


Undrained Bearing Capacity For Strip Footing On Two Layered Elastic-Plastic Clay By Using FEM

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Abstract

In this paper a program using FEM is developed and used as a tool to determine the undrained bearing capacity for a strip footing on layered clay. The clay soil is considered as elastic-plastic material. Results of the program for one and two layers are compared with available solution and give a good agreement. Different ratios of undrained shear strength for two layers are used with different ratios of thickness to estimate the ultimate bearing capacity in addition to the displacement that occurs in the media. The important cases that were studied here are: case of strong layer on soft layer and case of soft layer on strong layer. The Study was conducted to assess the depth of layer at which the soft layer doesn't affect the undrained bearing capacity. Undrained bearing capacity factors found depend on the ratio of the first layer to width of the footing and the ratio of undrained shear strength for the two layers. Contour lines are used extensively to demonstrate the displacement in soil at failure.

قابلية التحمل لاساس مستمر على تربة مرنة-لدنة ذات طبقتين باستخدام طريقة العناصر المحددة

الخلاصة

في هذا البحث تم تطوير برنامج مبني على طريقة العناصر المحددة واستخدامه في ايجاد قابلية التحمل لاساس مستمر على تربة متطبقة. تصرفت التربة باعتبار (Elastic-Plastic). النتائج التي تم الحصول عليها من البرنامج بعد تطبيقه على طبقة واحدة وطبقتين من التربة اظهرت توافقا جيدا مع مجموعة من الحلول المتوفرة. تم ايجاد قابلية التحمل -اضافة الى الازاحات المتولدة- لطبقتين من التربة مختلفتين في المقاومة والسمك. الحالات المهمة التي تم دراستها هي حالة كون الطبقة الاولى اقوى من الثانية وحالة كون الطبقة الثانية اقوى من الاولى. الدراسة اجريت لبيان عمق الطبقة الضعيفة التي عندها لا يوجد تأثير في قابلية التحمل. تم ايجاد معاملات قابلية تحمل التربة بالاعتماد على نسبة سمك الطبقة الاولى الى سمك الاساس علاوة على نسبة تحمل الطبقة الاولى الى الثانية. تم استخدام الرسوم الكنتورية لايضاح الازاحات التي تحصل في التربة عند الفشل.

Introduction

The performance of foundation depends on two factors, the bearing capacity and the settlement of the soil beneath the foundation. Bearing capacity of a clay and dense soil is essentially influenced by the undrained shear strength property. Calculating the bearing capacity is achieved by applying the limit equilibrium to the assumed shear failure surface. Bearing shear factors

(N_c) for the case of constant undrained shear strength with depth based on assumed shear failure surface are found different as follows, 5.7, 5.4, 5.62, 5.14.(Terzagi, 1943; Prandtl, 1921; Meyerhof, 1951). Almost all-natural soils are highly variable in their properties and rarely homogeneous. An overview of the different techniques developed to deal with soil heterogeneity has been presented by (Tamer et al 2002).

Undrained shear strength is usually not constant and may increase with depth. Several researchers have incorporated this property which is the increasing of shear strength with depth to find the bearing capacity of the soil under footing (Davis and Booker 1973; Martin and Houlsby 2002). The soil is often found as layers. Undrained shear strength is different at each layer. A method of finding the average between layers was used by Bowles, 1988). Many researchers have used the limit equilibrium techniques to compute the undrained shear factor when the soil is two layers Button, (1953); Merifield, (1999). Brown and Meyerhof (1969) found an empirical equation for bearing capacity which depends on the experimental studies. Finite element method has been used by some to analyze the problem using small and large strain theory (Carter and Wang, 2001). Another has used finite difference to analyze the problem of improving the first layer by reinforcement (Yin, Zhan,1997). In this research the soil is assumed as elastic plastic and the media are two layers different in strength and thickness. A failure criterion such is assumed as Von Mises criterion and the strain is viscoplastic strain. Research was achieved to study different cases of meshes and layers as presented next.

Problem definition

The bearing capacity analysis uses an elastic plastic stress strain law with a Von Mises failure criterion. The elastic plastic soil is described by three parameters namely the elastic properties (Young's modulus, E and Poisson's ratio, ν) and the undrained shear strength C_u . Figure 1 show a

sketch of the problem of a flexible strip footing of 4m width on a surface of two layered clay. In the present study the modulus of elasticity and Poisson's ratio are held constant at (100000 Kpa, 0.3 respectively) while C_u is taken as constant at each layer but it has different values in the two layers. (Young's modulus governs the initial elastic response but doesn't affect the bearing capacity (Fenton and Griffiths, 2003). The used program that is based on the finite element method uses eight-nod quadrilateral element. For more details about the program and the theoretical principles of the problem the reader is referred to (chapter six of the text by Smith and Griffiths, 1998). It was developed to take into account two layers with two values of C_u by including them in the part of viscoplastic analysis in the program. A uniform load was applied incrementally until failure occurs. The outputs of the program are the bearing capacity at each stage of loading and the vertical and horizontal displacement at each point in the mesh of soil media. Two cases of soil media dimensions are taken. Figure 4 shows the first case while Figure 5 shows the second.

Verification

At first the program was used to compute the bearing capacity for the case of constant undrained shear strength. Figure 4 shows the mesh used where the depth is $1.25B$ and it is laterally extended to $3B$. The mesh consists of 512 elements and 1633 nodes. High numbers of element are concentrated close to the footing where the effect of strip loading is increasing at these zones. Half of the problem is analyzed because of

symmetry. The results obtained from the program execution are compared with the three solutions as listed in the "introduction". Figure 2 shows the relation between the undrained shear strength for values $C_u=10, 20, \dots, 100$ and the bearing capacity Q where $Q = C_u N_c$. The results of the computer solution is closer to the Prandtl solution $Q=5.14 C_u$ than the other. The computer solution presented here is of two forms the first is of 250 iterations while the second is at iteration less than 250. The solution of Prandtl is located between these two forms therefore it can be used the solution of 250 iteration with acceptable error.

Two layered clay soil model a

The program is used to compute the ultimate bearing capacity for two cases of layered soil a- layer with low shear strength on a layer of strong shear strength b- layer with strong shear strength on a layer of low strength.

a- For the first case the program was implemented for shear strength ratio $C_{u1}/C_{u2} = 30/100, 50/100, 70/100, 10/100$, depth ratio $H1/H=1.25/5, 2.5/5, 3.75/5, 5/5$, and the thickness of the first layer to the width of footing $H1/B=1.25/4, 2.5/4, 3.75/4, 5/4$. In this model of mesh where the depth is $H=1.25B$ the depth ratio $H1/H$ should be considered. The load was applied incrementally started at load=1 with increment one at each stage of loading until failure occurs. Data obtained from the numerical solution are the bearing capacity and the displacement at each point. Curves of the relation between the bearing capacity at each loading with the displacement at the centerline point is presented. Also the vertical displacement with depth at

centerline is presented. It is necessary to understand the behavior of soil under loading, therefore the vertical displacement at surface of soil from the centerline of loading is presented in curves. For each case the vertical and horizontal displacement are obtained for each point in mesh.

b- For the second case, the program has been used to calculate the undrained bearing capacity for $C_{u1}/C_{u2} = 100/70, 100/50, 100/30$ where $H1/H=1.25/5, 2.5/5, 3.75/5, 5/5$. The depth to the width of footing is varied as $H1/B=1.25/4, 2.5/4, 3.75/4, 5/4$. Relations between the undrained bearing capacity with vertical displacement at failure stage are presented for each case. Also curves of displacement with depth and with surface are drawn. A good presenting for the displacement at each point in mesh is achieved by using contours.

Model b

In this model the depth of the analyzed soil is $7B$ while the laterally width is $10B$. The depth of the first layer to width of the footing is important and affects the bearing capacity instead of the depth of the first to the depth of second layer. The program is used to calculate the bearing capacity for different ratio of undrained shear strength varied as follows

$C_{u2}/C_{u1}=100/10, 100/20, \dots, 100/90$ and $H1/B=0.25, 0.5, 1, 1.5, 2, 2.5, 2.75$. The computer solution is compared with four solution (Button, 1953; Mirifield, 1999 and Carter 2001); as presented by Carter (2001) and the solution of (Brown and Meyerhof, 1969) according to the following semi empirical equation

$$N_c = 1.5 \frac{H}{B} + 5.14 \frac{C_{u2}}{C_{u1}}$$

Figure 21 shows the relation between the undrained bearing capacity and the ratio of the undrained shear strength for the second layer to that of the first C_{u2}/C_{u1} compared with four solutions. The curves are for two cases of H/B (0.5,1). The result of the computer program is close to the solution of (Carter and Wang, 2001) for small strain analysis. As it can be seen from Figs.6a, 7a, 8a the existence of a soft layer under the strong layer will decrease the bearing capacity factor N_c . Fig. 22 shows the depths of soft layer that is located under the strong layer at which the N_c factor does influence ratios of undrained shear strength C_{u2}/C_{u1} between 0.1-0.9. Contours were used to state the N_c factor value at any depth and for any ratio of undrained shear strength that can be observed in Fig 23.

Discussion

Figure 2 shows the increase in ultimate bearing capacity with the increase undrained in shear strength as it is expected. The plastic behavior of clay depends on C_u , therefore the deformation is increased with high C_u before failure occurs. Figs.6a, 7a show that the existence of soft clay layer under the strong layer will decrease the bearing capacity and as the thickness of soft layer increases the bearing capacity decreases. Also it can be seen that the increase in the thickness of the strong layer H_1/B increases the bearing capacity. For example at $H_1/B=1.25/4$, for $C_{u1}/C_{u2}=100/70$, $BC = 435$ Kpa while at $H_1/B=3.75/4$, $H_1/H=3.75/5$ for C_{u1}/C_{u2}

$= 100/70$, $BC=528$. The effect of the thickness of the strong layer on the bearing capacity is different as the shear strength ratio is different. As the ratio is close to $C_{u1}/C_{u2}=10$ the effect becomes high. From Figs.6a, 8a it can be noted that the difference between bearing capacity for $C_{u1}/C_{u2}=100/70$, at $H_1/B= 1.25/4$ and $H_1/B=3.75/4$ is $528-435= 93$, while for $C_{u1}/C_{u2}=100/70$ at the same thickness it is $425-154=271$. Figs.6b, 7b and 8b show the vertical displacement at surface with the distance from the center of loading. The points under footing moves down while the points besides the footing rise above and the rising decreases with the distance from the center of loading. This means that the mode of failure is general shear failure. Figs.6c, 7c, 8c show the relation between the vertical displacement and the centerline depth. The vertical displacement decreases with depth. For two layers the behavior of the soil under footing is very complex. For the case of soft layer on strong layer, the soft layer which is near to the footing governs the bearing capacity in spite of the thickness of the strong layer. In other words the depth factor H_1/B does affect the bearing capacity, see Figs.9a, 12a and 14a. According to this result the soft layer under footing in this case of research should be released or improved. The failure occurs at the first layer as you see from Fig.19a compared with Fig.20a, that the displacement is concentrated in the first layer. The figure shows the depth of the soft layer at which there is no effect on the bearing capacity, this means that the curve of the bearing factor of 5.14. The designer can determine the effect of soft layer on the bearing capacity by

using Fig.22. Another curve-Fig.23-states the bearing factor according to the depth factor and the undrained shear strength ratio.

Conclusions

Finite element method is a reliable method used to analyze the bearing capacity for a strip footing on two layers. The results of the computer program that is based on FEM is in agreement with available solution as presented last. According to the result of the research, the following notes can be concluded.

a- For the case of two layers where the first layer has a small undrained shear strength –which is close to the footing-compared to the second layer the bearing capacity of the clay governed by the layer of the small shear strength in spite of the strength of the second layer, therefore the first layer should be removed or improved or the footing is driven under this layer if possible.

b- The thickness ratio of the first layer to the second layer H_1/H has an effect on the bearing capacity when the dimension of the mesh is $1.25B$, $3B$ while there is no effect in the second model of $7B$, $10B$.

c- For the two layers where the first layer is stronger than the second layer the bearing capacity decreases with the decrease in the strength of the second layer. For all ratio of C_{u2}/C_{u1} the effect on the bearing capacity is vanished at depth $2.75B$.

d- For each ratio of C_{u2}/C_{u1} there is a depth factor H_1/B at which the effect of the soft layer on the bearing capacity is vanishing ($N_c=5.14$) so at $C_{u2}/C_{u1}=0.8$ the depth factor H_1/B is

Undrained Bearing Capacity For Strip Footing On Two Layered Elastic-Plastic Clay By Using Fem

0.75 , and at $C_{u2}/C_{u1}=0.5$ the depth factor H_1/B is 1.5 and so on as stated in Fig.23.

e- The bearing capacity depends on two factors H_1/B and C_{u2}/C_{u1} . Figures are presented to estimate the bearing capacity depending on these two factors.

References

- 1- Button, S.J., "The Bearing Capacity of Footings on A Two Layers Cohesive Subsoil", Proc., 3rd Int. Conf. on Soil Mechanics and Foundation Engineering, Zurich, Vol.,1, pp. 332-335(1953),
- 2- Bowles, J. E., Foundation Analysis and Design, McGraw-Hill, New York. (1988),
- 3- Brown, J. D. and Meyerhof, G. G., "Experimental Study of Bearing Capacity In Layered Clays", Proc., 7th Int. Conf. on Soil Mechanics and Foundation Engineering, Mexico, Vol.2, PP. 45-51. (1969),
- 4- Carter, J. P., and Wang, C., (2001), "Deep Penetration of Strip and Circular Footings Into Layered Clays" Research Report No. R807 (1-45) on web Site <http://www.civil.usyd.edu.au>.
- 5- Davis, E. H., and Booker, j. R., "The Effect of Increasing Strength with Depth on The Bearing Capacity of Clays, Geotechnique Vol. 23, No. 4, PP. 551-563. (1973),
- 6- Elkateb, T., Chalaturnyk, R. and Robertson, P. K., (2003), " An Overview of Soil Heterogeneity: Quantification and Implications on Geotechnical Field Problems", Ca. Geotech. J. 40:1-5; NRC Research Press Web Site at <http://cgj.nrc.ca>

- 7- Fenton, G. A., and Griffiths, D.V., "Bearing Capacity Prediction of Spatially Random $c-\phi$ Soils", *Can. Geotech. J.* 40 : 54-65; NRC Research Press Web Site at <http://cgj.nrc.ca>, (2003).
- 8- Meyerhof, G. G., "The Ultimate Bearing Capacity of Foundations", *Geotechnique*, Vol.2, No.4, PP.301-332, (1951).
- 9- Merifield, R. S., Sloan, S. W., and Yu, H.s "Rigorous Plasticity Solutions for the Bearing Capacity of Two-layered Clays", *Geotechnique*, Vol.49, No.4, PP. 471-490, (1999).
- 10- Smith, I. M. and Griffiths, D.V. *Programming the Finite Element Method* Third Edition, Jon Wiley & Sons, England, (1998).
- 11- Toulby, G. T., and Martin, C. M., "Undrained Bearing Capacity Factors for Conical Footings on Clay", *Geotechnique* Vol.53, No.5, PP.513-520, (2003).
- 12- Terzaghi, k. *Theoretical Soil Mechanics*, New York: John Wiley & Sons, (1943),
- 13- Yin, J. H., Zhan, C., "Elasto Plastic Modelling of Geosynthetic Reinforced Soft Soils", *Conference Numerical Models in Geomechanics*, pp.589-593, (1997).

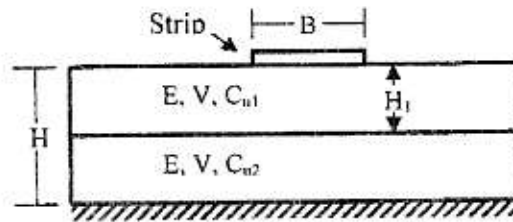


Fig. 1 Schematic of the problem

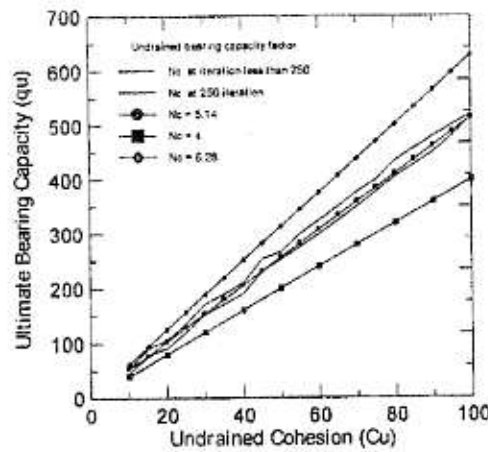


Fig. 2 Relation between ultimate bearing capacity and the undrained shear strength for four solutions

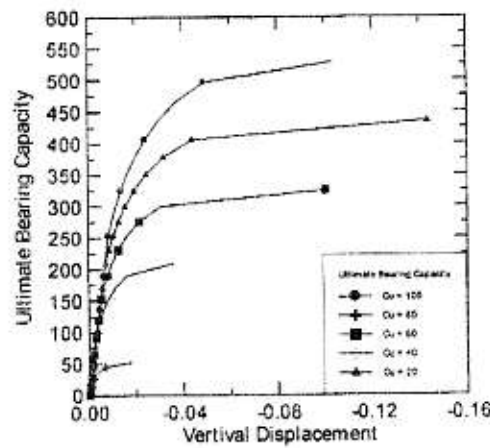


Fig. 3 Ultimate Bearing Capacity versus vertical settlement for various value of C_u

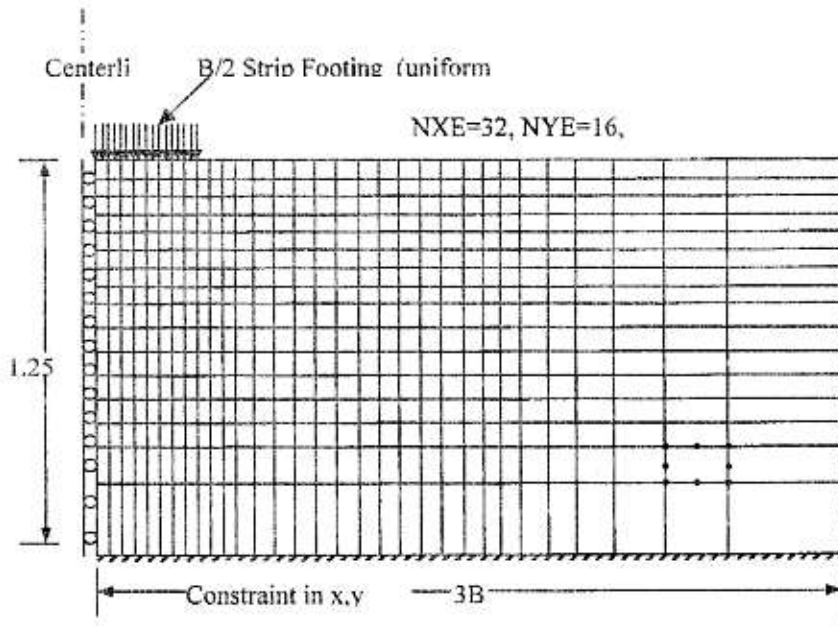


Fig. 4 Mesh of the problem case 1

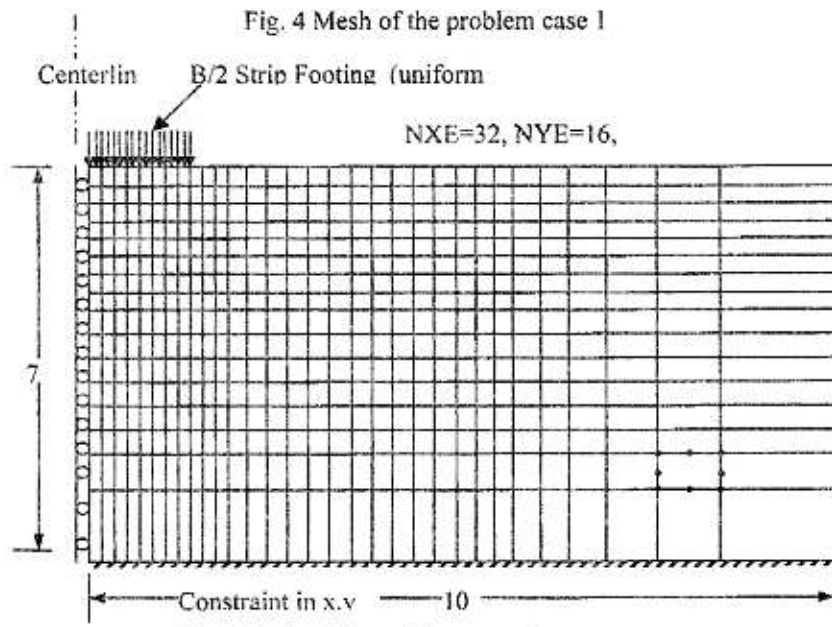


Fig. 5 Mesh of the problem case 2

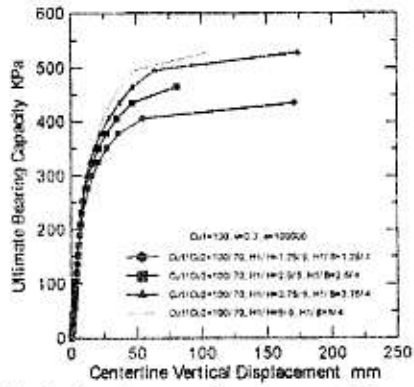


Fig. 6a Bearing capacity versus vertical displacement at the center of loading for $C_{u2}/C_{u1}=100/70$

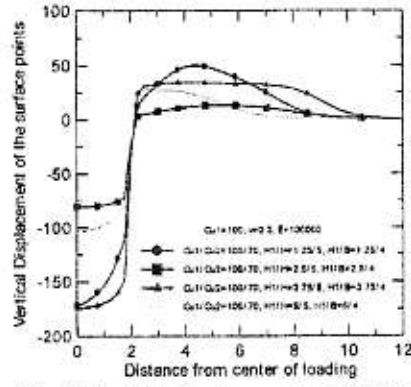


Fig. 6b Vertical displacement at surface versus distance from the center of loading for $C_{u2}/C_{u1}=100/70$

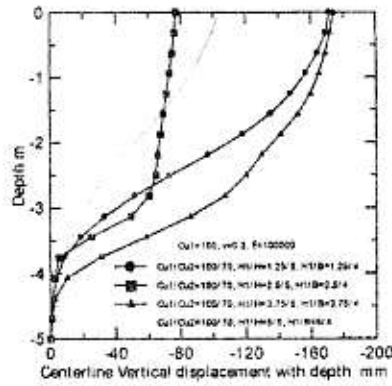


Fig. 6c Centerline Vertical displacement with depth $C_{u2}/C_{u1}=100/70$

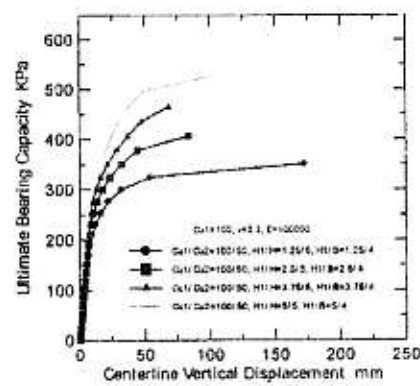


Fig. 7a Bearing capacity versus vertical displacement at the center of loading for $C_{u2}/C_{u1}=100/50$

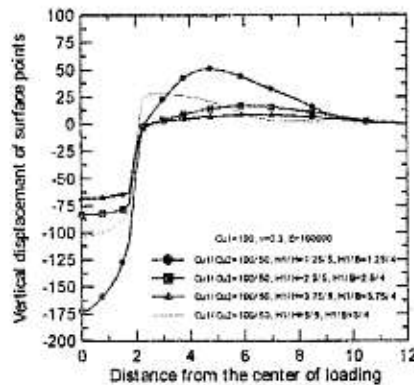


Fig. 7b Vertical displacement at surface versus distance from the center of loading for $C_{u2}/C_{u1}=100/50$

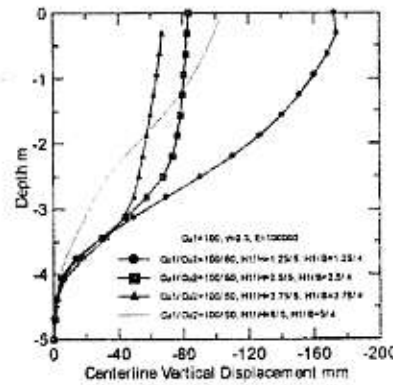


Fig. 7c Centerline Vertical displacement with depth $C_{u2}/C_{u1}=100/50$

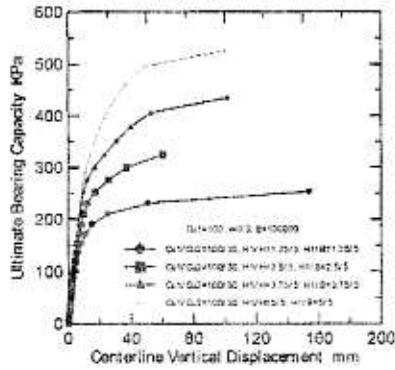


Fig. 8a Bearing capacity versus vertical displacement at the center of loading for $C_{u2}/C_{u1}=100/30$

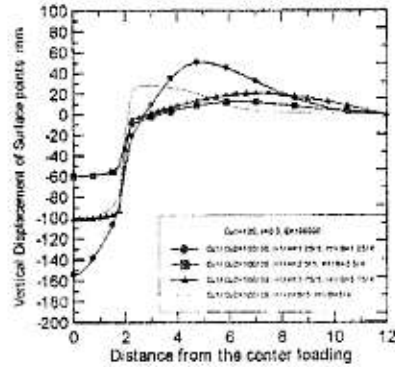


Fig. 8b Vertical displacement at surface versus distance from the center of loading for $C_{u2}/C_{u1}=100/30$

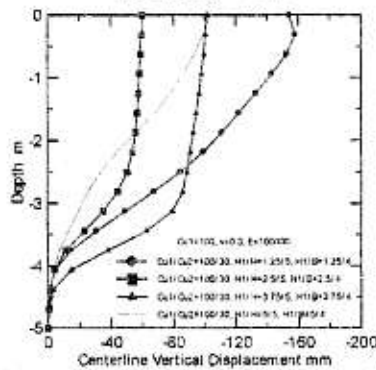


Fig. 8c Centerline Vertical displacement with depth $C_{u2}/C_{u1}=100/30$

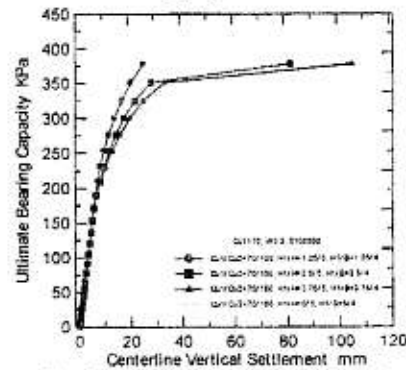


Fig. 9a Bearing capacity versus vertical displacement at the center of loading for $C_{u2}/C_{u1}=70/100$

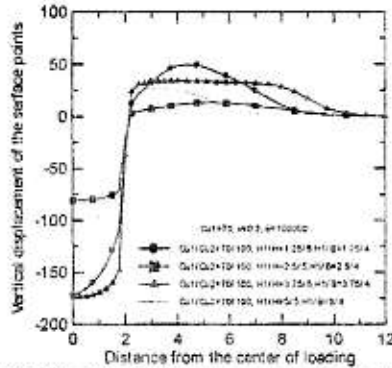


Fig. 9b Vertical displacement at surface versus distance from the center of loading for $C_{u2}/C_{u1}=70/100$

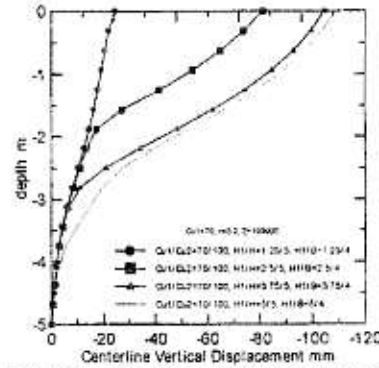


Fig. 9c Centerline Vertical displacement with depth $C_{u2}/C_{u1}=70/100$

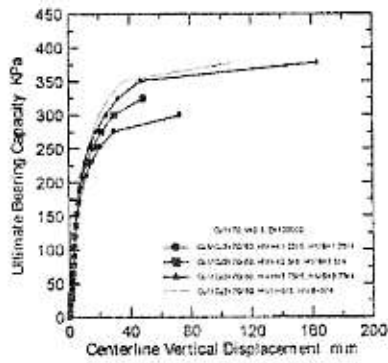


Fig. 10a Bearing capacity versus vertical displacement at the center of loading for $C_{u2}/C_{u1}=70/50$

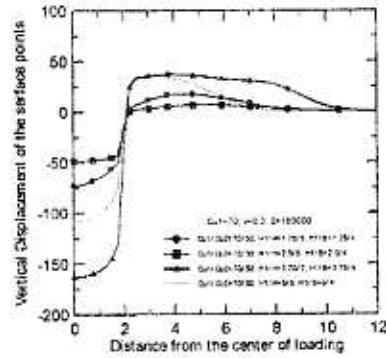


Fig. 10b Vertical displacement at surface versus distance from the center of loading for $C_{u2}/C_{u1}=70/50$

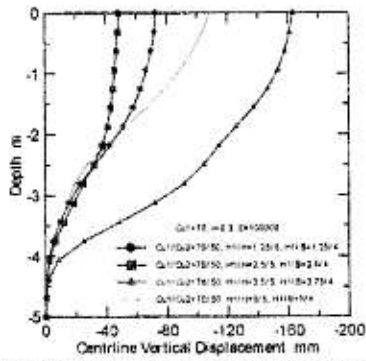


Fig. 10c Centerline Vertical displacement with depth $C_{u2}/C_{u1}=70/50$

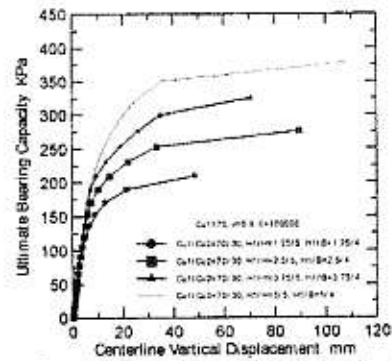


Fig. 11a Bearing capacity versus vertical displacement at the center of loading for $C_{u2}/C_{u1}=70/30$

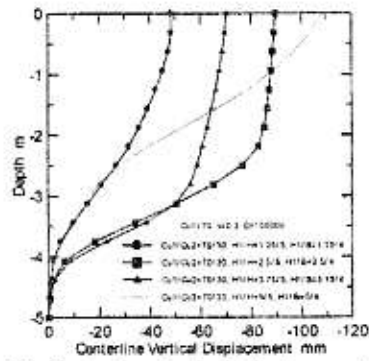


Fig. 11c Centerline Vertical displacement with depth $C_{u2}/C_{u1}=70/30$

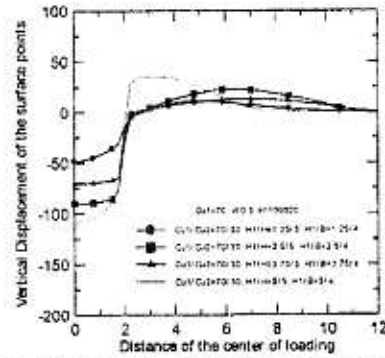


Fig. 11b Vertical displacement at surface versus distance from the center of loading for $C_{u2}/C_{u1}=70/30$

Undrained Bearing Capacity For Strip Footing On Two Layered Elastic-Plastic Clay By Using Fem

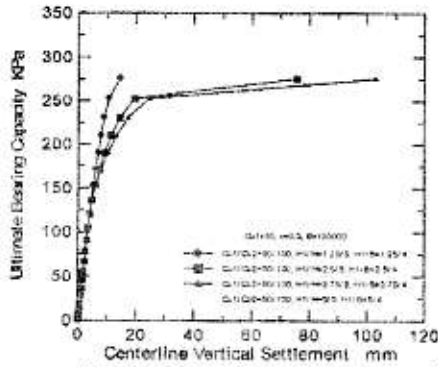


Fig. 12a Bearing capacity versus vertical displacement at the center of loading for $C_{u2}/C_{u1}=50/100$

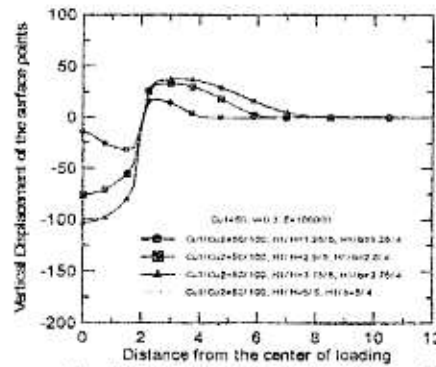


Fig. 12b Vertical displacement at surface versus distance from the center of loading for $C_{u2}/C_{u1}=50/100$

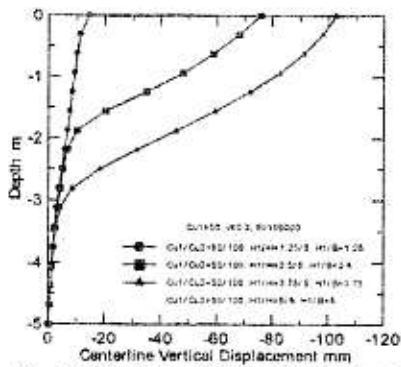


Fig. 12c Centerline Vertical displacement with depth $C_{u2}/C_{u1}=50/100$

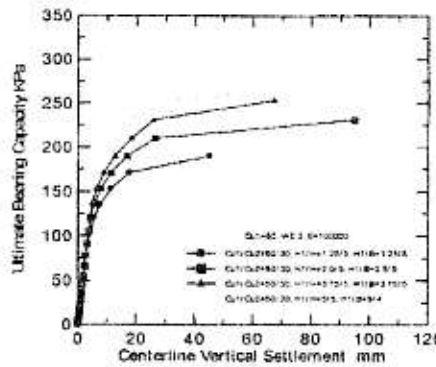


Fig. 13a Bearing capacity versus vertical displacement at the center of loading for $C_{u2}/C_{u1}=50/30$

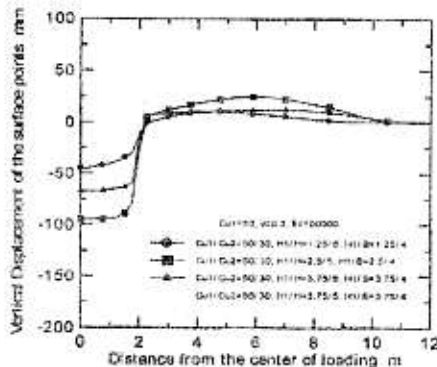


Fig. 13b Vertical displacement at surface versus distance from the center of loading for $C_{u2}/C_{u1}=50/30$

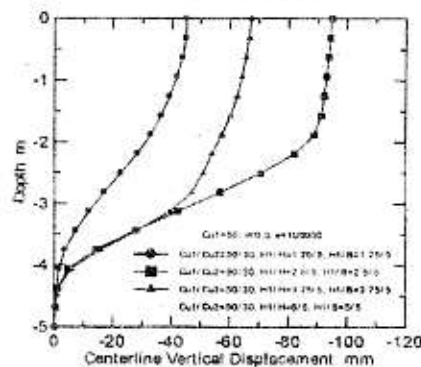


Fig. 13c Centerline Vertical displacement with depth $C_{u2}/C_{u1}=50/30$

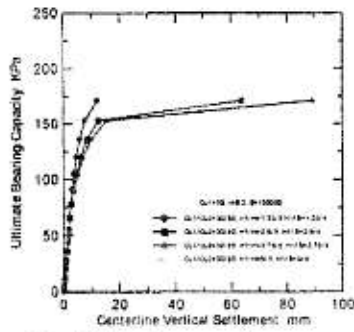


Fig. 14a Bearing capacity versus vertical displacement at the center of loading for $C_{u2}/C_{u1}=30/50$

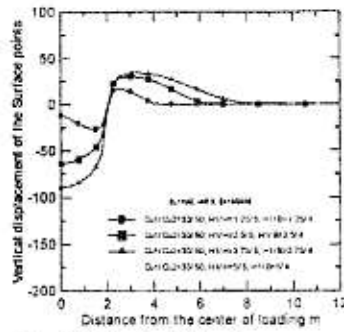


Fig. 14b Vertical displacement at surface versus distance from the center of loading for $C_{u2}/C_{u1}=30/50$

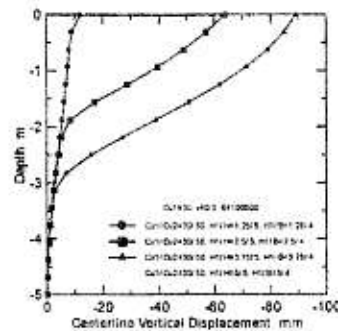


Fig. 14c Centerline Vertical displacement with depth $C_{u2}/C_{u1}=30/50$

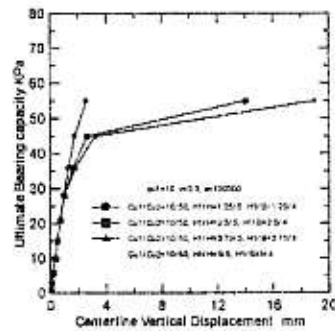


Fig. 15a Bearing capacity versus vertical displacement at the center of loading for $C_{u2}/C_{u1}=10/50$

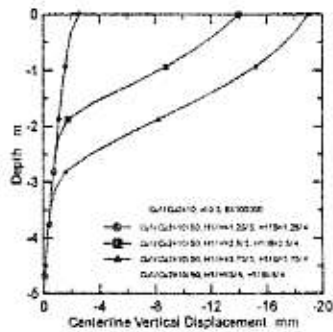


Fig. 15c Centerline Vertical displacement with depth $C_{u2}/C_{u1}=10/50$

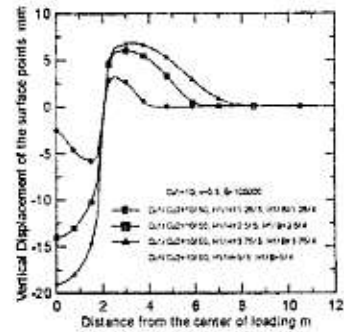


Fig. 15b Vertical displacement at surface versus distance from the center of loading for $C_{u2}/C_{u1}=10/50$

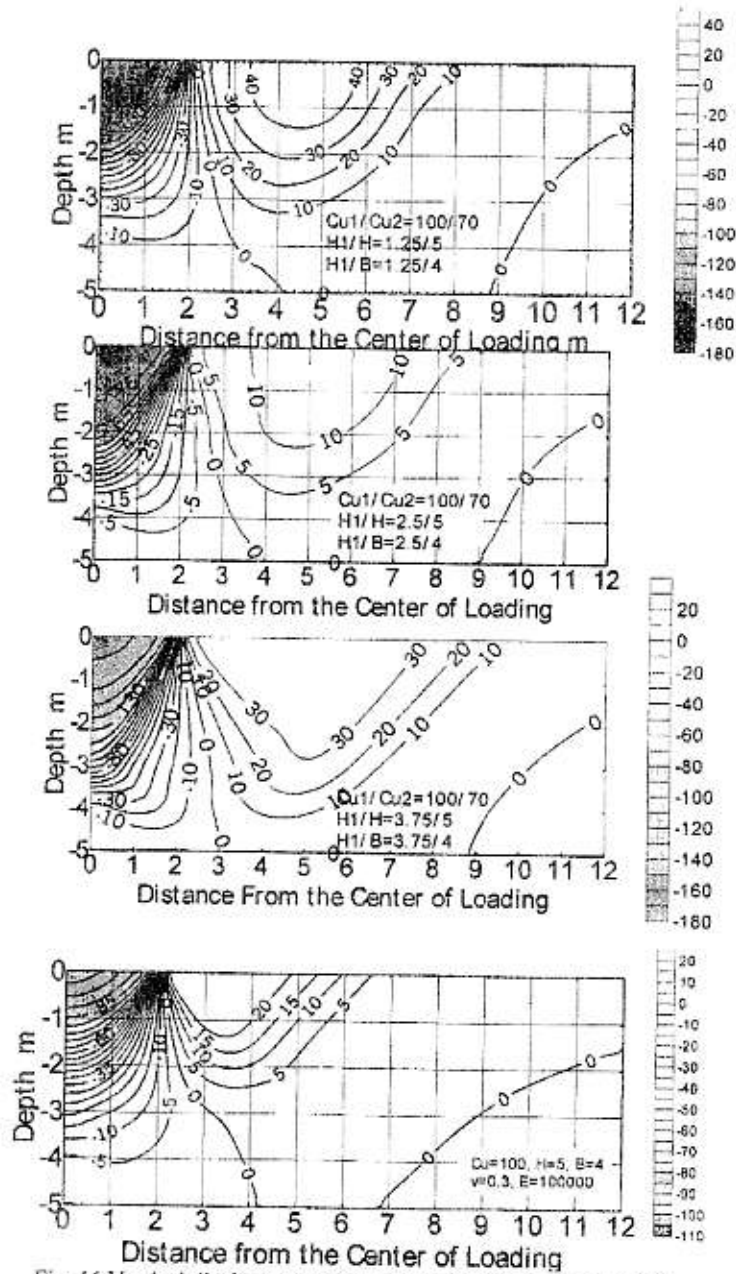


Fig. 16 Vertical displacement pattern at failure for $C_{u2}/C_{u1}=100/70$

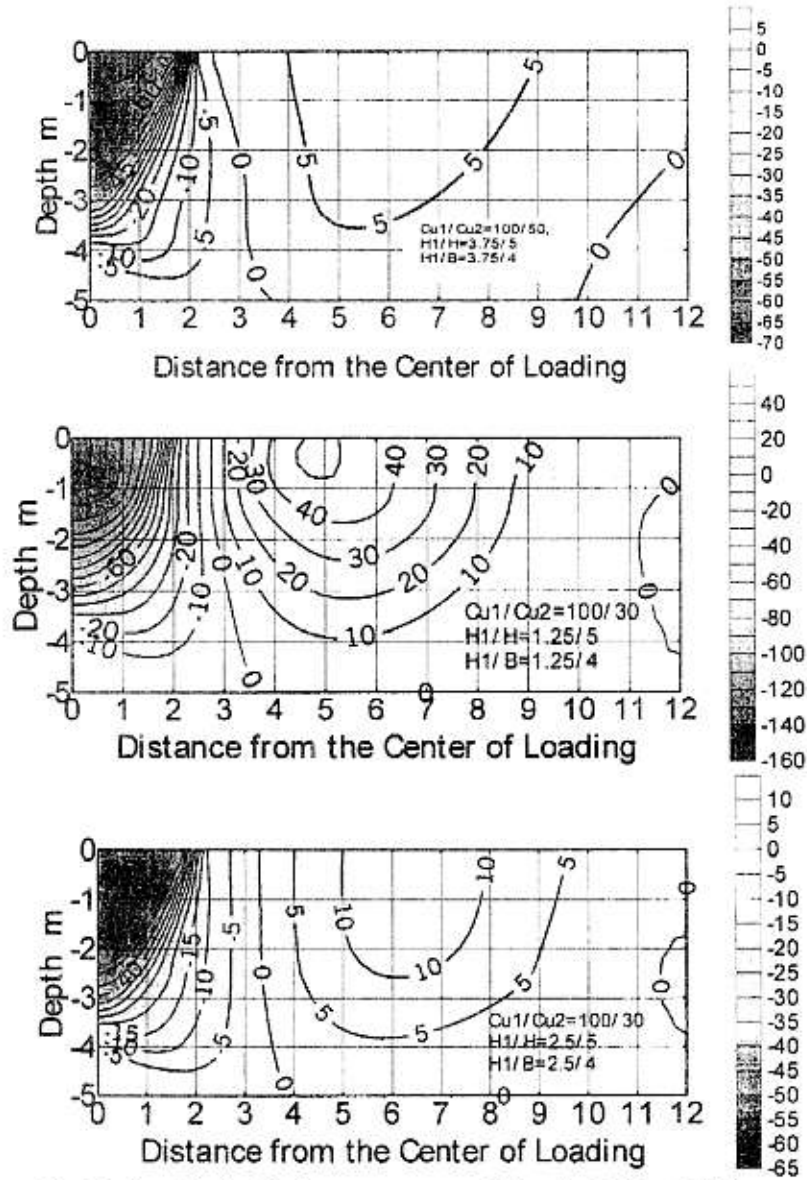


Fig. 17 a, b, c Vertical displacement pattern at failure for $C_{u2}/C_{u1}=100/50$

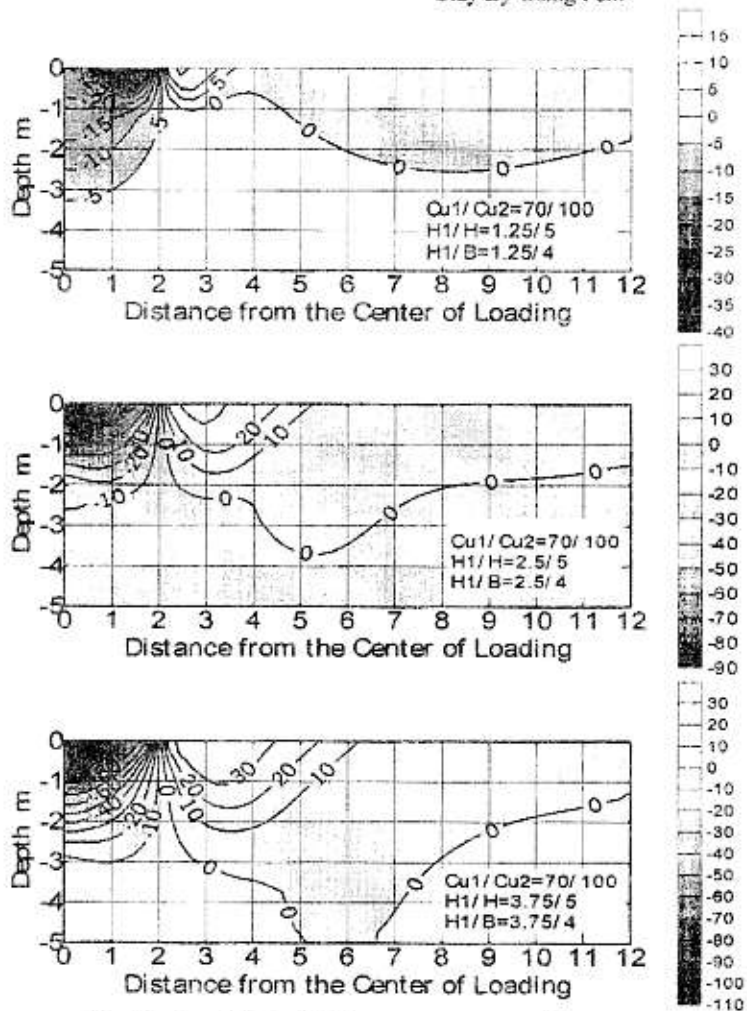


Fig. 18 a, b, c Vertical displacement pattern at failure for $C_{u2}/C_{u1}=100/30$

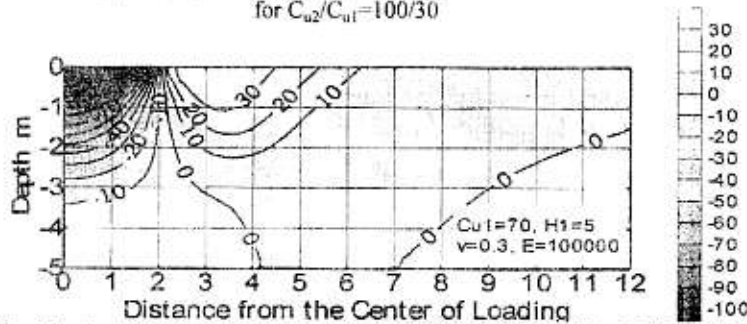


Fig. 19 a, b, c Vertical displacement pattern at failure for $C_{u2}/C_{u1}=70/100$

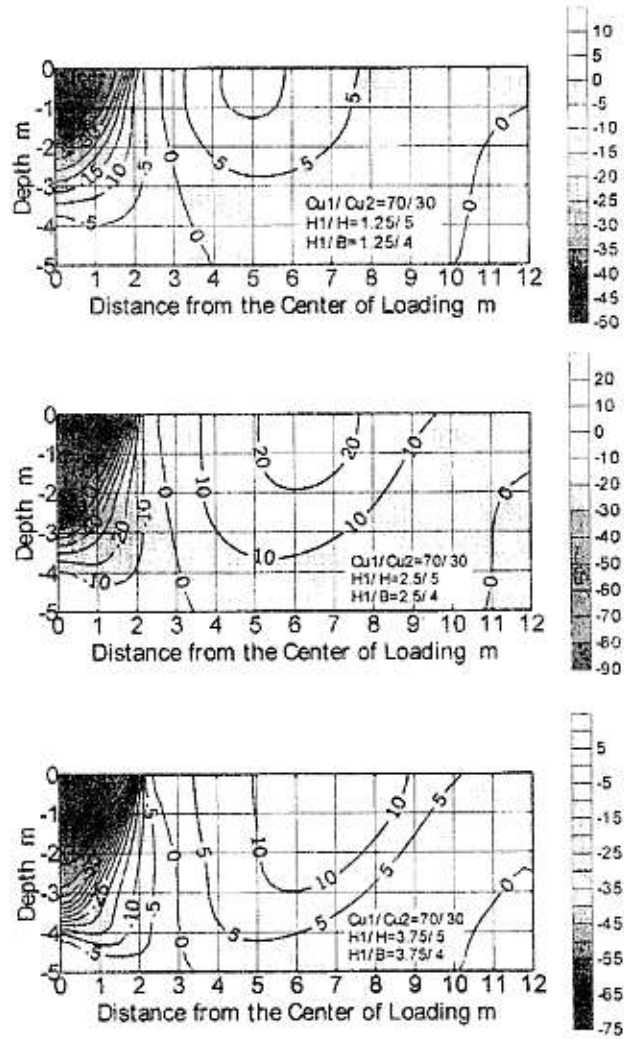


Fig. 20 a, b, c Vertical displacement pattern at failure for $C_{u2}/C_{u1} = 70/30$

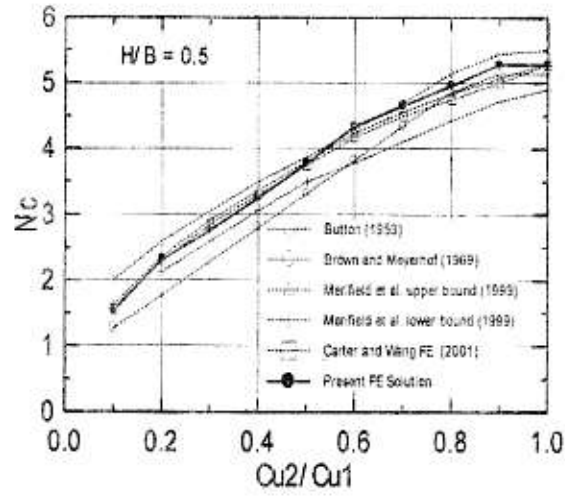


Fig. 21a Bearing capacity factor versus undrained shear strength ratio where $H_f/B=0.5$ for different solution

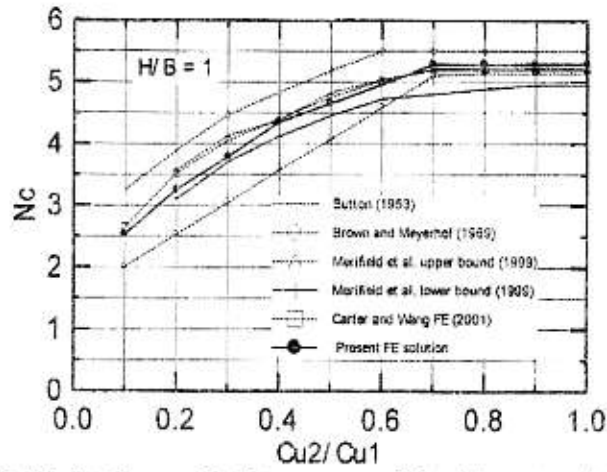


Fig. 21b Bearing capacity factor versus undrained shear strength ratio where $H_f/B=1$ for different solution

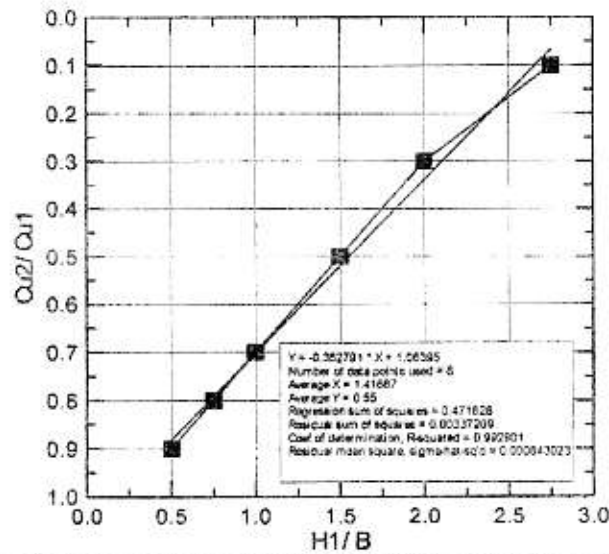


Fig. 22 Undrained shear strength ratio versus H/B for bearing factor $N_c=5.14$

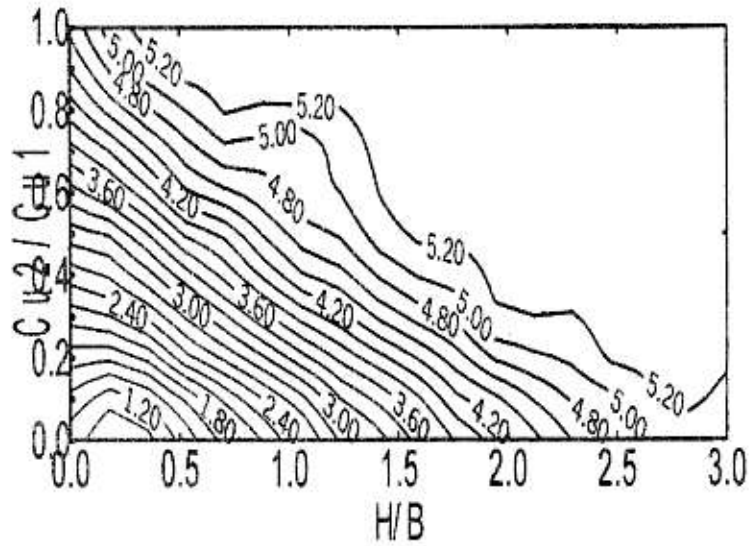


Fig. 23 Undrained shear strength ratio versus H/B for different bearing factor