Bond Strength of Tension Bars in High-and Normal Strength Concrete Beams

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Abstract

Sixty beams, obtained from the literature, have been studied in this work. All these beams, failed in development of deformed bars in tension. Most ultimate bond stress equations are based on normal strength concrete tests.

Regression analysis led to a proposed design equation for bond strength of deformed bars in tension. This method is shown to be safe and at the same time giving a coefficient of variation (COV) of 13.56 present. This value is lower than all the (COV) obtained for the 5 existing code methods.

Keywords: beams, bond stress, development, high strength concrete, normal strength concrete.

Introduction

Most methods used for development of deformed bars in tension are based on normal strength concrete. Recent advances in technology have led to production of high strength concrete. Thus it is
necessary to study development in high strength concrete.

Research significance

The paper reviews the development of tension bars in high and normal strength concrete beams based on 5 methods: ACI-02(1), Zsutty(2), Orangun et. al.(3), Kemp and Wilhelm(4), and Darwin et. al.(5). These methods are based on normal strength concrete tests to find the ultimate bond stress equations. This work aims at finding suitable equation to predict bond strength for high and normal strength concrete beams. A proposed simple design method, which is based on regression analysis, is introduced.

Experimental Results

All available tests of development of tension bars obtained from the literature are used in this work. Table (1) gives the range of variables of these 60 beams. These variables are: Compressive strength of concrete $f'_c$, development length $L_{db}$, width of concrete section $b$, diameter of anchored bar $d_b$, two clear cover one of them in x direction ($C_x$) while the other in y direction ($C_y$) and average bond stress in tests $U_{test}$. These beams are obtained from references (6-9).

Evaluation of Experimental Results

Existing development of tension bars design equations:

The following five existing methods considered in this work are applied to the experimental results of beams failing in development of deformed bars in tension.

1. **ACI 318M-02 code method**

$$U = \frac{1}{3.6 \sqrt{f'_c}} \left( \frac{C_a + K_{fr}}{d_b} \right) \frac{1}{\alpha \beta \gamma}$$  \hspace{1cm} (1)

where:

- $A$ reinforcement location factor,
- $B$ coating factor = 1.0 for uncoated bars,
- $A$ reinforcement size factor = 0.8 for $d_b \leq 19$ mm; = 1.0 for $d_b \geq 20$ mm,
- $\Gamma$ light weight aggregate concrete factor = 1.0 for normal weight concrete,
2. Zsutty method^{(2)}

\[ U = 5.07 f' C \left( \frac{d_b}{L_b} \right) \left( \frac{C}{d_b} + 2r \right)^{\frac{1}{2}} \] ...(3)

where:  \[ r = 100 \frac{A_{tr}}{S b} \]

\[ C = \text{the smaller of } C_b \text{ or } C_x \]

\[ C_s = \text{the smaller of } C_x \text{ or } 0.5*S \]

\[ \left( \frac{c}{d_b} + 2r \right) \leq 3 \]

3. Orangun et. al.^{(3)}

\[ U = \sqrt{f'_c} \left[ \frac{A_{tr} f_{yt}}{415.52 S d_b} \leq 0.25 \text{ and } \frac{C}{d_b} \leq 2.5 \right] \]

\[ \left( 0.1 + 0.25 \frac{C}{d_b} + 41.5 \frac{d}{L_b} + \frac{A_f}{tr \ y_t} \right) \left( 0.1 + 0.25 \frac{C}{d_b} + 41.5 \frac{d}{L_b} + \frac{A_f}{tr \ y_t} \right) \]

\[ \left[ \frac{C}{d_b} + 2r \right] \leq 3 \]

4. Kemp and Wilhelm^{(4)}

\[ U = \sqrt{f'_c} \left[ 0.546 + 0.241 \frac{C}{d_b} \right] \]

\[ + 0.191 \left( \frac{A_{tr} f_{yt}}{S d_b} \right) \] ...(4)

where:

\[ \frac{A_{tr} f_{yt}}{S d_b} \leq 12.4 \text{ and } \frac{C}{d_b} \leq 3.0 \]
5. Darwin et. al. \textsuperscript{(5)}

\[ U = \sqrt{f_c} \left[ \left( 0.088 + 0.176 \frac{C}{d_b} \right)^* \right. \]

\[ \left. \left( 0.92 + 0.08 \frac{C_{\text{max}}}{C_{\text{min}}} \right) + 6.228 \frac{d_b}{L_d} \right] \text{(5)} \]

\begin{align*}
C_{\text{max.}} &= \text{maximum value of } C_n \text{ or } C_b, \\
C_n &= \text{the smaller of one - half the clear spacing between bars} \left( S' \right) \text{ plus } 6.35 \text{ or } C_x \\
C_{\text{min.}} &= \text{minimum value of } C_n \text{ or } C_b.
\end{align*}

Statistical evaluation of existing design method

Table (2) indicates the values of the results of the 60 tested beams, compared with predicted strength \( (U_{\text{test}}/U_{\text{calc.}}) \). These values show a range of 1.06-1.62 for the mean of this ratio. It can be seen that the Kemp and Wilhelm method is the one with the greatest amount (27 specimens) of unsafe predictions-based on a value of \( (U_{\text{test}}/U_{\text{calc.}}) < 1 \). The lowest ratio for this method is 0.60. In contrast, the ACI code method is the most conservative with all 60 beams being on

the safe side. This method has the highest mean value (1.62).

The coefficient of variation (COV) gives a good indication as a measure of the relevance of the method for prediction of the ratio \( (U_{\text{test}}/U_{\text{calc.}}) \). From Table (2), it can be seen that the Zsutty method has the highest COV (at 22.87 percent). The best COV of all 5 existing methods is in the Orangun et. al. method (at 20.29 percent).

Regression analysis of test results

By using the regression analysis, the 60 test results were analyzed by a personal computer. The aim is to obtain a simple and conservative design method for development that gives the lowest possible COV values of the ratio \( (U_{\text{test}}/U_{\text{calc.}}) \). This has led to the following prediction equation for \( U_{\text{prop.}} \).
Table (2) shows a summary of statistical evaluation of the proposed design method.

To illustrate the relevance of the proposed design method the ratio of \( \frac{U_{\text{test}}}{U_{\text{calc}}} \) has been compared by this method with that of the available design code procedure-Eqn. (1) by ACI 318-02. These are shown in Figs. (1, 2, 3 and 4).

The comparison in Fig. (1) between the ACI-02 method and the proposed method shows, as expected from Table (2), a large scatter in the ACI-02 method, as compared to the proposed Eqn. (6). In addition, the proposed method gives satisfactory safe prediction.

Similar conclusions regarding the much greater scatter by the ACI-02 method can be seen in Figs. [2 (influence of \( L_d \)), 3 (influence of \( b \)) and 4 (influence of \( d_b \))]. A slight rise of safety with increasing \( L_d, b, \) and \( d_b \) can be noticed indicating that this method (ACI-02) tends to the rise conservatively with increasing \( L_d, b, \) and \( d_b \); i.e. a positive slope is obtained from results of \( \frac{U_{\text{test}}}{U_{\text{calc}}} \) versus \( L_d, b, \) and \( d_b \). Similar relationships using the present new equation are shown in Figs. (2, 3 and 4). These figures show improvement in the obtained results and the best fit line has a negative slope with a relative capacity strength value of \( \frac{U_{\text{test}}}{U_{\text{calc}}} \).

**Conclusions**

Based on this work, the following conclusions are made:

1- Table (2) shows that the COV of the ratio \( \frac{U_{\text{test}}}{U_{\text{calc}}} \) was in descending order 22.87, 21.59, 20.77, 20.74 and 20.29 respectively using Zsutty, Kemp and Wilhelm, Darwin et al., ACI-02, and Orangun et. al. methods.

2- Most results of (ACI-02) indicate conservative prediction of strength with high arithmetic mean of \( \frac{U_{\text{test}}}{U_{\text{calc}}} \), while the proposed method led to improve results compared to
(ACI-02) are shown in Table (2).

3- Fig. (1) shows that the safety of prediction by the (ACI-02) and proposed methods are essentially unchanged within the range of $f_c'$. A large scatter in the ACI-02 method, as compared to the proposed Eqn. (6).

4- Figs. (2, 3 and 4) show a slight rise of safety factor with rising $L_d$, $b$, and $d_b$ values based on ACI-02, in contrast with proposed method (but large scatter for ACI-02 versus much less scatter for the proposed method). In addition, the proposed method gives satisfactorily safe prediction.

Acknowledgment

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Future Research

The following suggestions may be considered as an extension of the present work:

1- Development length of top bars in high strength concrete (bars confined by transverse reinforcement).

2- Investigating the local bond stress-slip behaviour of reinforcing bars embedded in fiber in high strength concrete beams.

References

1- ACI Committee 318, “Building Code Requirements for Reinforced Concrete and Commentary (ACI-318-02/ACI-318R-02),” American Concrete Institute, Detroit, MI, 2002.


Notation

- $A_r$: area of transverse reinforcement crossing the potential plane of splitting adjacent to a single anchored reinforcement, $mm^2$
- $b$: width of concrete section, $mm$
- $C_a$: the smaller of $C_x$ or $C_{xs}$
- $C_b$: clear bottom cover to main reinforcement, $mm$
- COV: coefficient of variation
- $C$: maximum value of $C_n$ or $C_{n,\max}$, $mm$
- $C_n$: minimum value of $C_n$ or $C_{n,\min}$
- $C_{xs}$: the smaller of one-half the clear spacing between bars plus 6.35 or $C_x$
- $C_x$: is one-half the bar diameter plus the smaller of $C_x$ or one-half the clear spacing between the bars in the layer ($S'$), $mm$
- $C_c$: clear cover measured along the line through the layer of bars, $mm$
- $d_b$: diameter of anchored bar, $mm$
- $f_m$: yield strength of transverse reinforcement, MPa
- $f_c$: compressive strength of concrete, MPa
- $L_{cd}$: development (embedding) length, $mm$
- $U$: average bond stress, MPa
- $U_{calc}$: calculated average bond stress, MPa
- $U_{res}$: average bond stress in tests, MPa
- $S$: center to center spacing of transverse reinforcement, $mm$
- $S'$: clear spacing between anchored bars, $mm$
- $N$: number of anchored bars
- $\alpha$: reinforcement location factor
- $\beta$: coating factor
- $\lambda$: reinforcement size factor
- $\gamma$: light weight aggregate concrete factor
- $\bar{X}$: mean value of $U_{res}/U_{calc}$
Table 1- Range of variables for the 60 tested beams.

<table>
<thead>
<tr>
<th>Detail</th>
<th>$f'_c$, MPa</th>
<th>$L_d$, mm</th>
<th>$b$, mm</th>
<th>$d_b$, mm</th>
<th>$C_b$, mm</th>
<th>$C_s$, mm</th>
<th>$U_{test}$, MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>16.41</td>
<td>140</td>
<td>149.3</td>
<td>12.7</td>
<td>17.5</td>
<td>69.9</td>
<td>2.46</td>
</tr>
<tr>
<td>High</td>
<td>83.7</td>
<td>1143</td>
<td>461.2</td>
<td>35.8</td>
<td>68.3</td>
<td>221.8</td>
<td>14.52</td>
</tr>
</tbody>
</table>

Table 2- Statistical analysis of the ratio of ($U_{test}/U_{calc}$).

<table>
<thead>
<tr>
<th>Detail</th>
<th>ACI-02</th>
<th>Zsutty</th>
<th>Orangun et. al.</th>
<th>Kemp and Wilhelm</th>
<th>Darwin et. al.</th>
<th>Proposed equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{x}$</td>
<td>1.62</td>
<td>1.13</td>
<td>1.22</td>
<td>1.06</td>
<td>1.12</td>
<td>1.21</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.336</td>
<td>0.258</td>
<td>0.248</td>
<td>0.229</td>
<td>0.232</td>
<td>0.164</td>
</tr>
<tr>
<td>COV %</td>
<td>20.74</td>
<td>22.87</td>
<td>20.29</td>
<td>21.59</td>
<td>20.77</td>
<td>13.56</td>
</tr>
<tr>
<td>Number&lt;1</td>
<td>0</td>
<td>22</td>
<td>13</td>
<td>27</td>
<td>22</td>
<td>6</td>
</tr>
</tbody>
</table>

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Compressive strength of concrete $f_c'$, MPa

a- ACI-02

b- Proposed method

Figure (1): Influence of compressive strength of concrete $f_c'$ on test results.
Figure (2): Influence of development embedment length $L_d$, mm on test results.

Development (embedment) length $L_d$, mm

- ACI-02
- Proposed method
Proposed method

Figure (3): Influence of beam width $b$ on test results.
Diameter of anchored bar $d_b$, mm

a- ACI-02

b- Proposed method

Figure (4): Influence of diameter of anchored bar $d_b$ on test results.