellular structure of square plan form.

<table>
<thead>
<tr>
<th>% of web opening</th>
<th>Simply supported at four edges</th>
<th>Simply supported at two edges</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Finite Element Method</td>
<td>Grillage Method</td>
</tr>
<tr>
<td>Defl. (mm)</td>
<td>Norm. Stress (MPa)</td>
<td>Defl. (mm)</td>
</tr>
<tr>
<td>0</td>
<td>1.20</td>
<td>35.18</td>
</tr>
<tr>
<td>12.9</td>
<td>1.38</td>
<td>37.34</td>
</tr>
</tbody>
</table>
Table:

<table>
<thead>
<tr>
<th>25.5</th>
<th>2.15</th>
<th>89.31</th>
<th>2.34</th>
<th>98.55</th>
<th>2.77</th>
<th>113.02</th>
<th>3.17</th>
<th>125.39</th>
</tr>
</thead>
<tbody>
<tr>
<td>36.3</td>
<td>2.65</td>
<td>107.32</td>
<td>2.90</td>
<td>116.94</td>
<td>3.01</td>
<td>120.70</td>
<td>3.38</td>
<td>134.62</td>
</tr>
</tbody>
</table>

(a) 25.5% web opening in transverse direction.

(b) 25.5% web opening in transverse direction.

(c) 36.3% web opening in transverse direction.

Fig. (3) Dimensions of openings in the transverse diaphragms depending on the least depth of the structure.
Fig. (4) Variation of effective torsional constant coefficient $k_T$ with percentage of web openings. [6]
The results are shown in Fig.(6) for the vertical deflection of the simply supported structure at four edges with no openings, 12.9% web opening, 25.5% and 36.3% opening.

The results in Table.(2) show a significant reduction in structure stiffness due to introduction of web openings. The deflection increases by (14.4%) for structures with 12.9% opening and (44% and 54%) increase in deflection are noted for (25.5%) and (36.3%) openings in webs respectively.

For structure of square plan form and supported at four edges (grillage method), the normal stresses increase by (7.4%) for 12.9% opening. The increase reaches (61.4% and 67.45%) for opening percentages from (25.5%) to (36.3%) respectively.

The Finite Element mesh involved (144) elements for top plates and the same for bottom plate. (32) Elements for each transverse plate which contained openings. (36) Elements for each longitudinal plate. So the total elements are (888) elements.

Results were also obtained for the square cellular structure simply supported structure at two opposite edges and subjected to equal vertical loads at the internal web intersection points. Table (2) shows that the deflection increases by (16%) for 12.9% opening, (42%) increase in deflection for 25.5% opening and the last increase is (45.5%) for 36.3% opening in web plates.
The normal stresses for the cellular structure simply supported at two edges increase by (45%) for (12.9%) opening, (75.6%) increase in normal stresses for (25.5%) opening and (77.3%) for (36.3%) opening.

The results which are obtained from Table (2) show that the change in stresses due to introduction of web opening is much greater than the change in deflection. This is true as the stresses depend on the derivatives of displacements and thus liable to higher errors.

Fig. (6) Variation of vertical deflection at section (A-A) of the cellular structure simply supported at four edges.
Fig.(7) Variation of vertical deflection at section (A-A) of the cellular structure simply supported at two opposite edges.

Fig.(8) Variation of central deflection of the square cellular structure.

- Fig.(8) shows the variation of the central deflection against the percentage web opening for both support conditions. This increase in deflection is due to mainly to the reduction in the
transverse shear rigidity of the web.

Results For a Cellular Structure of Rectangular Plan Form:
- The rectangular cellular structure has $b/d = 4$, $a/d = 2$, $b/t = 300$, $t = 10$ mm, $d = 750$ mm at one edge and the other is 1500 mm. The structure is subjected to equal vertical loads at the internal web intersection points. The dimensions of the structure are shown in Fig. (9)
- The dimensions of the openings for each case are shown in Fig. (3).
- Table (3) shows the results for the structure of rectangular plan form with two support conditions. In the first support condition, the structure is supported at four edges. In the second condition of supporting, the structure supported at two opposite edges.
- The values of shear area coefficient $k_s$ and torsional coefficient $k_T$ are presented in Table (2). These values are taken from Fig.(4) and Fig.(5). And used in the grillage analysis.
- For the structure supported at four edges, the maximum deflection increases by $34\%$ for $12.9\%$ opening, $54\%$ and $68.2\%$ increase in deflection for $25.5\%$ and $36.3\%$ opening respectively. The normal stress increase by $54.2\%$ for $12.9\%$ opening. The increase reaches $68\%$ and $77\%$ for $25.5\%$ and $36.3\%$ of opening.
- In the rectangular cellular structure supported at two opposite edges, the increase in deflections are $(41\%, 44.8\%$ and $49.7\%)$ for percentage of opening $(12.9\%, 25.5\%$ and $36.3\%)$ respectively. While the normal stresses increase from $(60.5\%, 63.4\%$ and $66.5\%)$ for the same stepping of openings.
- The characteristics noted in the results for the square cellular structure are again presented. Although there is little change in the longitudinal and transverse stresses,
- The deflection increases significantly as a result of the reduction in the stiffness arising from the introduction of openings. The grillage stresses tend to exceed those obtained from the finite element method; this is acceptable for a method intended for use as a design tool.
Table (3). Results for rectangular cellular structure of b/d=4
a/d=2 and b/t =300

<table>
<thead>
<tr>
<th>% of web opening</th>
<th>Simply supported at four edges</th>
<th>Simply supported at two edges</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Finite Element Method</td>
<td>Defl. (mm)</td>
</tr>
<tr>
<td></td>
<td>Grillage Method</td>
<td>Defl. (mm)</td>
</tr>
<tr>
<td>0</td>
<td>1.29</td>
<td>19.39</td>
</tr>
<tr>
<td>12.9</td>
<td>1.98</td>
<td>43.86</td>
</tr>
<tr>
<td>25.5</td>
<td>2.87</td>
<td>63.22</td>
</tr>
<tr>
<td>36.3</td>
<td>4.09</td>
<td>89.34</td>
</tr>
</tbody>
</table>

E = 0.202 × 10^6 N/mm²
v = 0.3
G = 77692.31 N/mm²

Fig. (9) Dimensions of rectangular cellular structure.
Results For a Larger Cellular Plate Structure of Rectangular Plan Form

Another rectangular cellular plate structure of \( a/d = 4, \) \( b/d = 2, \) and \( b/t = 150. \) The structure is subjected to the same loading as in the previous example. The dimensions of openings for each case are \((\text{width}=b_1=1161 \text{ mm}, \text{height}=h_1=250 \text{ mm})\) for 12.9% opening, \((b_1=1530 \text{ mm}, h_1=375 \text{ mm})\) for 25.5% opening and \((b_1=2178 \text{ mm}, h_1=375 \text{ mm})\) for 36.3% opening. Characteristics similar to those noted earlier are again observed. The results in Table (4) show the maximum vertical deflections and normal stresses. The grillage values of the deflections and the predicted stresses are slightly in excess of the finite element results for the girders with openings.

The reduction in torsional stiffness is again apparent and the reduction becomes most rapid for the models with larger openings. Fig.10) and Fig.(11) present the variation of maximum deflection and normal stresses for the larger rectangular cellular structure.

Table (4) Results For Rectangular Cellular Structure of \( b/d=2, \)
\( a/d=4 \) and \( b/t=150. \)

<table>
<thead>
<tr>
<th>% of web opening</th>
<th>Simply supported at four edges</th>
<th>Simply supported at two edges</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Finite Element Method</td>
<td>Grillage Method</td>
</tr>
<tr>
<td></td>
<td>Defl. (mm)</td>
<td>Norm. Stress (MPa)</td>
</tr>
<tr>
<td>0</td>
<td>1.62</td>
<td>29.81</td>
</tr>
<tr>
<td>12.9</td>
<td>2.52</td>
<td>106.08</td>
</tr>
<tr>
<td>25.5</td>
<td>3.03</td>
<td>123.40</td>
</tr>
<tr>
<td>36.3</td>
<td>3.21</td>
<td>129.28</td>
</tr>
</tbody>
</table>
Fig. (10) Variation of vertical deflection for the larger rectangular cellular structure
CONCLUSIONS

- The introduction of web openings leads to a considerable increase in deflections.

Theses increases in deflections arise mainly from a reduction in the transverse shear and torsional stiffness of the web plates. Such effect can be taken into account by the use of appropriate reduction coefficients, the values of which may be obtained from the empirical curves in Fig. (4) and (5). Theses values were needed in a grillage analysis of the cellular structure in which the effect of warping restraint is included implicitly through the torsional stiffness of the grillage member.

- The structure of linearly varying depth is analyzed under different proportions, with different support conditions and with different size of openings (12.9%, 25.5% and 36.3%). The results are comparable to those by the finite (plate/shell) elements.

- This study shows that the change in stresses due to the introduction of web openings is much greater than the change in deflection.
The two methods (grillage and finite element methods) take the same time for solution of the cellular structure due to efficient computer software.

REFERENCES