Theoretical Aspects of some Mechanical Properties of Composites

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Abstract
In the present work, two mathematical models are constructed in order to define the detailed nature of composite. The first one is based on the classical Rule of Mixtures (RoM) which is normally rooted from the ordinary strength of materials. The second model is based on the theory of elasticity, which deals with the detailed response of the internal macrostructure of the composite. A virtual composite was assumed to be formed of a number of matrices (Epoxy resin & Nickel) containing various inclusions (Carbon fibres & powder, E-glass fibres & powder, and Kevlar fibres) in sequential permutations. In general, the elasticity model $E$, exhibited various degrees of superiority to the RoM depending on the mechanical parameters in question and the mechanism by which it influences the internal details of the material.

Keywords: composites, modelling

Introduction
The use of the completely different materials combination with low weight and thin structural members requires accuracy and through out analysis that becomes even more complicated when an anisotropic materials are used. The material's combination is decided according to the structural needs and the relative importance of various properties; i.e., specific applications. Nevertheless, one must keep in mind that the combinations of materials that enhance a particular property, composites, often involve the degradation of another property; so, all relevant properties must be considered, since the properties of the composite's constituents and their distribution and the quality of interactions among them strongly

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influence the properties of a composite material.

The role of the reinforcement in a composite material is fundamentally one of increasing the mechanical properties of the neat resin system. All of the different fibres used in composites have different properties and so affect the properties of the composite in different ways. The result of putting strong fibres into a weak polymer can be a very strong and lightweight material. The matrix allows designers to apply the high strength fibres to real-life situations. Such materials allow the designer to customize a material so that it will properly react with the stresses placed upon it, [1 & 2]. The fundamentals of composite design based on:

1. The materials used for the matrix,
2. The materials used as fibres.
3. The fibres lengths compared to their diameters,
4. The fibres arrangement in the matrix.

Therefore, the mechanical properties of the composite will be dominated by the contribution of the fibre to the composite. The surface interaction of fiber and resin is controlled by the degree of bonding that exists between them, which is heavily influenced by the treatment given to the fiber surface. The four main factors that govern the fibre's contribution are:

1. The basic mechanical properties of the fibre itself,
2. The surface interaction of fibre and resin (the interface),
3. The amount of fibre in the composite (Fibre Volume Fraction),
4. The fibre’s orientation with respect to the direction of stress within the composite.

In general, fibre reinforced thermosetting (it has been estimated that over three-quarters of all matrices used in FRP’s are thermosetting polymers, [4]), where polymers do not exhibit the ductile failure mechanisms associated with metals, instead the brittle nature of most fibres and thermosets tends to generate a brittle mode of failure. It is this fundamental difference, which gives rise to the very distinct energy absorption characteristics of FRP’s.

2) Theoretical Concepts of Composite Micromechanics:

There are two approaches in micromechanics to predict the mechanical properties of composite materials:

1. The mechanics of material approach to stiffness based on classical strength of material theory for the combined behaviour of each component, i.e. Rule of Mixtures, RoM.

2. The elasticity approach to stiffness based on the variation energy principles of classical Theory of 2-

1) the Rule of Mixtures, RoM:

The material assumed to be transversely isotropic and the RoM expressions for the elastic properties of a unidirectional fibrous composite are:

\[ E_{11} = cE_f + (1-c)E_m \]  
\[ E_{22} = \frac{E_f \times E_m}{(1-c)E_f + cE_m} \]  
\[ v_{12} = (1-c)\nu_m + cv_f \]  
\[ G_{12} = \frac{G_f \times G_m}{(1-c)G_f + cG_m} \]
2-2) the Elasticity Approach:
The engineering significance of the reliable analysis of properties is quite different for particulate composites from that for fibre composites. For the former, such capability is desirable, while for the latter it is crucial, since the range of them and the ability to control the internal geometry are quite different in the two cases. Here, the Young’s modulus of matrix with spherical particles, which is isotropic, will depend on the volume fractions, \[5\]. Here, the classical field equations of elasticity, assumed valid for the composite materials with effective properties replacing the usual homogeneous properties and that know as the classical approximation.

2-2-1) Particulate Composites:
The analysis involves a single particle in an infinite medium, known as dilute material, violates the condition that the representative volume’s element, is small compared to the scale of the problem of intended application. The definition of Dilute is that the state of strain in any one particle in the composite body under homogeneous boundary conditions does not affected by all the other particles. However, the physical meaning of this idealization is simply that the particles may neglect, no matter what the size of the representative volume’s element may be.

The composite spheres model introduced by Hashin, [6], by composing of a gradation of sizes of spherical particles embedded in continuous matrix phase. However, the size distribution is not random, but has a very particular characteristic, as shown in Fig (1), i.e. the ratio of \[\alpha/b\] is constant for each composite sphere, and independent of its absolute size. Thus, there must be a specific gradation of sizes of particles such that each composite sphere has \[\alpha/b = \text{constant}\], while still having a volume filling configuration. This distribution requires particle sizes down to infinitesimal.

According to that, Hashin found the effective bulk modulus \(K\), which applies to the entire volume element:

\[ k = k_m + \frac{c(k_f - k_m)}{1 + (1 + c)A_1} \]  

Where:

\[ A_1 = \left[ \frac{k_f - k_m}{k_m + \frac{4}{3}G_m} \right] \]

He also obtained the shear modulus \(G\) as with bulk modulus \(K\):

\[ \frac{G}{G_m} = 1 - \frac{G_{II}}{15(1 - \nu_m)}(1 - c) \]  

Where:

\[ G_{I} = \left(1 - \frac{G_{m}}{G_i} \right) \]

\[ G_{II} = \left[7 - 5\nu_m + 2(4 - 5\nu_m) \frac{G_{I}}{G_{m}} \right] \]

Hence, the Young Modulus and the Poisson’s ratio can be evaluated using equation (5) and equation (6):

\[ E = \frac{9KG}{3K + G} \]

And

\[ \nu = \frac{1}{2} - \frac{E}{6K} \]  

embedded in a matrix, and the specimens are in general cylindrical with fibres in direction 1.
The composite is consequently transversely isotropic which implies
\[ v_{23} = v_{21} \quad \text{and} \quad \frac{v_{12}}{E_{11}} = \frac{v_{23}}{E_{22}} \quad \ldots \ldots (8) \]
\[ E_{22} = \frac{4G_{23}K_{23}}{K_{23} + G_{23} + A_2} \quad \ldots \ldots (9) \]
\[ v_{23} = \frac{K_{23} - G_{23} - A_2}{K_{23} + G_{23} + A_2} \quad \ldots \ldots (10) \]
\[ v_{21} = \frac{4v_2G_{23}K_{23}}{E_{11}(K_{23} + G_{23}) + 4v_2G_{23}K_{23}} \quad \ldots \ldots (11) \]

Where:
\[ A_2 = 4v_2G_{23} \frac{K_{23}}{E_{11}} \]

The cylindrical model of the composite material, given by Hashin and Rosen, [7], consists of an infinitely long circular cylinders of fibres embedded in a continuous matrix phase. Each individual fibre of radius \( a \), associated with a matrix material of radius \( b \) and each individual cylinder combination referred as a composite cylinder such that the absolute values of (\( a, b \)) vary with each one and the volume filling configuration obtained. The ratio \( [a/b] \) is required to be constant for all the individual cylinders.

In addition, the effective uniaxial modulus for a single composite cylinder:
\[ E_{11} = cE_f + (1-c)E_m + \frac{4c(1-c)v_f(v_f - v_m)}{MR} G_m \]

Where:
\[ MR = \frac{(1-c)G_m}{K_f + \frac{G_m}{3}} + \frac{cG_m}{K_m + \frac{G_m}{3}} + 1 \]

And it had been found [8,9]:
\[ v_{12} = (1-c)K_m + cv_f + \frac{c(1-c)(v_f - v_m)RR}{MR} \]

\[ \ldots \ldots (12) \]

Where:
\[ RR = \left[ \frac{G_m}{K_m + \frac{G_m}{3}} - \frac{G_m}{K_f + \frac{G_m}{3}} \right] \]
\[ K_{23} = K_m + \frac{G_m}{3} \frac{c}{K_f - K_m} + K_2 \]

\[ \ldots \ldots (13) \]

Where:
\[ K_{1} = K_m + \frac{1}{3}(G_f - G_m) \]
\[ K_{2} = \frac{(1-c)}{K_m + \frac{4}{3}G_m} \]
\[ \frac{G_{12}}{G_m} = \frac{G_f - c_1 + G_m - c_2}{G_f - c_2 + G_m - c_1} \quad \ldots \ldots (14) \]

Where:
\[ c_1 = (1+c) \quad \text{and} \quad c_2 = (1-c) \]

The estimation of the transverse shear modulus, \( G_{23} \), had found by R. M. Christensen and K. H. Loas, [10]:
\[ \frac{G_{23}}{G_m} = 1 + \frac{c}{G_f - G_m} + K_{\alpha m}G_{\alpha m} \]

Where:
\[ K_{\alpha m}G_{\alpha m} = \left[ \frac{K_m + \frac{7}{3}G_m}{2 K_m + \frac{8}{3}G_m} \right] \]

\[ \ldots \ldots (15) \]

2-2-3) Randomly Oriented Fibre Composites

The solution of a composite plate made of fibres randomly orientated in continuous matrix phase, founded by Boucher, [13], as shown below:
\[ KK = \frac{c}{1 + \frac{4}{3} \frac{G_f}{K_m}} \]  
\( \text{...(18)} \)

\[ GG = \frac{c}{1 + \frac{6(K_f + 2G_f)}{G_m}} \]  
\[ G_m + \left[ G_f \left( 9K_f + 8G_f \right) \right] \]  
\[ \text{...(19)} \]

Where:

\[ KK = \frac{K_f - K_m}{K_f - K_m} \]

\[ GG = \frac{G_f - G_m}{G_f - G_m} \]

However, Christensen, [13], used Eq's (2-33) to predict the properties of the randomly oriented system:

\[ K = \frac{1}{9} \left[ E_{11} + 4(1 + \nu_{12}) \nu_{23} \right] \]  
\[ \text{...(20)} \]

\[ G = \frac{1}{15} \left[ E_{11} + (1 - 2\nu_{12}) \nu_{23} + 6(G_{12} + G_{23}) \right] \]  
\[ \text{...(21)} \]

\[ E = \frac{C_{11} - C_{22}}{3[2E_{11} + C_{33} + 2(G_{12} + G_{23})]} \]  
\[ \text{...(22)} \]

Where:

\[ C_{11} = \left[ E_{11} + 4(1 - \nu_{12}) \nu_{23} \right] \]

\[ C_{22} = \left[ E_{11} + (1 - 2\nu_{12}) \nu_{23} + 6(G_{12} + G_{23}) \right] \]

\[ C_{33} = \left[ 8\nu_{12}^2 + 12\nu_{12} + 7 \right] \nu_{23} \]

\[ \nu = \frac{E_{11} + 2(2\nu_{12} + 8\nu_{12}^2 + 3)\nu_{23} - 4(G_{12} + G_{23})}{2[2E_{11} + C_{33} + 2(G_{12} + G_{23})]} \]  
\[ \text{...(23)} \]

3) Results and Discussions

Based on the theoretical considerations discussed earlier, the mechanical properties were evaluated for unidirectional fibres, chopped fibres, and dispersed powder composites using both methods: theory of Elasticity and the Rule of Mixtures, (RoM). Carbon inclusions dispersed in Epoxy resin Matrix, table (1), has been used as an example to emphasize the expected differences. Despite the use of different approaches in the evaluation of \( E_{11} \), Fig (3a), the difference lies in the \( 3^{rd} \) term which basically used the shear modulus and bulk modulus of both the fiber and the matrix, the difference is not very clear but the difference is very clear with the powder and the chopped inclusions. This result, in fact, gives the reason for using the elasticity-based formula instead of using the RoM, where the latter gives an approximate value, while the first gives the accurate value.

The value of \( E_{22} \), Fig (3b), is not available for the chopped fibres and the powder dispersions. The elasticity based formula uses the shear modulus \( G_{23} \), bulk modulus \( K_{23} \), Poisson’s ratio \( \nu_{12} \), in addition to \( E_{11} \), while the RoM uses only the Young modulus for both matrix and fiber, \( E_m \) & \( E_f \), respectively. Hence, the RoM uses only the properties of the matrix and the fibre, while in the elasticity based way, it depends on the mechanical properties of the composite material. Here, the mechanical properties are the same in all directions for both the chopped fibers and the spherical inclusions, i.e. there is no value of \( E_{22} \). The evaluating of \( G_{52} \), Fig (3c), the chopped fibres have the highest values, while the RoM have the lowest, where it used only the properties of the fibre and the matrix, while the elasticity based formula for chopped fibres uses the internal properties of the composite bulk, i.e. \( E_{11}, \nu_{12}, K_{23}, G_{12}, \) and \( G_{23} \). In addition, it is clear that for the elasticity based formula of powder dispersion, the
value of the ratio, $\frac{G_m}{G_i}$ or $\frac{G_i}{G_m}$, the ratio between the shear moduli of the matrix and the inclusion, plays a major role in the equation. For Poisson’s ratio, $\nu_{12}$ as shown in Fig (3d), it is clear that shear modulus and the bulk modulus of both the fibre and the matrix are taken into consideration, where the first have the higher values than the latter. In addition, the powder dispersion has the highest values and the chopped fibres have the lowest, since the powders are working as voids in the matrix, and there is no continuity between the particles, each one works by its own, while in chopped fibres, as mentioned earlier, have three components of working planes, each one has the third of the total. One can notice that the differences are very clear between the evolutions of the three types of inclusions, using RoM and the elasticity based formula where the equations are completely different. One can see from the figures mentioned that the values of $E_{11}$ are very close in the RoM and the elasticity-based formula for the unidirectional, while the other properties, $E_{22}$, $G_{12}$, and $\nu_{12}$ are not. In addition, it is clear that the values of $E_{11}$, and $G_{12}$ of the chopped fibres are higher than that in the powder dispersions of the same constitutes, while it’s the lowest for $\nu_{12}$, where adding a chopped fibres to Epoxy resin reduces the value of the Poisson’s ratio, while it increases for powder dispersions.

The mechanical properties of different types of fibers and matrices used in this study are summarized in table (2) in order to study the differences in the behaviour. The existence of fibres in Epoxy resin matrix, fig (4a) increase its young modulus, while decrease it for the Nickel matrix, fig (4b), accept for the Carbon Fibers, which is obvious, since the carbon's Young modulus is close in value to the Nickel. The use of unidirectional Carbon Fibres increases the young modulus of both epoxy Resin and Nickel matrices. On the other hand, using a unidirectional E-glass Fibres increase the young modulus of the Epoxy matrix, fig (4c) and decrease it in the Nickel matrix, fig (4d), since the E-glass has the lower value of young modulus relative to the Nickel matrix. The behaviour of the different inclusions was investigated in order to study their effects on the mechanical properties of the composite materials. In the Epoxy resin matrix, the behaviour of the $E_{11}$ of chopped Carbon fibres and powder dispersions have overlapped values at a volume fraction of 13%, fig (5a), where the increase of chopped fibers in the matrix tend to strength it, while the increase of powders tend to weaken it since it would behave as voids in the matrix. On the other hand, in the Nickel matrix, the Powder dispersions coincide with unidirectional fibers, fig (5b), while the chopped fibres, and almost have no effect because of the high value of the Nickel Young modulus. In addition, the presences of E-glass inclusions in the Epoxy resin matrix enhances the young modulus values, fig (5c), in the same manner as the carbon inclusions, but without an overlap values since the E-glass has a low value of Young modulus, more flexible, relative to the carbon. This low value in Young modulus tends to decrease the one of the Nickel matrix, fig (5d).
The behaviour of the Poisson's ratio, \( v_{12} \), seems to be in a different manner. In the Carbon/Epoxy Resin Matrix, fig (6a), it's clear that the Poisson's ratio has the higher values for powder dispersions, nevertheless, the chopped fibers overlapped in values with the unidirectional at a volume fraction less than 35\%. On the other hand, for Carbon/Nickel composite, fig (6b) the chopped fibers have the higher values than that of the others, and the unidirectional have the lowest. However, both types of composites, shared in the decrease of the Poisson's ratio values. This behaviour can also be noticed in the E-glass inclusions, where the powder dispersions are higher in value and the chopped fibers are the lowest with an overlapped value at a volume fraction of 35\% with unidirectional fibers, fig (6c). However, in the Nickel matrix, the unidirectional has the higher values and the powder dispersions have the lowest values with sharper degradation starting at a volume fraction of 15\%, fig (6d). These results lead to the idea, that the impurity in any materials reduces the Poisson's ratio \( v_{12} \), and that leads to a reduction in the value of \( \varepsilon_{22} \), or increase in \( \varepsilon_{11} \).

The behaviour of the Shear modulus, \( G_{12} \), increases in Carbon/Epoxy, Carbon/Ni, and E-glass/Epoxy composites, but decreases in E-glass/ Ni composite, as shown in fig (7). The chopped fibers, always, have the higher values, and the powder dispersions have the lowest. Nevertheless, in a different behaviour, the E-glass inclusions in Nickel matrix tend to decrease the shear modulus values, since it has a lower value of \( G_{12} \), than that of the Nickel. Here, all types of inclusions coincide with each other until a volume fraction of 9\%, where a separation occurs, and the powder dispersions degraded away at volume fraction 30\%, from the others and the unidirectional almost coincide with the chopped fibres.

A comparison between the theoretical calculations and the experimental results obtained from previous published works are summarized in table (3), for the case of unidirectional Fibers. As some data had not found in these references, only the theoretical results are exhibits.

For the Young modulus, \( E_{11} \), it is clear that the values that are observed experimentally, seems to be lower than the theoretically predicted except for those obtained from reference (14), and the RoM and the elasticity based formula are very close. This result may relate to the higher volume fraction that been used, while for the flexural young modulus, \( E_{22} \), the values of the experimental results are somewhat midway, except those from reference (15).

4) Conclusions
Based on that, the design process of composite material may develop, using the values of different properties of constitute, Young modulus of matrix and inclusion, and the volume fraction. In addition, the RoM is not expected to give the most accurate results as the elasticity based formula had for the chopped fibers and the dispersed powder inclusions, because of the different approaches in evaluations. Moreover, the results obtained from each mathematical model directly reflect the presumption philosophy of its individual approximation. In addition, the role played by the different shapes of inclusions, (particles or fibres), is mainly decided by the mode of the
exerted stresses. The differences between the theoretically estimated results and those obtained from experimental results may rise from the following factors:

1. The Incorrect control of the temperature/pressure cycle, leads to an incorrect state of resin curing and flow. As a result, a poor consolidation, incomplete filling of the mould, failure of separated flow streams in moulds, and, non-uniform filling of the mould cavity taken place.

2. Incorrect overall fiber volume fraction, this is due to the presence of the porosity resulting from bubbles during moulding, and as a solution, one can use the vacuum system.

3. The stresses in the fiber and matrix are equal.

4. Misaligned or broken fibres

5. The fibres dispersion randomly at any cross-section of the composite, i.e., non-uniform fiber distribution, and this leads to matrix-rich regions.

6. Resin cracks or transverse cracks take place due to the thermal mismatch stresses where most composites consist of materials of widely different thermal expansion coefficients. This develops residual stresses during the cure process sufficiently high to crack brittle matrix. Additives to the resin as a modification may eliminate this problem

5) Recommendations

The following suggestions can be recommended for future works:

1. All the above results are for single layer composite, a comprehensive study is needed for multilayered composite.

2. A mathematical model is needed to describe the relation between the interfacial shear strength between the fibre and the matrix with the shear strength evaluated from the plates theory due to impact event, and then, if possible, carrying out an experimental study.

3. One can make a benefit of the Elasticity based approach in the design of a hybrid composite.

References


**Abbreviations**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>FRPs</td>
<td>Fibre Reinforcement Polymers</td>
</tr>
<tr>
<td>FVF</td>
<td>Fibre Volume Fraction</td>
</tr>
<tr>
<td>a</td>
<td>the radius of the inclusion = matrix’s inner radius, μm</td>
</tr>
<tr>
<td>b</td>
<td>the matrix’s outer radius, μm</td>
</tr>
<tr>
<td>c</td>
<td>the fiber volume fraction, %</td>
</tr>
<tr>
<td>d</td>
<td>diameter of the fiber, μm</td>
</tr>
<tr>
<td>E_f</td>
<td>Yong modulus of the fiber, GPa</td>
</tr>
<tr>
<td>E_m</td>
<td>Yong modulus of the matrix, GPa</td>
</tr>
<tr>
<td>V_f</td>
<td>the volume percentage of the fiber in the composite, %</td>
</tr>
<tr>
<td>V_m</td>
<td>the volume percentage of the matrix in the composite, %</td>
</tr>
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</table>

**Table (1) Mechanical properties of composite constitute in use, [13].**

<table>
<thead>
<tr>
<th>Composite Components</th>
<th>Young Modulus (GPa)</th>
<th>Poisson’s Ratio</th>
<th>Density (gm/cc)</th>
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<tbody>
<tr>
<td>Carbon Inclusion</td>
<td>220</td>
<td>0.15</td>
<td>1.7</td>
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<tr>
<td>Epoxy Resin</td>
<td>3.3</td>
<td>0.37</td>
<td>1.3</td>
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Theoretical Aspects of some Mechanical Properties of Composites

Table (2) Mechanical properties of composites constitute, [13]

<table>
<thead>
<tr>
<th>Constitutes</th>
<th>Young Modulus (GPa)</th>
<th>Poisson’s Ratio</th>
<th>Shear Modulus (GPa)</th>
<th>Density (gm/cc)</th>
</tr>
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<tbody>
<tr>
<td>Carbon Fibres</td>
<td>220</td>
<td>0.15</td>
<td>90</td>
<td>1.7</td>
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<tr>
<td>Steel Fibres</td>
<td>207</td>
<td>0.292</td>
<td>80</td>
<td>3.85</td>
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<td>E-glass Fibers</td>
<td>73</td>
<td>0.22</td>
<td>30</td>
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<td>Kevlar Fiber</td>
<td>135</td>
<td>0.35</td>
<td>50</td>
<td>1.44</td>
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<td>3.3</td>
<td>0.37</td>
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<td>1.3</td>
</tr>
<tr>
<td>Nickel</td>
<td>207</td>
<td>0.31</td>
<td>79</td>
<td>3.7</td>
</tr>
</tbody>
</table>

Table (3) a comparison results between the experimental results and the theoretical predicted Data.

<table>
<thead>
<tr>
<th>Constitutes</th>
<th>E11</th>
<th>Experimental</th>
<th>Elasticity</th>
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<th>RoM</th>
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<tr>
<td>26 % E glass Fibers in Polyester, Ref.(16), 2005</td>
<td>E11</td>
<td>17</td>
<td>19.76</td>
<td>+16.23</td>
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<tr>
<td></td>
<td>E22</td>
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<tr>
<td></td>
<td>G12</td>
<td>N.A.</td>
<td>1.21</td>
<td></td>
<td>0.978</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ν12</td>
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<td>0.33</td>
<td>-8.34</td>
<td>0.331</td>
<td>-8.05</td>
</tr>
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<td>20 % S glass Fibers in Polyester, Ref.(15), 2003</td>
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<td>19.79</td>
<td>18.7</td>
<td>-5.5</td>
<td>18.68</td>
<td>-5.6</td>
</tr>
<tr>
<td></td>
<td>E22</td>
<td>4.895</td>
<td>3.1</td>
<td>-36.67</td>
<td>2.485</td>
<td>-49.23</td>
</tr>
<tr>
<td></td>
<td>G12</td>
<td>N.A.</td>
<td>1.08</td>
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<td>0.914</td>
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<tr>
<td></td>
<td>ν12</td>
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<td>0.3225</td>
<td>+29</td>
<td>0.328</td>
<td>+31.2</td>
</tr>
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<td>50 % E glass Fibers in Epoxy Resin, Ref.(17), 1983</td>
<td>E11</td>
<td>38.27</td>
<td>38.174</td>
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<td></td>
<td>G12</td>
<td>3.72</td>
<td>3.267</td>
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<td>-37.9</td>
</tr>
<tr>
<td></td>
<td>ν12</td>
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<td>0.2895</td>
<td>+3.4</td>
<td>0.295</td>
<td>+5.36</td>
</tr>
<tr>
<td>50 % Carbon Fibers in Epoxy Resin, Ref.(18), 2006</td>
<td>E11</td>
<td>109.34</td>
<td>111.68</td>
<td>+2.1</td>
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<td></td>
<td>E22</td>
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<td>+7.14</td>
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<td>ν12</td>
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<td>+8.46</td>
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<td>55 % E glass Fibers in Epoxy Resin, Ref.(19), 1999</td>
<td>E11</td>
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<td>0.26</td>
<td>0.2822</td>
<td>+8.46</td>
<td>0.287</td>
<td>+10.57</td>
</tr>
</tbody>
</table>

N.A.: not available.
RoM: Rule of Mixtures
Figure (1) Composite sphere assemblage

Figure (2) Unidirectional Fibre Composite Assemblage
Theoretical Aspects of some Mechanical Properties of Composites

Figure (3) Mechanical properties comparison for Carbon Inclusions in Epoxy Resin Matrix
Figure (4) $E_{11}$ comparison of different unidirectional fibers embedded in different matrices
Figure (5) $E_{11}$ comparison of different inclusions embedded in different matrices
Figure (6) Poisson’s Ratio comparison of different inclusions embedded in different matrices
Figure (7) Shear Modulus comparison of different inclusions embedded in different matrices