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Analytic Technique For Active Mode-Locked Fiber Lasers

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Abstract

In this work, Amplitude modulation mode-locked fiber laser is studied,by using Ytterbium Doped Fiber Laser, single mode fiber, operating with 1055 nm wavelength with 976 nm optical pump and AM Mode-Locked by optical modulators. A grating pair is used to compensate the normal dispersion. The effect of both normal and anomalous dispersion regimes on output pulses are investigated. Master equation of the Mode-locking fiber laser is introduced .Pulse shapes for both dispersion regimes are assumed after modifying (Ginzburg-Landu equation) GLE which is essentially Generalized Nonlinear Schrödinger equation GNLSE and by applying the moment method ,a set of five rate equations for pulse energy ,pulse width ,frequency shift ,temporally shift and chirp ,which solutions described the pulse from round trip to the next and how they approach to steady state values. To solve these equations numerically fourth order, Runge-Kutta method is performed through Mat-Lab 7.0 computer program. Result shows that, the output pulse width from the AM mode-Locked equals to τ=0.8ps in anomalous and τ=1ps in normal regimes. The study shows the stability of working in anomalous dispersion regime is better than normal regime.

Keywords: Fiber Laser, Ytterbium Doped Fiber, AM Mode Locking, Moment Method, Pulse Chirp, Pulse Energy, Pulse Width.

تقنيةالتحليل الرياضي لقفل النمط السعوي لليزر اليف البصري

الخلاصة

في هدا البحث تم دراسة قفل النمط للتضمين السعوي لليزر الليف البصري المشاب بعنصر اليوتربيومز. تم دراسة هذه النماذج باستعمال ليف احادي النمط ويعمل بطول موجي 1055 نانو متر مشوب لتربيوم ويضخ بليزر976 نانومتر . استعملت تقنية زوج من المحززات لمعادلة التشتت العادي. تم تقديم معادلة ليزر الليف لقفل النمط الرئيسة وباستخدام إشكال النبضة المقترحةو بعد تعديل معادلة (كايزن بيرغ) والتي هي أساسا معادلة شرودنكر اللاخطية العامة وباستعمال طريقة العزم تم تقديم خمس معادلات تصف طاقة النبضة,الانحراف الترددي,الانحراف الزمني,الزقزقة وعرض النبضة تصف هذه المعادلات نشأة معاملات النبضة خلال ل دورة . وتحل هذه المعادلات رقميا باستخدام طريقة رينج كتا وباستعمال برنامج ماث لاب الإصدار السابع.حصلنا على عرض النبضة = 1بيكو ثانية في النمط الاعتيادي و 0.8 بيكو ثانية بالنظام الشاذ. أظهرت الدراسة استقرارية العمل عند نظام التشتت الشاذ وان تأثير الانضغاط هو أفضل من

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النظام العادي بسبب التأثيرات الممزوجة لكل من تشتت سرعة المجموعة السالب واللاخطية لكلا نظامي

Introduction

Ultra-short pulses are very important research field. Today short pulsed laser systems find numerous applications in areas of fundamental research as well as for medical and industrial applications, depending on the wavelength and pulse width. As early as 1970, an analytic theory was developed for determining pulse parameters and shape in actively mode-locked solid-state lasers [1]. It is possible to include the effect of dispersion on the pulse shape and the nonlinear effects within the cavity become important, analytic investigations begin more accurate. This is frequently the case in fiber lasers where both fiber dispersion and nonlinearity are important.

In this work, we used tool for investigating mode-locked lasers by essentially treating the mode-locked pulses as particles with a fixed analytic shape. This approach allows us to simplify the governing nonlinear partial differential equation, often called the master equation of mode-locking, into a set of coupled ordinary differential equations. The resulting equations are similar to the rate equations commonly used to describe continuously operating lasers. They can be solved quickly using standard techniques and have the added benefit that under steady-state conditions they reduce to algebraic equations describing pulse energy, width, and chirp. These algebraic equations indicate the trade-offs associated with the different laser parameters and overcome any issues associated with computation time. Our approach represents an application of the moment method [2], a technique used extensively within the field of telecommunications [3, 4], to the case of a mode-locked laser.

Pulse shape is invariably close to Gaussian or hyperbolic secant, depending on the type of mode-locking employed, the cavity dispersion (normal or. anomalous), and the strength of nonlinearities. This observation is at the root of our approach since the moment method requires a knowledge of the pulse shape.

2. The master equation of mode-locking

If the dispersive and nonlinear effects are relatively weak over a single round trip, the temporal shape and width of the pulse change little during this period (assuming the mode-locker's effect on the field is weak and discrete losses are minimal). Although an approximate treatment, it is fair to model such a system by the master equation of mode-locking [5] obtained by averaging over the round-trip cavity length L_R. This equation takes the form [5, 6]

$$T_R\frac{\partial A}{\partial T}+\frac{i}{2}\left(\bar{\beta}_2+i\bar{g}T_2^2\right)L_R\frac{\partial^2 A}{\partial t^2}-\frac{\bar{\beta}_3}{6}L_R\frac{\partial^3 A}{\partial t^3}=\bar{\gamma}L_R|A|^2 A+\frac{1}{2}(\bar{g}-\bar{\alpha})L_R A+M(A,t) \quad \text{.....(1)}$$

where $T = z/Vg$, Vg is the group velocity, and $A(T, t)$ is the slowly varying envelope of the electric field. As in Refs. [5, 7,8,9], we have assumed second-order dispersion dominates; therefore higher-order dispersive effects have been ignored in Eq. (1). It is important to note there are two time scales in this equation; the time *t* measured in the frame of the moving pulse and the propagation time *T*. Since we averaged over a single round trip, *T* is measured by the round-trip time $T_R = L_R/Vg$. It is assumed that the time scale associated with the pulse is sufficiently smaller than T_R so the two times are essentially decoupled. This treatment is valid for most lasers for which T_R exceeds 1 ns and pulse widths are typically less than 100 ps.

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In rare-earth-doped fibers, the gain medium responds on a time scale much slower than the round-trip time, and the saturated gain may be approximated by[7]

$\bar{g} = \bar{g}_o/(1+P_{ave}/P_{sat})$ Where, P_{sat} represents the saturation power of the gain medium, g_o the average small-signal gain, and, P_{ave} the average power over one pulse slot of duration T_m, which could be calculated by this equation:[7]

$$P_{ave} = \frac{1}{T_m}\int_{-T_m/2}^{+T_m/2} |A(t,z)|^2 dt \quad \text{....(2)}$$

In Eq. (2) The term $A(t,z)$, represents the slowly varying envelope of the electric field . The pulse slot is calculated by this equation: $T_m = 1/F_r = T_R/N$ Where F_r is the mode-locked frequency, F_m is the modulation frequency. N is a harmonic at which the laser will mode locked ($N \geq 1$). T_R , is the time of one roundtrip. The overbar in Eq. (1) means the averaged value of the parameter over a round trip. More specifically, $\bar{\beta}_2$ represents the averaged second-order dispersion of the cavity elements, while $\bar{\gamma}$ takes into account the averaged nonlinear parameter and $\bar{\alpha}$ represents the averaged losses. The finite gain bandwidth is assumed to have a parabolic filtering effect with a spectral full width at half maximum (FWHM) given by $\Delta w = 2/T_2$.

In the absence of the mode-locker, Eq. (1) reduces to the well-known Ginzberg–Landau equation, which supports a shape-preserving solution in the anomalous-dispersion regime known as the autosoliton [6]

$A(T,\ t)\ =\ a[\operatorname{sech}(t/\ \tau)]1+iq\ \exp[i f(T)]$ .... (3)

Where the pulse parameters a, τ, q, and $\phi(T)$ are determined by the parameters appearing in Eq. (1). In the absence of a mode-locker, a stable pulse will neither form nor survive multiple round trips in the cavity. However, the active fiber will try to impose the autosoliton shape on any pulse circulating in such a laser.The pulse shortening in AM mode-locked lasers by using of dispersive and nonlinear elements [5]. In this work we assume that the laser is mode-locked with the autosoliton shape; we then seek to include the effects of the mode-locker on the pulse energy, width, and chirp. To extend our analysis to the normal-dispersion regime, we also consider a chirped Gaussian pulse

$A(T,\ t)\ =\ a[\exp(-t^2/2\tau^2)1+iq\ \exp[i\ \phi(T)]$ …..(4)

The moment method

In an effort to study the pulse evolution process under the influence of Eq. (1), without resorting to full numerical simulations, we have employed the moment method [2–4]. This approach allows us to develop ordinary differential equations that govern the evolution of the pulse parameters. These equations can be solved quickly, yielding the information of experimental interest. All of this, however, is based on a knowledge of the exact pulse shape. For this reason, one should also solve Eq. (1) to ensure that the actual pulse shape does not deviate much from the assumed pulse shape.

It is used to convert this third order partial differential equation to a set of rate equations that describe the pulse parameters during each roundtrip [2-4].These pulse parameters evolution equations are obtained by using the so-called moment method [4].

As seen in Eqs. (3) and (4), a mode-locked pulse is characterized by four parameters, amplitude a, width τ, chirp q, and phase $\phi(T)$. The amplitude can easily be related to energy and so we focus on pulse energy E, width τ,ζtemporally shift,Ωfrequency shift and chirp q.These parameters can be defined as moments of $A(T, t)$ [4]:

$$E(T)=\int_{-\infty}^{+\infty} |A(T,t)|^2\, dt \qquad \ldots(5)$$

$$\xi(T)=\frac{1}{E}\int_{+\infty}^{-\infty} t\,|A(T,t)|^2\, dt \qquad \ldots(6)$$

$$\Omega(T)=\frac{i}{2E}\int_{+\infty}^{-\infty}[A^*\frac{\partial A}{\partial t}-A\frac{\partial A^*}{\partial t}]dt \qquad \ldots(7)$$

$$q(T)-\frac{i}{E}\int_{+\infty}^{\infty}(t-\xi)[A^*\frac{\partial A}{\partial t}-A\frac{\partial A^*}{\partial t}]dt \qquad ..(8)$$

$$\tau^2(T)=\frac{2c_s}{E}\int_{+\infty}^{-\infty}(t-\xi)\,|A(T,t)|^2\, dt \qquad ....(9)$$

Mode-locking by amplitude modulation (AM) is one of the oldest techniques; we focus on it to illustrate our approach by using $M(A, t) = -\Delta_{AM}\ [1-\cos\ (\omega_m t)]A$ in Eq. (1), where Δ_{AM} is the modulation depth experienced by a pulse during a single round trip, $\omega_m = 2\ \pi/T_m$ is the modulation frequency (assumed to be identical to the repetition rate of the mode-locked pulse train), and the average loss of modulator has been incorporated into $\bar{\alpha}$ Assuming that the mode-locked pulses are much shorter than *Tm*, we approximate the effect of the AM mode-locker as $M(A, t) = -\Delta_{AM}\,\omega_m^2\,t^2/2A$. Using Eqs. (3) and (4) for the pulse shape we obtain the following equations:

$$\frac{T_R}{L_R}\frac{dE}{dT} = (\bar{g} - \bar{a})E - \frac{\bar{g}T_2^2}{2t^2}[C_0(1+q^2) + 2\Omega^2 t^2]E + \frac{2\Delta_{AM}}{L_R}\Psi_0\cos[w_m(x-t)]E \quad \text{.....(10)}$$

$$\frac{T_R}{L_R}\frac{dx}{dT} = \bar{b}_2\Omega - \bar{g}T_2^2 q\Omega + \frac{\bar{b}_3}{4t^2}[C_0(1+q^2) + 2\Omega^2 t^2] - \frac{\Delta_{AM}t}{L_R}\Psi_1\sin[w_m(x-t_s)] \quad \text{..... (11)}$$

$$\frac{T_R}{L_R}\frac{d\Omega}{dT} = -[C_0(1+q^2) + 2\Omega^2 t^2] - \frac{\Delta_{AM}}{L_R}q\Psi\sin[wm(x-ts)] \quad \text{..... (12)}$$

$$\frac{T_R}{L_R}\frac{dq}{dT} = \frac{\bar{b}_2}{t^2}[C_0(1+q^2)+2\Omega^2t^2] - \frac{\bar{g}T_2^2}{t^2}q[C_1(1+q^2)+2\Omega^2t^2] + C_2\frac{\bar{g}E}{\sqrt{2p}t} + \frac{\bar{b}_3\Omega}{t^2}\left[\frac{3}{2C_0}(1+q^2)+\Omega^2t^2\right] - \frac{\Delta_{AM}t}{L_R}\Psi_1\{qk\cos[w_m(x-t_s)]+2\Omega\sin[w_m(x-t_s)]\} \quad \text{......(13)}$$

$$\frac{T_R}{L_R}\frac{dt}{dT} = C_3\frac{\bar{b}_2}{t}q + C3\frac{\bar{b}_3}{t}q\Omega + C_0C_3\frac{\bar{g}T_2^2}{2t}(C_4 - q^2) - \frac{\Delta_{AM}t}{2L_R}\Psi_2\cos[w_m(x-t_s)] \quad \text{...... (14)}$$

Where the constants *Cn* (*n* = 0 to 4) are introduced such that they all equal 1 for a Gaussian pulse. In the case of an autosoliton, $C0 = 2/3$, $C1 = 1/63$, $C2 = \sqrt{2}\,\pi/3$, $C3 = 6/\pi2$, $C4$ =

2.The $\Psi_0 = \exp(-w_m^2 t^2/4)$, $\Psi_1 = w_m t\Psi_0$ and $\Psi_2 = w_m t^2\Psi_0$.

To illustrate pulse dynamics ,we solve the five pulse parameters elevation equations numerically for AM type of mode locked ,equations(10-14), MATLAB 7.0 program has been written by using fourth-fifth order Runge-kutta method which uses the function ODE-45.This method is used to solve ordinary differential equations numerically.

MATLAB 7.0 program uses the constants in table 1,with the following initial values for pulse parameters [4] .

$E(0)=1$fJ, $\zeta(0)=0$, $\Omega(0)=0$, $q(0)=0$, $\tau(0)=0.5$ ps

To illustrate pulse dynamics, we solve Eqs. (10)–(14) for a fiber laser using realistic parameter values. More specifically, we use $\beta_2 = \pm 0.014$ ps2/m, $\bar{\gamma} = 0.012$ W−1/m, $g_0 = 0.55$ m−1, $\bar{\alpha} = 0.17$ m^{-1}, $T_2 = 47$ fs/rad, $P_{sat} = 12.5$ mW, $F_{rep} = 10$ GHz, $L_R = 4$ m, $T_R = 40$ ns, and $\Delta_{AM} = 0.3$.

Results and Discussion

Fig (1) shows the approach to steady state in both the normal and anomalous-dispersion regimes for change in chrip ,tempril shift and frequncy shfit over multiple round trips. It reveals that the pulse converges quickly in the anomalous dispersion regime but takes > 1000 round trips before converging in the normal dispersion regime. Although the rate of convergence depends on the initial conditions used (*E* =1 fJ, *q*=0, and τ= 0.5 ps), this type of behavior is expected since the nonlinear effects are weaker in the normal dispersion regime. In the normal-dispersion regime the nonlinear effects add to the effect of dispersion and broaden the pulse to $\tau = 3$ ps, thus reducing its peak power and the role played by nonlinearity shown in fig (2). In the anomalous-dispersion regime the interplay between dispersion and nonlinearity prolongs the convergence. Energy reaches its maximum value $E_{max} \approx 2.793$ p J in about (75) roundtrips as shown in Fig (2) (while in normal regime in about 100 roundtrips).Then damping oscillation is going over thousands of roundtrips We also note the initial energy used is more than 2700 times smaller than the steady state value obtained as shown in Fig(2). In fact it is obvious that number of roundtrip are needed for anomalous regime to achieve steady-state is $RT_{ss}=2000$, while for normal regime $RT_{ss}=2500$. From plots of pulse width evolution as in Fig.(2) , a broadening in pulse width is introduced with maximum width τ_{max} =1.8ps in first (50) roundtrips in anomalous (where in normal τ_{max} =3ps in first (100) roundtrips), then exhibits damped oscillation over thousands of roundtrips decreasing to steady-state value τ_{ss}=0.8ps , (in normal regime τ_{ss} =1ps) where in anomalous less number of roundtrips than in normal regime is needed $RT_{ss}>2500$ .While in anomalous regime ,shown the convergence of the pulse width to its steady state value.

The behavior is a consequence of the interplay between dispersion and nonlinearity. In the anomalous regime, the two effects produce chirps with opposite signs, which partially cancel one another, whereas, the chirps add in the normal-dispersion regime [6,8,9].Table (1) shoes the max and min values for pulse parameter.

Conclusions

By applying the moment method to the master equation of mode-locking, we have derived a set of five ordinary differential equations for pulse energy, width, temporal shift, frequency shift and chirp. These equations play the role of rate equations for mode-locked lasers. Their solution shows that, although a steady state is eventually reached after sufficiently large number of round trips the approach to steady state can be quite different depending on whether the average cavity dispersion is normal or anomalous.

The rate equations reduce to five algebraic equations in the steady state, which were used to study the dependence of pulse width and chirp on cavity dispersion and nonlinearity. We also verified that in the absence of dispersive and nonlinear effects, our analytic result reduces to that obtained in Ref. [1]. Although we have focused on the case of AM mode locking, our approach is quite general and can be applied to all actively and passively mode-locked lasers.

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Table (1) Pulse Parameters Database for both Dispersion Regimes (Fr=10GHz)AM mode locked

	Normal Regime							Anomalous Regime						
PRAMETER	**Max**	**RT max**	**Min**	**RT min**	**∆**	**SS**	**RT SS**	**Max**	**RT max**	**Min**	**RT min**	**∆**	**SS**	**RT_{ss}**
E	2.794 p J	100	2.778 p J	200	0.016 p J	2.79 p J	2500	2.793 pJ	50	2.752 PJ	75	0.041 P J	2.785 p J	2000
ζ	+2 fs	50	-2fs	300	0 fs	0	>2500	+2.2 fs	100	- 1.5fs	200	0.7fs	0.25	2000
Ω	9 GHz	300	+2.0 GHz	500	7 GHz	5	>2500	+9 GHz	125	2 GHz	200	7 GHz	6.5 GHz	2000
q	+2	175	-2	25	0	0	>2500	+0.9	25	-0.7	100	0.2	0	2000
τ	3ps	100	0.25ps	175	2.75ps	1ps	>2500	1.8ps	50	0.5ps	80	1.3ps	0.8ps	2000

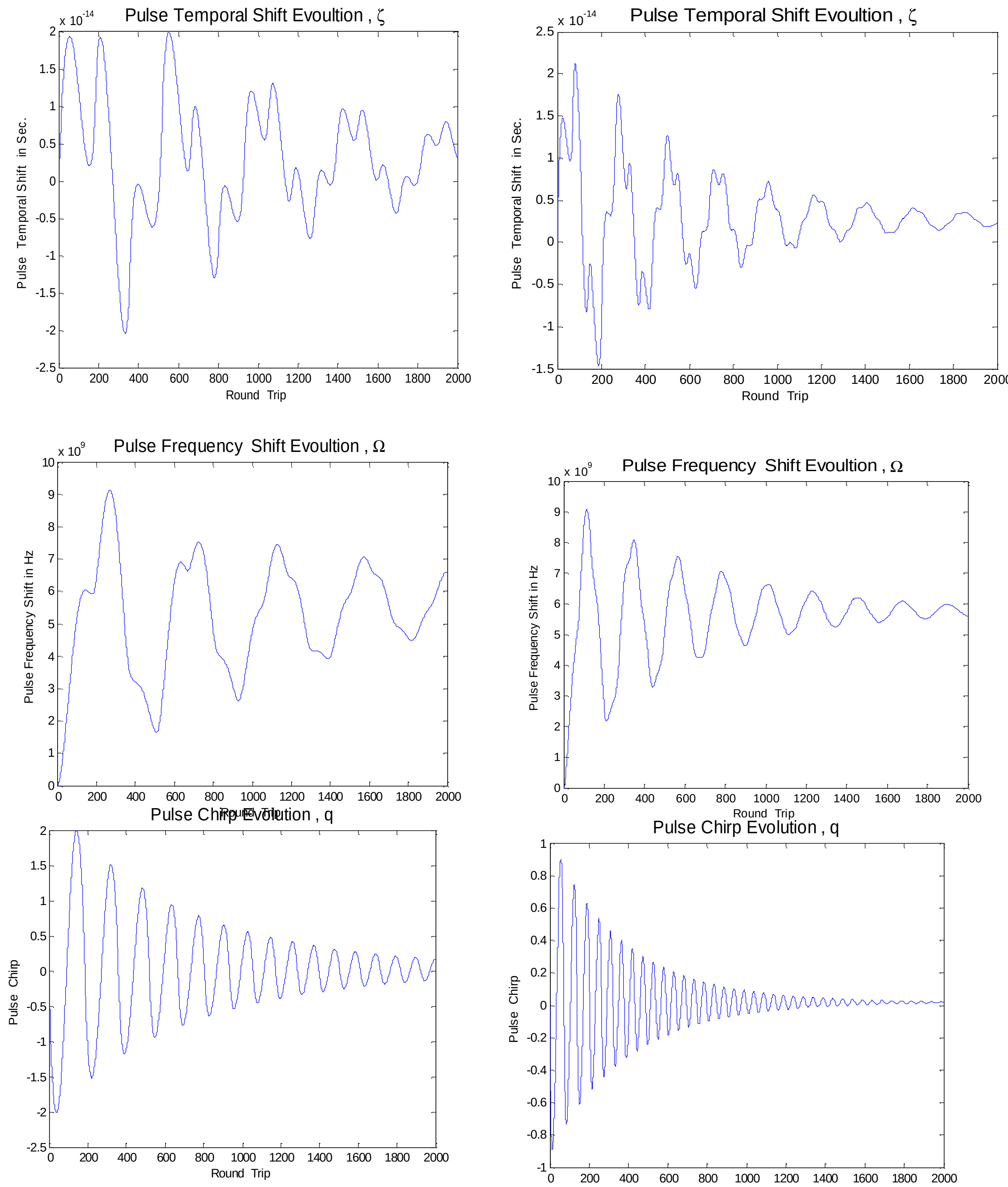

Figure (1)Change in chrip ,tempril shift and frequnicy shfit over multiple round trips in normal dispersion and anomalous dispersion

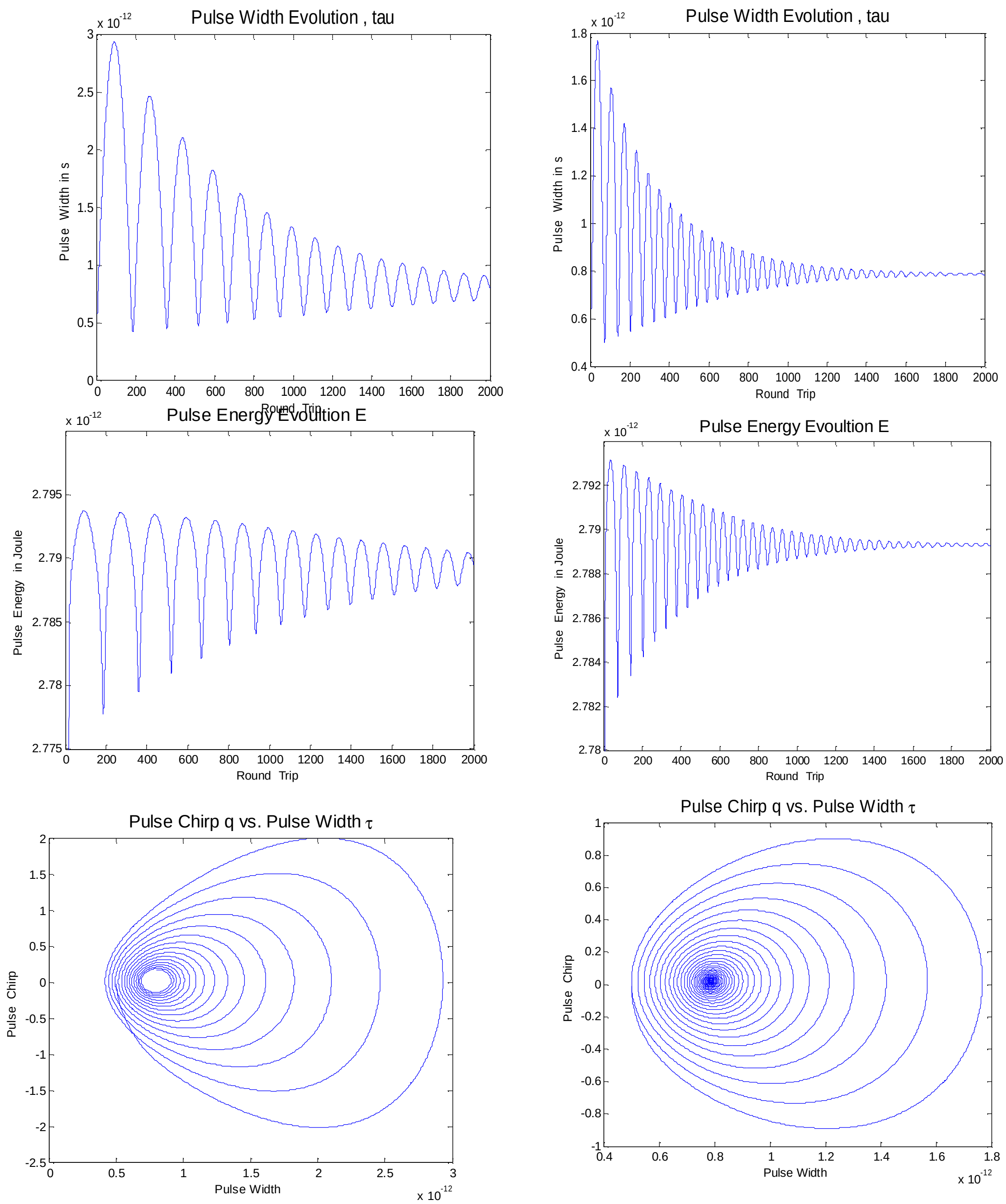

Figure (2) Change in pulse energy and pulse width over multiple round trips in normal dispersion and anomalous dispersion