Critical Speed of Hyperbolic Disks and Eccentricity Effects on The Stability of The Rotor Bearing System

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Abstract

The critical speed of hyperbolic disks for the selected model is calculated in three different cases, anisotropic material and thermal case, isotropic material and thermal case finally isotropic material and thermal with generated heat case. The second case is found to be the best because the critical speed occurs above maximum speed values.

For recognition of the eccentricity (e) effects on the stability, the original bearings of the selected model were tilting pads bearing replaced by adjustable hydrodynamic pads bearing. The results proved that stability criteria (Sc), critical mass ($\bar{M}_c$), whirling frequency ratio ($\frac{f}{nf}$) of variable rotational speed of the adjustable hydrodynamic pads bearing are stable for the range of eccentricity (e) about 0.02 mm to 0.09 mm. Finally the thermohydrodynamic (THD) analysis of eccentricity ratio (e) has effects on the stability of the rotor-bearing system.

Keywords: hyperbolic disk, rotor-bearing system, critical speed, stability criteria

Notation

- $a$, $b$: outer and inner radius of the disk respectively, mm
- $B_c$: bearing centre
- $B_c^*$: new bearing centre after tilting angle
- $C_r$: radial clearance of the bearing (bearing radius-journal radius), mm

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Introduction

The rotordynamics of turbomachinery encompasses the structural analysis of rotors (shafts and disks) and the design of fluid film bearings that determine the best dynamic performance given the required operating conditions. This best performance is denoted by well-characterized natural frequencies and critical speeds with amplitudes of synchronous dynamic response within required standards and demonstrated absence of subsynchronous vibration instabilities, [1]. Lewis et al.,(1987), [2] have developed a spilt bearing with a new configuration, this bearing surfaces consists of a five pad tilting pad fluid bearings with one pad having a mean for being moved radially. This radial motion causes changes in the proportionality constants, the stiffness and the damping capabilities of the bearing which in turn changes the critical speeds of the rotor system Wang and Shin, (1990), [3] improved the stability of rotor bearing system by optimization technique used to find the optimum diameters of shaft elements so that the optimized rotor can sustain maximum fluid leakage excitation. Chivens and Nelson (2000),[4] presented an analytical investigation into the influence of disk flexibility on the transverse bending natural frequencies and critical speeds of a rotating shaft-disk system. The partial differential equations governing the system motion and the associated exact
solution form had been developed. Numerical solutions are presented covering a wide range of non-dimensional parameters and general conclusions are drawn. Krodkiewski and Sun (2002),[5] presented the modeling and analysis of a rotor-bearing system with a new type of active oil bearing, the active bearing is supplied with a flexible sleeve whose deformation can be changed during operation of the rotor. The flexible sleeve is also a part of a hydraulic damper whose parameters can be controlled during operation as well, the self exciting vibration can be eliminated or greatly reduced during operation by properly controlled deformation of the flexible sleeve and optimal choice of the hydraulic damper parameters. In this paper the topic has been divided into two parts, the first is confined the evaluation of the critical speeds of hyperbolic disk by thermoelastic relations in three operating cases, while the second contains the replacement of the two tilting bearings type used in the selected model with two hydrodynamic adjustable pads bearings and also study the effects of the eccentricity on the stability criteria at high speeds, and the effect of thermohydrodynamic (THD) analysis on the stability of rotor-bearing system.

**Theoretical Analysis of The Model**

It takes similar characteristics of centrifugal compressor, it's consisting of three disks (hyperbolic profile) clamped on a shaft which is supported by two tilting pads bearings driven by steam turbine using in "southern refineries company", all dimensions in origin specifications as shown in figure (1).

**Mechanical Specification**

(i)-Type of compressors RB43B; Serial No. 33F3047

(ii)-shaft properties:
- total length: 1411 mm
- shaft material; all properties as shown in table (1).
- maximum diameter: 142 mm
- minimum diameter: 108 mm at bearing and 63 mm (at shaft end).

(iii)- Two tilting pads bearing (right and left).

(iv)-Numbers and type of disks; three hyperbolic profile variable thickness disk, some whose dimensions:
- inner and outer diameters respectively: 139mm - 420mm
- disk thickness: 100 mm (at inner diameter)
- disk material; all properties as shown in table (1)

**Operating Data**

(i)-Suction pressure: 11 kg/cm²
(ii)-Discharge pressure: 12.5 kg/cm²
(iii)-Process gas: recycle gas
(iv)-Suction temperature: 45 °C
(v)-Discharge temperature: 70 °C
(vi)-Normal speed without suction (8000 r.p.m - 10670 r.p.m)
(vii)-Rated speed: 11670 r.p.m
(viii)-Maximum speed: 12000 r.p.m
Rotating Disks with Variable Thickness and Inhomogeneity

Disks of hyperbolic axial thickness with profile shown in figure (2) and described by [6].

\[ h(r) = h_2 (r / a)^{-\beta} \]  

(1)

where:

- \( h_2 \) = outer thickness at radius \( a \)
- \( \beta \) = constant in variable thickness expression

The governing equations for thermoelastic analysis of an inhomogeneous, rotating disk with arbitrary varying thickness, small deformations and rotational symmetry are assumed. If the thickness of the disk is small compared to its radius, the axial stress is negligible and the disk is then in a state of plane stress. The material is assumed to be isotropic but inhomogeneous along the radial direction. That is the Young’s modulus \( E = E(r) \), the density of the disk material \( \rho = \rho(r) \), the coefficient of linear expansion \( \alpha = \alpha(r) \), and the Poisson’s ratio \( \nu = \nu(r) \).

The temperature field is steady, axisymmetric but inhomogeneous, that is, \( T = T(r) \) where \( T \) is the temperature. From equilibrium consideration of an element as shown in figure (2),[7].

\[
d(F, r, \theta) + \rho \frac{\partial^2 u}{\partial t^2} h dr d\theta - F \frac{\partial u}{\partial r} dr d\theta = 0 \]  

(2)

The exact elastic displacement in radial direction \( u(r) \) can be written as:

\[ u(r) = (81.26r + \frac{221711}{r} - 0.028 \times 10^{-3}) \times 10^{-3} \text{ mm} \]  

(3)

Now the exact value of radial and tangential forces per unit length components are then obtained as follows:

\[ F_r = (1.784 \times 10^3 \times \frac{1}{r^2} - 2011 \times 10^3 r^3) \times 10^3 \text{ N/m} \]  

(4)

\[ F_\theta = (1.784 \times 10^3 \times \frac{1}{r^2} - 11583 \times 10^3 r^3) \times 10^3 \text{ N/m} \]  

(5)

Thermal Effect on The Critical Speed of Height-Speed Disk

Anisotropic elastic disk of radius assumed to be spinning uniformly about its polar axis with an angular velocity \( \omega \) in a steady-state heat flow, for such a disk, following stress-strain relations, [8].

\[
\begin{align*}
\sigma_r &= C_{11}E_r + C_{12}E_\theta - (C_{11}E_r + C_{12}E_\theta)\Delta T \\
\sigma_\theta &= C_{12}E_r + C_{22}E_\theta - (C_{12}E_r + C_{22}E_\theta)\Delta T \\
\tau_{r\theta} &= 0
\end{align*}
\]  

(6)

\[
\omega^2 = \frac{T (C_{11}C_{22} - C_{12}^2)(3\alpha_\theta - \alpha_r)}{(3C_{11} + C_{12})\rho a^2} \]  

(7)

where: \( C_{11}, C_{12}, C_{22} \)= elastic constants

\( \Delta T = \) differential temperatures \(^\circ\text{C} \)

\( T = \) operating temperature \(^\circ\text{C} \)

\[
\begin{align*}
\frac{C_{22}}{C_{11}C_{22} - C_{12}^2} &= \frac{1}{E_1} \\
\frac{C_{11}}{C_{11}C_{22} - C_{12}^2} &= \frac{1}{E_2} \\
\frac{C_{12}}{C_{11}} &= \frac{u_r}{\omega} \\
\frac{C_{12}}{C_{22}} &= \frac{u_\theta}{\omega}
\end{align*}
\]  

(8)
The critical speed ($\omega_c$) depends on thermo-mechanical anisotropy of the disk material, size of the disk and temperature at center of the disk, so that the selection of material and size of the disk become important for design consideration.

Equation (9) can be written for disk material which is isotropic material:

\[
\omega_c = \left[ \frac{T \cdot E_0 (\alpha_r - \alpha_{\theta})_r}{(3 + u_r) \cdot \rho \cdot a^2} \right]^{1/2}
\]

where:
- $\alpha_r, \alpha_\theta$ = thermal coefficient of expansion in radial and tangential direction respectively.
- $E_1, E_2$ = Young’s modulus in radial and tangential directions respectively.
- $\nu_r, \nu_\theta$ = radial and tangential Poisson’s ratio respectively.

If we consider such a thermal situation that rate of generation of heat is fixed amount “Q” per unit volume, the critical speed:

\[
\omega_c = \left[ \frac{E_1 \cdot \alpha_r \cdot Q}{2K^* \cdot \rho \cdot (3 + u_r)} \right]^{1/2}
\]

where: $K^*$=thermal conductivity (W/m°C.)

Q=generated heat in disk (W/m$^3$)

**Adjustable Hydrodynamic Pads Bearing**

This bearing has four pads arranged as shown in Fig.(3), [9] these pads were designed to be adjustable, i.e each pad could be tilted about its leading edge in a clockwise (cw) or counter clockwise (ccw) direction. The running surface of each pad, at zero tilt, fits into a circle which corresponds to the bearing surface circle in a conventional bearing.

The stability margins of the rotor bearing system, in terms of critical mass ($M_c$), threshold speed( $\Omega$), and whirl frequency ($f_w$), can be established by applying *Routh’s criteria* to the dynamic coefficients as stiffness and damping coefficients,[10].

\[ K_{xx} = \frac{W}{e} \cos \phi \]  \hspace{1cm} (12)

\[ K_{xz} = W \frac{\partial \phi}{\partial e} \cos \phi + \frac{\partial W}{\partial e} \sin \phi \]  \hspace{1cm} (13)

\[ K_{zx} = -\frac{W}{e} \sin \phi \]  \hspace{1cm} (14)

\[ K_{zz} = \frac{\partial W}{\partial e} \cos \phi - W \sin \phi \frac{\partial \phi}{\partial e} \]  \hspace{1cm} (15)

\[ C_{xx} = -\frac{2}{\omega} \left[ e \frac{W}{e} \sin \phi \right] \]  \hspace{1cm} (16)

\[ C_{xz} = C_{zx} = \frac{2}{\omega} \left[ \frac{W}{e} \cos \phi \right] \]  \hspace{1cm} (17)

\[ C_{zz} = \frac{2}{\omega} \left[ e \frac{W}{e} \cos \phi + \frac{\partial W}{\partial e} \sin \phi \right] \]  \hspace{1cm} (18)

\[ \frac{K_{xx}}{K_{xz}}, \frac{K_{xz}}{K_{zz}, \frac{K_{zx}}{K_{zz}} = \left( K_{xx}, K_{xz}, K_{zx}, K_{zz} \right) C^3}{(\omega \mu F^2)} \]  \hspace{1cm} (19)
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where:

\[ K_{xx}, K_{xz}, K_{zx}, K_{zz} = \text{Stiffness coefficients in xx, xz, zx, zz directions} \]

\[ C_{xx}, C_{xz}, C_{zx}, C_{zz} = \text{Dynamic damping coefficients in xx, xz, zx, zz directions} \]

\[ K_{xx}, K_{xz}, K_{zx}, K_{zz} = \text{Dimensionless stiffness coefficients in xx, xz, zx, zz directions} \]

\[ C_{xx}, C_{xz}, C_{zx}, C_{zz} = \text{Dimensionless damping coefficients in xx, xz, zx, zz directions} \]

A rotor-bearing system is asymptotically stable when the rotor mass \( M_c \) is less than critical mass \( \bar{M}_c \) for operating condition determined by bearing geometry and rotor speed, also stable when the operating speed of the rotor is less than the speed defined by threshold speed parameter \( \Omega \), a negative value of critical mass \( M_c \) means the rotor bearing system will be stable at all the values of rotor mass \( \bar{M}_c \).

Similarly, a negative value of the whirling frequency ratio \( \frac{-2}{f_v} \) implies the absence of whirl.[10].

(i). Critical mass of rotor \( (M_c), (\text{Kg}) \)

\[
M_c = \frac{\bar{M}_c}{r} \mu \omega C_f^4 \quad \text{and}
\]

\[
M_r = \frac{\bar{M}_r}{r} \mu \omega C_f^4 \quad (23)
\]

(ii). The computation of the threshold speed \( \Omega \) including the thermal effects is as follow [11].

**Step 1** - Compute the Sommerfeld number \( S \) with viscosity of oil evaluated at the supply temperature

\[
S = \frac{\mu \omega l_c l_s^3}{\pi \dot{W} C_r^2}
\]

**Step 2** - Compute the eccentricity ratio \( \varepsilon \), as function of Sommerfeld number,[12].

\[
\varepsilon = -0.36667 + \frac{0.49737}{\sqrt{\lambda}} - 0.20986 \ln(S)
\]

**Step 3** - Compute the temperature rise parameter \( \Psi \)

\[
\Psi = \frac{\beta \mu \omega}{\rho c_v} \left( \frac{r}{C_f} \right)^2
\]

**Step 4** - Compute the threshold speed \( \bar{\Omega} \) corresponding to the temperature rise parameter computed in.
step 3 and eccentricity ratio computed in step 2.

\[ \bar{W} = W \cdot C_r^2 \]

\[ \mu \cdot \omega \cdot r^4 \]

\[ \bar{\Omega} = \sqrt{\frac{M \cdot c}{W}} \]

\[ \bar{\Omega} = \omega \sqrt{\frac{g}{C_r}} \] (24)

(iii). whirl frequency \( f_w \)

\[ f_w = f_w \cdot \omega \] (25)

where:

\( \bar{M} = \) dimensionless critical mass

\( f_w^2 = \) whirl frequency ratio

\( \bar{\Omega} = \) dimensionless threshold speed;

\( g = \) gravitational acceleration

\[ 9.81 \text{ m/sec}^2 \]

\( w = \) dimensionless load-carrying capacity.

Study The Rotor Bearing System

Parametric Stability Analysis

Method of analysis based on Liapunov's direct method is used for studying the behavior of the nonlinear system of differential equations governing the motion of a rotor-bearing system in the neighborhood of its equilibrium point. Among the results reached in Liapunov’s direct method is the demonstration of the roles played by, [13].

Applying Sylvester's Theorem to test the positive definiteness of \( \bar{U}_{ik} \), its successive principal minor determinants must be positive definite, thus yielding the following sufficient conditions for system whirl stability.

\[ \frac{\omega^2}{\bar{U}_{ik}} \]

1 (27)

\[ \frac{\omega^2}{\bar{U}_{ik}} < 1 \]

\[ \frac{\omega^2}{\bar{U}_{ik}} > 1 \]

\[ \alpha^2 \] (28)

\[ \frac{\omega^2}{\bar{H}} \]

\[ \bar{H} \] (29)

and

\[ \bar{S}\text{c} = \left[ \frac{\omega^2}{\bar{H}} \right] \left[ \frac{\omega^2}{\bar{H}} - 1 \right] \] (30)

where:

\[ \bar{S}\text{c} = \text{Stability criteria} \]
\[ \frac{\omega^2}{\omega_0^2}, \frac{\omega^2}{\omega_0^2}, -\alpha, \beta = \]

dimensional parameters on the whirl stability

Discussion of Results

The results of this paper are illustrated below. Fig. (4) show the relationship between temperature rise (suction and discharge temperatures) and increasing value of critical speed \( \omega_c \) for three cases, also shows the critical speed \( \omega_c \) as a variable quantity. The second case is found to be the best case because the critical speed occurs at over the range of the high rotational speed disk \( N=12000 \) r.p.m. \((\text{centrifugal compressor driven by steam turbine})\).

It is quite easy to observe the dependence of critical speed \( \omega_c \) on the thermo-mechanical anisotropy of the disk material (size of the disk and temperature at center of the disk). From the values of the critical speed \( \omega_c \) given in equations (9,10 and 11) it is noted that the critical speed \( \omega_c \) is independent of the variation of disk thickness that is the critical speed \( \omega_c \) will remain the same for all hyperbolic profiles. From Fig.(5), the rotor critical mass \( \overline{M_r} \) increases as the eccentricity \( \varepsilon \) increases to reach system stability. The rotor critical mass \( \overline{M_r} \) must be negative this means the curve no.3 is more stable especially at high values of eccentricity \( \varepsilon \). Fig.(6) shows the relationship between whirling frequency ratio \( \frac{\omega^2}{N^2} \) with change of eccentricity ratio \( \varepsilon \). The system becomes more stable at the negative value of this ratio as shown in curve (3) at rotational speed equal 12000 r. p.m.. Whirl begins with the rotor operating relatively close to center of the bearing. When the journal is close to the center of the bearing (the eccentricity ratio is small) the bearing stiffness is much lower than the shaft stiffness. When the journal is located relatively close to the bearing wall (the eccentricity ratio is near 0.9) the bearing stiffness is typically much higher than rotor shaft stiffness. One can improved the stability of rotor bearing system by selecting softer surface material of bearings.

It can be seen from Fig.(7) the thermohydrodynamic \( \text{THD} \) analysis effects on the threshold speed at all eccentricity ratios \( \varepsilon \). The best value of threshold speed \( \Omega \) at \( \varepsilon=0.4 \) is in the stable region. This figure shows the increase of temperature rise parameter \( \psi \) with increase of eccentricity ratio \( \varepsilon \) when the maximum rotational speed \( N=12000 \) r.p.m the value of \( \varepsilon=0.66 \) at \( \psi=0.2 \) but when \( \varepsilon=0.696 \) at \( \psi=0.26 \). Table (2) shows the relationship between the stability criteria with change of eccentricity \( \varepsilon \). It also shows the stability criteria increases with the increase of eccentricity, but the values of stability criteria must be larger than \( \alpha^2 \).
Conclusions
From the results of this work the following conclusions can be obtained:
1- The critical speed is increasing as the operation temperature increase.
2- When eccentricity increases the stability criteria increases as shown in Table (2). This means the stiffness and damping coefficients are improved.
3- The best value of threshold speed is at eccentricity ratio equal 0.421 for all values of temperature rise parameter when using thermohydraulic analysis.
4- When the adjustable hydrodynamic bearing is used the rotor-system becomes more stable. The critical speed occurs at over rotational speed range.
5- Elastic deformation of bearing surface is useful to reduce the instability whirl. This is controlled by selecting softer bearing material.

References
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Table (1) Mechanical Properties of the Shaft and Hyperbolic Disk, [1]-[5].

<table>
<thead>
<tr>
<th></th>
<th>Shaft Material Properties (High Strength Steel cgs)</th>
<th>Disk Material Properties (High Strength Steel mk)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Properties</td>
<td>Properties</td>
</tr>
<tr>
<td>I</td>
<td>Stiffness values</td>
<td>Stiffness values</td>
</tr>
<tr>
<td></td>
<td>Young modulus (E)</td>
<td>Young modulus (E)</td>
</tr>
<tr>
<td></td>
<td>199.95 GPa</td>
<td>1.5×10¹²·r⁰.¹ (N/m²)</td>
</tr>
<tr>
<td></td>
<td>Shear modulus (G)</td>
<td>Shear modulus (G)</td>
</tr>
<tr>
<td></td>
<td>75.842 GPa</td>
<td>74.376GPa</td>
</tr>
<tr>
<td></td>
<td>Poisons ratio (ν)</td>
<td>Poisson's ratio (ν)</td>
</tr>
<tr>
<td></td>
<td>0.32</td>
<td>0.3</td>
</tr>
<tr>
<td>II</td>
<td>Limit stress values</td>
<td>Limit stress values</td>
</tr>
<tr>
<td></td>
<td>Tension 148.24 MPa</td>
<td>Tension 145.37 MPa</td>
</tr>
<tr>
<td></td>
<td>compression 165.47 MPa</td>
<td>Compression 162.27 MPa</td>
</tr>
<tr>
<td></td>
<td>Shear 107.56 MPa</td>
<td>Shear 105.48 MPa</td>
</tr>
<tr>
<td>III</td>
<td>Thermal values</td>
<td>Thermal values</td>
</tr>
<tr>
<td></td>
<td>Expansion coefficient (α)</td>
<td>Expansion coefficient (α)</td>
</tr>
<tr>
<td></td>
<td>11.88x10⁻⁶ 1/ºC</td>
<td>12.34x10⁻⁶ 1/ºC</td>
</tr>
<tr>
<td></td>
<td>Conductivity (k*)</td>
<td>Conductivity (k*)</td>
</tr>
<tr>
<td></td>
<td>0.0868095 W/m. ¹C</td>
<td>0.0943211 W/m. ¹C</td>
</tr>
</tbody>
</table>
Table (2) Effects of The Nondimensional Cross-Coupling Parameter on The Stability Condition in the \( \left( Sc > \bar{\alpha}^2 \right) \)

<table>
<thead>
<tr>
<th>Eccentricity, ( e ) (mm)</th>
<th>( \bar{\alpha}^2 )</th>
<th>( Sc=\left[ \frac{\omega^2_{xx}}{\omega^2_{22}} - 1 \right] \left[ \frac{\omega^2_{22}}{\omega^2_{22}} - 1 \right] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>0.8913</td>
<td>0.9989961</td>
</tr>
<tr>
<td>0.03</td>
<td>0.8451</td>
<td>0.9990153</td>
</tr>
<tr>
<td>0.04</td>
<td>0.7327</td>
<td>0.9990404</td>
</tr>
<tr>
<td>0.05</td>
<td>0.6911</td>
<td>0.9990703</td>
</tr>
<tr>
<td>0.06</td>
<td>0.6237</td>
<td>0.9991037</td>
</tr>
<tr>
<td>0.07</td>
<td>0.5541</td>
<td>0.99901392</td>
</tr>
<tr>
<td>0.08</td>
<td>0.4113</td>
<td>0.9991758</td>
</tr>
<tr>
<td>0.09</td>
<td>0.4026</td>
<td>0.9992124</td>
</tr>
</tbody>
</table>

Figure (1) Rotor – Bearing of Centrifugal type RB4-3B

Figure (2) Profile of The Hyperbolic Disk and The Force Acting on An Element,[8 ]
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Figure (3) The Adjustable Pads Bearing Center Position ($B^*_C$) After Tilting Angle ($\delta$),[9]

Figure (4) Variation Critical Speed ($\omega_c$) with Temperature ($T$) for Three Operating Cases

case (1)=anisotropic material and thermal
Case (2)=isotropic material and thermal
Case (3)=isotropic material, thermal and heat generated

$\omega_c=1250 \text{ rad/s}$
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Figure (5) The Eccentricity Effect on The Stability Criteria (dimensionless critical mass)

Figure (6) The Eccentricity ratio (ε) Effect on The Whirling Frequency Ratio ($\frac{f_w}{f_r}$)
Figure (7) The Eccentricity Ratio ($\varepsilon$) With Temperature Rise Parameter ($\psi$) Effect on The Stability