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Numerical Simulation of Buoyancy- Driven Laminar Flow Through An Air –Filled Isosceles Triangular Enclosure With A Differentially Heated Side Walls And An Adiabatic Bottom Wall

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Abstract

Thermal And Flow Fields Due To Laminar Free Convection In An Isosceles Triangular Enclosure Having Thick Conducting Sidewalls Have Been Investigated Computationally. Inclined Left And Right Side Walls Are Maintained At Isothermal Hot And Cold Temperatures Respectively While The Bottom Wall Is Considered Adiabatic. Problem Has Been Analyzed And The Non-Dimensional Governing Equations Are Solved Using Finite Volume Approach And Employing More Nodes At The Fluid–Solid Interface. Triangular Enclosure Is Assumed To Be Filled With An Air With A Prandtl Number Of **0.7**. Rayleigh Number Varies From $\mathbf{10^3}$ To $\mathbf{10^6}$ Where The Flow And Thermal Fields Are Computed For Various Rayleigh Numbers. Consequently, It Was Observed That The Stream Function And Temperature Contours Strongly Change With High Rayleigh Number. The Streamline And Isotherm Plots And The Variations Of The Average Nusselt Number At The Hot Left And The Cold Right Side Walls Are Also Presented. The Results Explained A Good Agreement With Another Published Results.

Keywords: Free Convection, Fvm, Buoyancy-Driven Flow, Isosceles Triangular Enclosure, Laminar Flow.

التمثيل العددي للجريان الطباقي المسّير بقوة الطفو خلال دهليز مثلث متساوي الارتفاع مملوء بالهواء وله جدران جانبية متغيرة الحرارة وجدار سفلي معزول حراريا

الخلاصـــــــة

انسياب الجريان والحرارة نتيجة الحمل الحر الطباقي خلال دهليز مثلث الشكل ومتساوي الارتفاع و بجدران جانبية سميكة وموصلة للحرارة تم تمثيلها عدديا. الجداران المائلان الأيسر و الأيمن يكونان عند درجة حرارة ساخنة وباردة بالتعاقب بينما الجدار السفلي يكون معزول حراريا. المشكلة تم تحليلها والمعادلات اللابعدية الحاكمة تم حلها باستخدام طريقة الحجوم المحددة مع مراعاة استخدام نقاط شبكية مكثفة عند مناطق تماس المائع بالجدار. الدهليز المثلثي مملوء بالهواء وبرقم براندل يساوي 0.7. رقم رايلي يتغير بمدى من 10^3 إلى 10^6 حيث مجالات الجريان والحرارة تم حسابها لقيم مختلفة من رقم رايلي .بناءا على ذلك لوحظ أن خطوط دالة الجريان والحرارة تتغير بشدة مع قيم رقم رايلي العالية.أشكال خطوط الجريان والحرارة وتغيّر رقم نسلت المتوسط عند الجدار الجانبي الأيسر الساخن و الجدار الجانبي الأيمن البارد تم عرضها أيضا. النتائج أوضحت تطابق جيد مع النتائج الأخرى المنشور

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1. Introduction

The Phenomenon Of Free Convection Heat Transfer And Fluid Flow Or Sometimes Called Buoyancy-Driven Flow In Enclosures Has Been Studied Extensively In Recent Years In Response To Energy-Related Applications, Such As Thermal Insulation Of Buildings Using Air Gaps, Solar Energy Collectors, Furnaces And Fire Control In Buildings. Most Of The Geometrical Configurations Of Enclosures In The Previous Studies Are Focused On Rectangular Or Square Enclosure. However, The Shape Of Enclosure Can Be In Different Configurations Such As, In Most Of The Related Engineering Situations Which Include Triangle, Parallelogram Or Trapezoidal. Modeling Natural Convection Inside A Triangular Enclosure, Previously Reported In The Literature Employed A Symmetry Condition At The Midplane And Performed The Simulations Using Only One Half Of The Physical Domain **[1]**. **Akinsete** And **Coleman [2] , 1982,** Used A Finite Difference Representation Of The Steady-State Stream Function, Vorticity And Energy Equations With An Adiabatic Boundary Condition For The Vertical Wall In A Right Triangular Enclosure. They Recognized The Existence Of Unbounded Heat Transfer At The Corner Of The Enclosure Where A Temperature Discontinuity Exists And Calculated The Limiting Value Of Nusselt Number. **Del Campo Et Al. [3], 1988,** Modeled Natural Convection In Triangular Enclosures Using Galerkin Finite Element Method With A Stream Function – Vorticity Formulation Of Steady State Equations Of Motion. Their Investigation Was Based On A Symmetric Boundary Condition For A System With Heating From Below. The Solutions Obtained Were Symmetric About Mid-Plane. **Jyotsna** And **Vanka [4], 1995,** Investigated The Steady Viscous Flow In A Triangular Cavity By Using Multigrid Calculation. They Found That The Bi-Quadratic Elements With Lesser Number Of Nodes Smoothly Capture The Non-Linear Variations Of The Field Variables Which Were In Contrast With Finite Difference / Finite Volume Solutions Available. **Asan** And **Namli [5], 2000,** Carried Out A Numerical Study For The Two-Dimensional Laminar Natural Convection In A Pitch Roof Of Triangular Cross-Section Under Summer Day Boundary Condition. They Investigated The Effects Of Height-Base Ratio And Rayleigh Number On The Flow Structure And Heat Transfer. They Found That A Considerable Proportion Of The Heat Transfer Across The Base Wall Of The Region Takes Place Near The Intersection Of The Cold Horizontal Wall And Hot Inclined Wall. **Chengwang** And **Patterson [6], 2002,** Carried Out Both Analytically And Numerically The Unsteady Natural Convection In A Triangular Domain Induced By The Absorption Of Solar Radiation. Their Work Consists Of Two Parts: A Scaling Analysis And A Numerical Simulation. The Scaling Analysis For Small Bottom Slopes Reveals That A Number Of Flow Regimes Were Possible Depending On The Rayleigh Number And The Relative Value Of Certain Non-Dimensional Parameters Describing The Flow. They

Classified The Flow Into A Conductive, A Transitional Or A Convective Regime Determined Merely By The Rayleigh Number. Proper Scales Have Been Established To Quantify The Flow Properties In Each Of These Flow Regimes. Their Numerical Simulation Was Verified The Scaling Results. **El Hassan Et Al. [7], 2005,** Performed A Numerical Computation Of Laminar Natural Convection In A Gamma Of Right-Angled Triangular Cavities Filled With Air. The Vertical Walls Were Heated And The Inclined Walls Were Cooled While The Upper Connecting Walls Were Insulated From The Ambient Air. The Defining Apex Angle Was Located At The Lower Vertex Formed Between The Vertical And Inclined Walls. The Finite Volume Method Was Used To Perform The Computational Analysis Encompassing A Collection Of Apex Angles Compressed In The Interval That Extends From **5°** To **63°**. The Height-Based Rayleigh Number, Being Unaffected By The Apex Angle Ranges From A Low $\mathbf{10^3}$ To A High $\mathbf{10^6}$. Numerical Results Were Reported For The Velocity Field, The Temperature Field And The Mean Convective Coefficient Along The Heated Vertical Wall. A Correlation Equation Was Constructed And Also A Figure Of Merit Ratio Between The Average Nusselt Number And The Cross Sectional Area Of The Cavity Was Proposed. **Roy Et Al. [8], 2008,** Used A Penalty Finite Element Analysis With Bi-Quadratic Elements To Investigate Natural Convection Flows Within An Isosceles Triangular Enclosure With An Aspect Ratio Of **0.5**. Two Cases Of Thermal Boundary Conditions Were Considered With Uniform And Non-Uniform Heating Of Bottom Wall. The Numerical Solution Of The Problem Was Illustrated For Rayleigh Numbers (***Ra***), $\mathbf{10^3} \le \boldsymbol{Ra} \le \mathbf{10^5}$ And Prandtl Numbers (**Pr**), $\mathbf{0.026} \le \mathbf{Pr} \le \mathbf{1000}$.They Found That The Intensity Of Circulation Was To Be Larger For Non-Uniform Heating At A Specific **Pr** And ***Ra***. Multiple Circulation Cells Were Found To Occur At The Central And Corner Regimes Of The Bottom Wall For A Small Prandtl Number Regime (**Pr** =**0.026-0.07**) Also They Observed The Oscillatory Distribution Of The Local Nusselt Number. In Contrast, The Intensity Of Primary Circulation Was Found To Be Stronger, And Secondary Circulation Was Completely Absent For A High Prandtl Number Regime (**Pr** = **0.7-1000**). Some Correlations Were Proposed For The Average Nusselt Number In Terms Of The Rayleigh Number For A Convection Dominant Region With Higher Prandtl Numbers. **Roy Et Al. [9], 2008,** Studied Numerically The Natural Convection In A Right-Angle Triangular Enclosure Filled With A Porous Matrix. A Penalty Finite Element Analysis With Bi-Quadratic Trapezoidal Elements Was Used For Solving The Navier-Stokes And Energy Balance Equations. Their Study Was Carried Out In Two Cases, Depending On Various Thermal Boundary Conditions. The First When The Vertical Wall Was Uniformly Or Linearly Heated, While The Inclined Wall Was Cold Isothermal And The Second When The Inclined Wall Was Uniformly Or Linearly Heated, While The Vertical Wall Was Cold Isothermal. In All Cases, The Horizontal Bottom Wall Was Considered Adiabatic, And The Geometric Aspect Ratio Was

Considered To Be **1.0**. It Has Been Found That At Low Darcy Numbers (**Da** $\leq$ **10**$^{-5}$), The Heat Transfer Was Primarily Due To Conduction, Irrespective Of The ***Ra*** And **Pr**. As Rayleigh Number Increases, There Was A Change From A Conduction-Dominant Region To A Convection-Dominant Region For **Da=10**$^{-3}$. The Results Were Presented In The Form Of The Stream Function And Isotherm Contours. It Was Observed That The Average Nusselt Number For The Vertical Wall Was $\sqrt{2}$ Times That Of The Inclined Wall For All Studied Cases. **Tanmay Et Al. [10] , 2008,** Used A Penalty Finite Element Analysis With Bi-Quadratic Elements To Investigate The Effects Of Uniform And Non-Uniform Heating Of Inclined Walls On Natural Convection Flows Within A Isosceles Triangular Enclosure. Two Cases Of Thermal Boundary Conditions Were Considered. The First Case When Two Inclined Walls Were Uniformly Heated While The Bottom Wall Was Cold Isothermal And The Second When The Two Inclined Walls Were Non-Uniformly Heated While The Bottom Wall Was Cold Isothermal. The Numerical Solution Of The Problem Was Presented For The Range Rayleigh Numbers (***Ra***), $10^3 \leq Ra \leq 10^6$ And The Prandtl Numbers (**Pr**), ($0.026 \leq Pr \leq 1000$). It Has Been Found That At Small Prandtl Numbers, Geometry Does Not Have Much Influence On Flow Structure While At **Pr =1000**, The Stream Function Contours Were Nearly Triangular Showing That Geometry Has Considerable Effect On The Flow Pattern. They Noticed That The Non-Uniform Heating Produced Greater Heat Transfer Rates At The Center Of The Walls Than The Uniform Heating.

The Main Object Of The Current Study Is To Compute The Thermal And Flow Fields Due To Laminar Free Convection In An Air-Filled Isosceles Triangular Enclosure Where Their Inclined Left And Right Side Walls Are Maintained At Isothermal Hot And Cold Temperatures Respectively While The Bottom Wall Is Considered Adiabatic. The Computations Are Carried Out For Different Rayleigh Number Ranging From **10**^{3} To **10**^{6}. The Current Study Is Based On The Configuration Of **Tanmay Et Al. [10], 2008**. The Main Difference Between **Tanmay Et Al. [10],** Work And The Present Work Is That The Present Work Deals With The Inclined Isothermal Hot And Cold Temperatures Side Walls While The Bottom Wall Is Considered Adiabatic Using Finite Volume Method Rather Than The Uniform And Non-Uniform Heating Of Inclined Walls While The Bottom Wall Is Cold Isothermal Using Finite Element Method As In **Tanmay Et Al. [10]** Work.

2.ProblemDescription, MathematicalAnalysisAnd Assumptions

The Geometry Under Investigation Is An Air–Filled Isosceles Triangular Enclosure Which Is Considered Insulated At The Bottom Wall. The Right Side Wall Of The Enclosure Is Maintained At A Uniform Hot Temperature (T_h) While The Other Left Side Wall Is Maintained At A Uniform Cold Temperature (T_c) .The Geometry Under Consideration Is Shown Schematically In **Figure (1)**. The Triangular Enclosure Has A Height (H) And A Base (W) And The

Working Fluid Is Chosen To Be Air With Prandtl Number **Pr = 0.7**. The Flow And Thermal Fields Are Described By Continuity, Momentum And Energy Equations. These Equations Are Written In A Dimensionless Form By Dividing All Dependent And Independent Variables By Suitable Constant Terms. The Solution Is Obtained Using A Finite Volume Scheme And The Following Assumptions Are Considered:-

1. The Flow Is Considered Steady, Laminar And Two-Dimensional.

2. The Fluid Properties Are Assumed Constant Except For The Density Variation Which Is Treated According To Boussinesq Approximation.

3. The Fluid Inside The Triangular Enclosure Is Assumed Newtonian While Viscous Dissipation Effects Are Considered Negligible.

The Viscous Incompressible Flow And The Temperature Distribution Inside The Enclosure Are Described By The Navier–Stokes And The Energy Equations, Respectively. The Governing Equations Are Transformed Into Dimensionless Forms Under The Following Non-Dimensional Variables **[10]**:

$$\theta = \frac{T - T_c}{T_h - T_c}, X = \frac{x}{W} \text{ And}$$

$$Y = \frac{y}{H}, P = \frac{pH^2}{\rho\alpha^2},$$

$$U = \frac{uW}{a} \text{ And } V = \frac{vH}{a} \quad ..(1)$$

The Dimensionless Forms Of The Governing Equations Of Steady Two-Dimensional Internal Laminar Natural Convection Are Expressed In The Following Forms **[10]**:-

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \qquad ......(2)$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \Pr\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) ..(3)$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \Pr\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + Ra\Pr\theta \qquad ......(4)$$

$$U\frac{\partial\theta}{\partial X} + V\frac{\partial\theta}{\partial Y} = \left(\frac{\partial^2\theta}{\partial X^2} + \frac{\partial^2\theta}{\partial Y^2}\right) \qquad (5)$$

Where *X* And *Y* Are The Dimensionless Coordinates Measured Along The Horizontal And Vertical Axes, Respectively, *U* And V Being The Dimensional Velocity Components Along X And Y Axes. Also, *Θ* Is The Dimensionless Temperature, *P* Is The Dimensionless Pressure, Pr Is The Prandtl Number And Ra Is The Rayleigh Number. The Previous Dimensionless Numbers Are Defined By **[11]**:-

$$\Pr = \frac{n}{a} \qquad .....(6)$$

$$Ra = \frac{gb(T_h - T_c)H^3 \Pr}{n^2} \qquad .....(7)$$

Where *b* Is The Volumetric Coefficient Of Thermal Expansion, *N* Is The Kinematic Viscosity , *A* Is The Thermal Diffusivity And **G** Is The Gravitational Acceleration. The Rate Of Heat Transfer Is Expressed In Terms Of Average Nusselt Number At The Bottom And Side Walls As Follows [**10**]:

$$Nu_{av} = \frac{1}{\sqrt{2}} \int_0^{\sqrt{2}} Nu_x \, dH \qquad ....(8)$$

Where , *Dh* Represents The Elemental Height Along The Left And Right Side Walls And Nu_x Is

The Local Nusselt Number Along These Side Walls.

3. Boundary Conditions

The Boundary Conditions Which Are Used In The Present Study Can Be Arranged As Follows:-

1. The Bottom Wall Is Thermally Insulated, So: - Y = 0

$, \frac{\partial q}{\partial Y} = 0$ And U = V = 0.. (9)

2. The Left Side Wall Is Kept At A Uniform Hot Temperature (T_h), So:-

X=0, $q = 1.0$ AndU=V=0…(10)

3. The Right Side Wall Is Kept At A Cold Temperature (T_c), So:-

X=1.0, $q = 0.0$ AndU=V=0 …(11).

4. Numerical Scheme

4. 1 Grid Generation

Numerical Grid Generation Has Now Become A Fairly Common Tool For Use In The Numerical Solution Of Partial Differential Equations On Arbitrarily Shaped Regions. The Coordinate Transformation Technique Advanced By Thompson Et Al. [12] Is Used For The Solution Of Problems Over Complex Geometries. The Transformation Is Obtained From The Solution Of Partial Differential Equations On The Regular Computational Domain. Mapping Is Done To Convert The Regions Having Irregular Shape (Physical Domain) Into The Computational Domain Where The Geometry Becomes Regular (Computational Domain) With A Suitable Transformation [13]. A Curvilinear Mesh Is Generated Over The Physical Domain Such That One Member Of Each Family Of Curvilinear Coordinate Lines Is Coincident With The Boundary Contour Of The Physical Domain. The Navier–Stokes Equations Are Then Solved On The Transformed Plane And The Solution Is Back-Transformed To The Physical Plane. The Transformation Is As Follows [13]:

$x=x(x,y)$; $h=h(x,y)$ (12)

And The Inverse Transformation Is Given By:

$x=x(x,h)$; $y=y(x,h)$ (13)

This Grid Is Obtained By Solving Non-Homogeneous 2-D Poisson Equations [14]:-

$$\xi_{xx} + \xi_{yy} = F(\xi,\eta) \quad (14-a)$$
$$\eta_{xx} + \eta_{yy} = Q(\xi,\eta) \quad (14-b)$$

Eqs. (14-A) And (14-B) Are Then Transformed To Computational Space By Interchanging The Roles Of The Independent And Dependent Variables. This Yields A System Of Two Equations Of The Form:

$$g_{11}x_{\xi\xi} - g_{21}x_{\xi\eta} + g_{22}x_{\eta\eta} = -J^2(Fx_\xi + Qx_\eta) \quad (15-a)$$
$$g_{11}y_{\xi\xi} - g_{21}y_{\xi\eta} + g_{22}y_{\eta\eta} = -J^2(Fy_\xi + Qy_\eta) \quad (15-b)$$

Where:

$$g_{11} = x_\eta^2 + y_\eta^2 \quad (15-c)$$
$$g_{21} = 2\left(x_\xi x_\eta + y_\xi y_\eta\right) \quad (15-d)$$
$$g_{22} = x_\xi^2 + y_\xi^2 \quad (15-e)$$
$$J = x_\xi y_\eta - x_\eta y_\xi \quad (15-f)$$

F And Q Are Functions That Provide Control Of The Mesh Concentration And Their Values Can Be Obtained From Ref [14]. The Values Of P And Q Have To Be Chosen Depending On The Clustering Of The Grid Required For The Problem In Hand. The Transformed Equations (15-A) And (15-B) Are Discretized Over The Computational Plane Using Second-Order Differencing And Then Solved Numerically. The Coefficients In Eq. (15) Are Computed At Each Grid Point. For The Present Study, The Grid Used Is Shown In Figure (2).

4. 2 Solution Procedure

The Preferred Method For The Present Numerical Simulation Is The Two-Dimensional Finite Volume Method Of (Patankar, 1980)[15]. In This Investigation, A Non-Uniformly Collocated Grid Procedure Is Used In Primitive Variables With A Power-Law Differencing Scheme For The Convection Terms Whereas A Central Differencing Is Used To Discretize The Diffusion Terms. The Steady State Governing Equations (2-5) Are Solved By The Finite Volume Method Using Patankar's Algorithm [15] .The Finite Volume Method Is A Method Of Discretization In Space Of The Entire Domain, Which Can Use A Mesh With Finite Number Of Volumes. The Enclosure Divided With Non-Uniform And Non-Orthogonal Grid On The Sub-Volumes. Each Of Control Volume Surround One Nodal Point In Its Center Called Discretization Point. The Pressure Correction Equation Is Derived From The Continuity Equation To Enforce The Local Mass Balance (Ferziger And Peric [16]). Linear Interpolation And Numerical Differentiation (Patankar, [15]), Are Used To Express The Cell-Face Value Of The Variables And Their Derivatives Through The Nodal Values. Discretized Momentum Equations Lead To Algebraic Equation Systems For Velocity Components *U* And *V* Where Pressure, Temperature, Fluid Properties Are Taken From The Previous Iteration Except The First Iteration Where Initial Conditions Are Applied. These Linear Equation Systems Are Solved Iteratively (Inner Iteration) To Obtain An Improved Estimate Of Velocity. The Improved Velocity Field Is Then Used To Estimate New Mass Fluxes, Which Satisfy The Continuity Equation. The Pressure-Correction Equation Is Then Solved Using The Same Linear Equation Solver And To The Same Tolerance. The Energy Equation Is Then Solved In The Same Manner To Obtain A Better Estimate Of The New Solution. Solutions Are Assumed To Converge When The Following Convergence Criterion Is Satisfied For Every Dependent Variable At Every Point In The Solution Domain:-

$$\frac{\sum_i \sum_j \left|\Phi^{new}_{i,j} - \Phi^{old}_{i,j}\right|}{\sum_i \sum_j \left|\varphi^{new}_{i,j}\right|} \leq 10^{-6} \qquad (16)$$

Where Φ Stands For The Dependent Variables *U*, *V*, *P*, And Θ. In This Study The **Sip**-Solver Based On Lower-Upper Decomposition (**Ilu**) [17] Is Used To Solve The Linear Equation Systems. To Avoid Divergence, An Under-Relaxation Parameter 0.55 Is Used For Velocity, 0.2 For Pressure And 0.85 For Temperature.

5. Grid Sensitivity Test

The Numerical Scheme Used To Solve The Governing Equations For The Present Work Is A Finite Volume Approach. It Provides Smooth Solutions At The Interior Domain Including The Corners. The Enclosure Is Meshed With A Non-Uniform Rectangular Grid With A Very Fine Spacing Near The Walls. As Shown In **Figure (2)**, The **2-D** Computational Grids Are Clustered Towards The Walls. The Location Of The Nodes Is Calculated Using A Stretching Function So That The Node Density Is Higher Near The Walls Of The Enclosure.

In Order To Obtain Grid Independent Solution, A Grid Refinement Study Is Performed For Uniformly Heated Left Inclined Wall, $\Theta(X,0) = 1$, Cold Inclined

Wall, Θ(X,Y) = 0 And Adiabatic Bottom Wall Inside Isosceles Triangular Enclosure With $Ra = 10^6$ And $Pr = 0.7$. In The Present Work, Eight Combinations (**40x40, 50x50, 60x60, 70x70, 80x80, 90x90**, **100x100** And **120x120**) Of Control Volumes Are Used To Test The Effect Of Grid Size On The Accuracy Of The Predicted Results. **Figure (3)** Shows The Convergence Of The Average Nusselt Number, $\overline{\mathbf{Nu}}_l$, At The Heated Surface Of The Inclined Wall With Grid Refinement. It Is Observed That Grid Independence Is Achieved With Combination Of **(90x90**) Control Volumes Where There Is Insignificant Change In $\overline{\mathbf{Nu}}_l$ .Therefore, It Is Shown That The High Mesh Refinement Provides A Good Result In The Present Model.

6. Numerical Results Verification

For The Purpose Of The Present Numerical Algorithm Verification, A Laminar Natural Convection Problem Inside The Same Tested Model As Obtained By **Tanmay Et Al.** [10], Where The Effects Of Uniform Heating Of Inclined Walls While The Bottom Wall Is Cold Isothermal On Natural Convection Flows Within A Isosceles Triangular Enclosure. The Comparison Is Made For Various Rayleigh (***Ra***), ($10^3 \leq \boldsymbol{Ra} \leq 10^6$) And Prandtl Number (***Pr***=0.026). The Developed Solution Algorithm Has Been Validated By Performing Calculations For The Average Nusselt Numbers Along The Uniformly Heated Inclined Walls, **Θ(X,Y) = 1** And Cold Bottom Wall, **Θ(X,0) = 0** Are Shown In **Table 1**. The General Agreement Between The Present Computation And The Previous Values By **Tanmay Et Al.** [10] Is Seen To Be Very Well With A Maximum Discrepancy Less Than 1**%.** Further Validation Is Performed By Using The Present Numerical Algorithm To Investigate The Same Problem Considered By **Tanmay Et Al.** [**10**] Using The Same Flow Conditions, Geometry, And The Boundary Conditions But The Numerical Scheme Is Different. Good Agreement Is Achieved Between **Tanmay Et Al.** [**10**] And The Present Numerical Scheme For Both The Streamlines And Temperature Contours As Shown In **Figure (4)**. These Validations Make A Good Confidence In The Present Numerical Model To Deal With The Same Isosceles Triangular Enclosure Configuration Problem For Calculating The Flow Field In The Present Configuration.

7. Results And Discussions:

The Properties Of The Laminar Flow And Thermal Fields Under Steady State Conditions In A Triangular Enclosure Which Is Heated From Left Side Wall Are Examined By Investigating The Effects Of The Rayleigh Number**Ra** .In The Current Numerical Investigation, The Following Domains Of The Dimensionless Groups Are Considered: Rayleigh Number (***Ra***), $\mathbf{10^3}$, $\mathbf{2x10^4}$, $\mathbf{10^5}$ And $\mathbf{10^6}$ While Prandtl Number (**Pr**) = **0.7**.

7.1 Effect of Rayleigh Number:

The Stream Lines And Isotherms At Different Rayleigh Number Ranging From $\mathbf{10^3}$ To $\mathbf{10^6}$ Are Shown In **Figures (5 And 6)** When The Left And Right Side Walls Are Maintained At Hot And Cold Temperatures Respectively While The Bottom Wall Is Considered Adiabatic. The

Temperature Of The Air Near The Hot Left Wall Of The Triangular Enclosure Is Greater Than The Temperature Of The Air Near The Cold Right Wall So The Air Near The Right Wall Have Higher Density Than Those Near The Hot One. Due To This Phenomena, The Air Near The Hot Left Wall Move To Upper Direction And Is Fallen Downward Near The Cold Right Wall Resulting A Rotating Vortices In The Triangular Enclosure. When The Values Of Rayleigh Number Are Small (I.E, When **Ra = 10^3** And **$2x10^4$)** As Shown In **Figure (5)**. The Temperature Contours Are Straight Lines And Approximately Parallel Referring That The Heat Inside The Triangular Enclosure Transfers By Conduction Heat Transfer .Also, It Can Be Noticed That The Temperature Contours Data Are Increased From The Small Values At Right Cold Side Wall To Large Values At Left Hot Side Wall Leading To Verify The Boundary Conditions Of The Present Work. When The Rayleigh Number Data Increases (I.E, When ***Ra*** **= 10^5** And **10^6)** As Shown In **Figure (6)**. The Air Circulation Inside The Triangular Enclosure Is Greatly Dependent On Rayleigh Number Where A Circulation Flow Region Of High Capacity Can Be Noticed Near The Upper Part Of The Hot Left Side Wall Or To The Lower Part Of The Cold Right Side Wall And Moves The Air Towards The Triangular Enclosure Core. Also, A Clear Confusion In Stream Line Contours Can Be Observed As Shown In **Figure (6)**. From The Other Hand, The Effect Of Natural Convection Is Also Appeared In The Temperature Contours Which Begin To Confuse And Move In The Direction Of The Central Part Of The Bottom Wall. Also, When ***Ra*** **= 10^6**, A Minor Vortices Can Be Noticed At The Upper Domain Of The Triangular Enclosure Due To Enhanced Convection From The Hot Left Side Wall Of The Enclosure. Finally, It Can Noticed In All Case Study, That The Stream Function Contours Are Almost Have A Triangular Shape Which Indicating A Higher Intensity Of Flows.

7.2 The Variation Of The Average Nusselt Number :

Figures (7 And **8)** Show The Average Nusselt Number Values Along The Hot Left And The Cold Right Inclined Side Walls Respectively For Different Rayleigh Numbers. From These Figures, It Is Seen That The Average Nusselt Number Increases Clearly With Rayleigh Number. It May Be Noticed Also That The Overall Heat Transfer Rate (Average Nusselt Number) Is Less For The Cold Right Inclined Side Wall As Compared To The Hot Left Inclined Side Wall. The Reason Of This Behaviour Is Due To Large Heat Input For The Hot Left Inclined Side Wall.

8. Conclusions

The Following Conclusions Can Be Found From The Results Of The Present Work:

1- When The Values Of Rayleigh Number Are Low. The Isotherms Are Straight Lines And Approximately Parallel Refering That The Heat Transfer Is Due To Conduction. Also, Rotating Vortices With Small Magnitudes Of The Stream Function Are Observed In The Triangular Enclosure.

2-When The Values Of The Rayleigh Number Increases. The Air Circulation Inside The Enclosure Is Greatly Dependent On Rayleigh

Number. From The Other Hand, The Effect Of Natural Convection Is Also Appeared In The Temperature Contours Which Begin To Confuse And Move In The Direction Of The Central Part Of The Bottom Wall.

3- When **Ra= 10^6**, A Minor Vortices Can Be Noticed At The Upper Domain Of The Triangular Enclosure. Also, It Can Noticed In All Case Study, That The Stream Function Contours Are Almost Have A Triangular Shape Which Indicating A Higher Intensity Of Flows.

4- The Average Nusselt Number Increases With Rayleigh Number. It May Be Noticed Also That The Overall Heat Transfer Rate Is Less For The Cold Right Inclined Side Wall As Compared To The Hot Left Inclined Side Wall.

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NOMENCLATURE:

Symbol	Description	Unit
F	*Control function*	
g	*Gravitational acceleration*	m/s^2
J	*Jacobian of residual equations*	
H	*Height of the triangular enclosure*	m
$\overline{Nu}_b$	*Average Nusselt number along bottom wall*	
$\overline{Nu}_l$	*Average Nusselt number along left side wall*	
$\overline{Nu}_r$	*Average Nusselt number along right side wall*	
P	*Dimensionless pressure*	
p	*Pressure*	N/m^2
Pr	*Prandtl number*	
Q	*Control function*	
Ra	*Rayleigh number*	
T	*Temperature*	℃
T_h	*Temperature of the hot surface*	℃
T_c	*Temperature of the cold surface*	℃
U	*Dimensionless velocity component in x-direction*	
u	*Velocity component in x-direction*	m/s
V	*Dimensionless velocity component in y-direction*	
v	*Velocity component in y-direction*	m/s
X	*Dimensionless Coordinate in horizontal direction*	
x	*Cartesian coordinate in horizontal direction*	m
Y	*Dimensionless Coordinate in vertical direction*	
y	*Cartesian coordinate in vertical direction*	m
W	*Width of the triangular enclosure*	m
Greek Symbols:		
α	*Thermal diffusivity*	m^2/s
β	*Volumetric coefficient of thermal expansion*	K^{-1}
θ	*Dimensionless temperature*	
Φ	*Variable vector*	
ν	*Kinematic viscosity of the fluid*	m^2/s
ρ	*Density of the fluid*	kg/m^3
ξ, η	*Dimensionless body-fitted coordinates*	

Table (1) Comparison of the present average Nusselt number along the uniformly heated inclined walls and cold bottom wall for triangular enclosure filled with air (Pr = 0.7) with those of previous studies.

Ra	Inclined Walls			Bottom Wall		
	Tanmay *et al.* [10]	Present work	Error %	Tanmay *et al.* [10]	Present work	Error %
1000	5.541062	5.530689	-0.187 %	3.7433	3.77002	0.714%
20000	5.553926	5.540587	-0.240%	3.7911	3.72665	-0.439%
100000	5.726385	5.732546	0.107%	3.8520	3.84333	-0.225%
700000	5.942065	5.930985	-0.186%	3.8602	3.86436	0.108%
1000000	5.982470	6.0181381	0.596%	3.8711	3.88259	0.297%

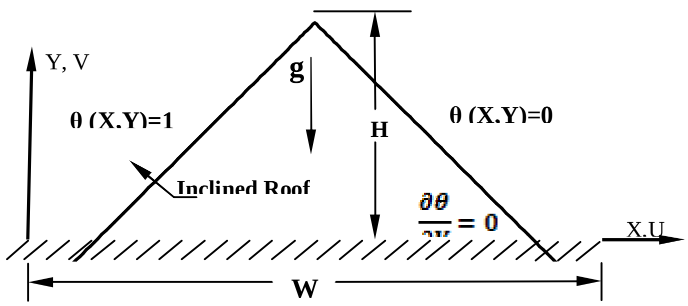


Figure (1) Physical configuration and boundary conditions of the triangular enclosure.

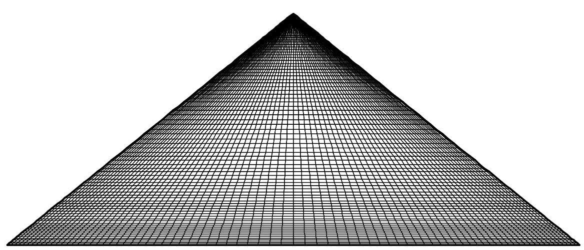


Figure (2) A typical 2D - grid distribution (90x90) with non-uniform and non-orthogonal distributions for the triangular enclosure.

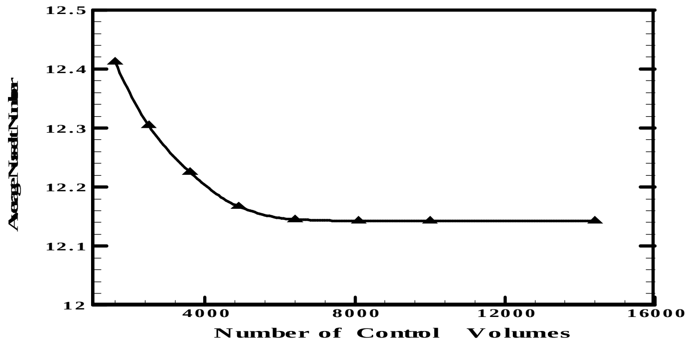
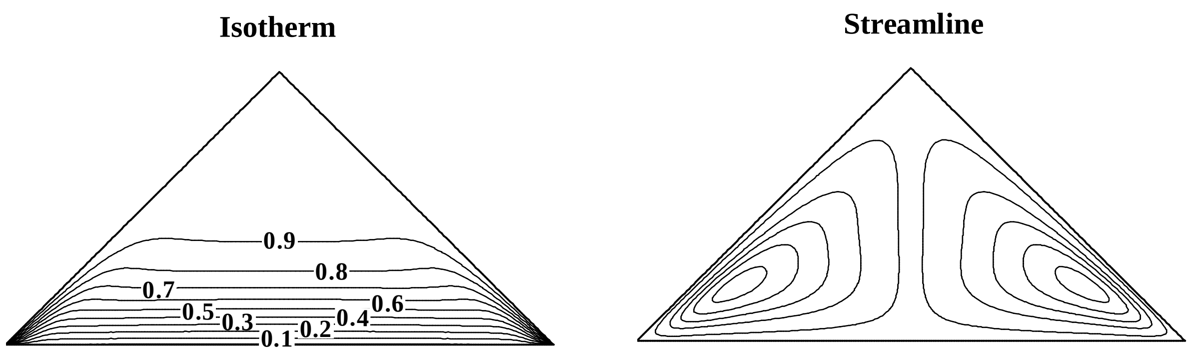
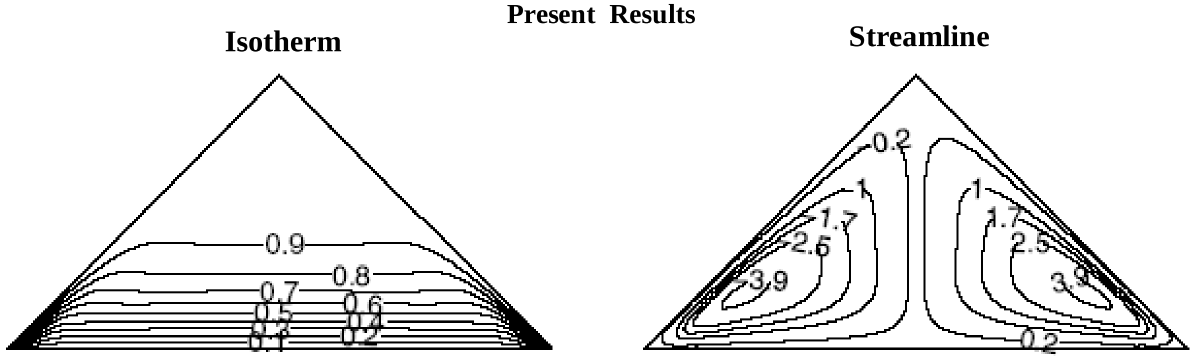


Figure (3) Variation of average Nusselt number along a heated left inclined wall as a function of control volume forIsosceles triangular enclosure with uniformly heated left inclined wall, cold right inclined walls and adiabatic bottom wall at *Ra* = 10^6 and Pr = 0.7.



Present Results



***Tanmy et al. Results* [10]**

Figure (4). Comparison of the isotherms and streamlines between the present work and that of Tanmy et al. [10] for cold bottom wall and uniformly heated inclined walls triangular enclosure filled with air (Pr = 0.7) at Ra = 10^6.

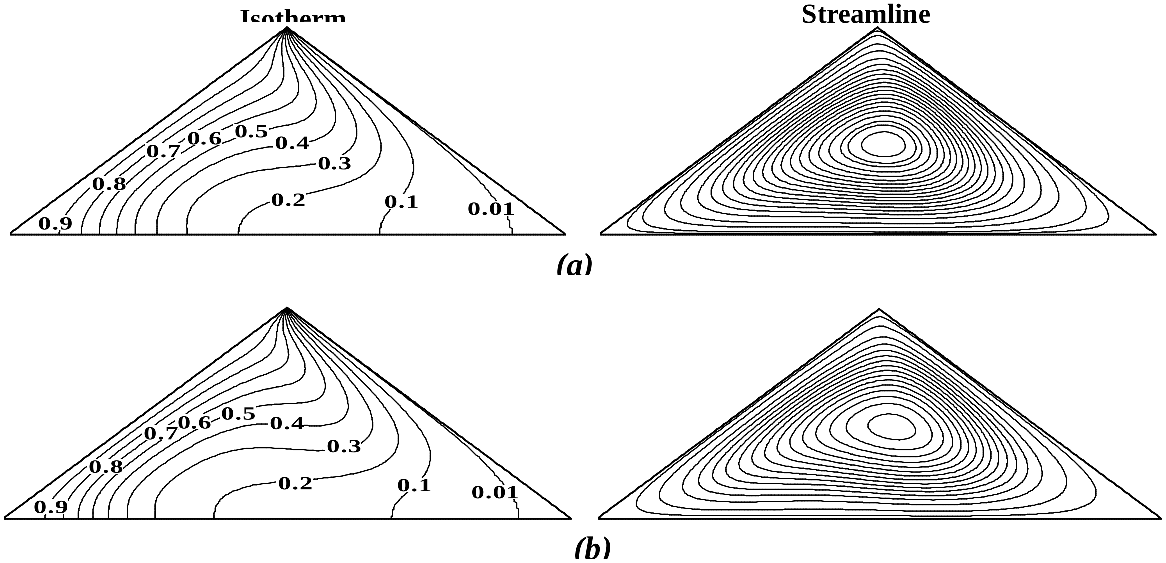
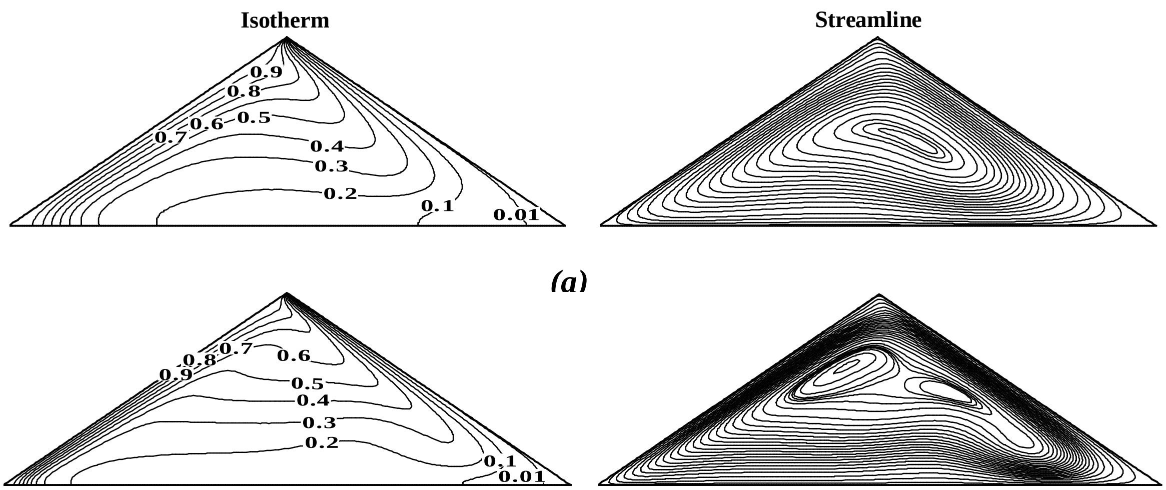


Figure (5) Temperature and stream function contours for uniformly heated left inclined wall, right cold inclined wall and adiabatic bottom wall inside triangular enclosure at *(a) Ra*=10^3, (b) *Ra*=$2x10^4$ and Pr =



(b)

Figure (6) Temperature and stream function contours for uniformly heated left inclined wall, right cold inclined wall, and adiabatic bottom wall inside triangular enclosure at (*a) Ra*=10^5, (b) *Ra*=10^6 and Pr = 0.7.

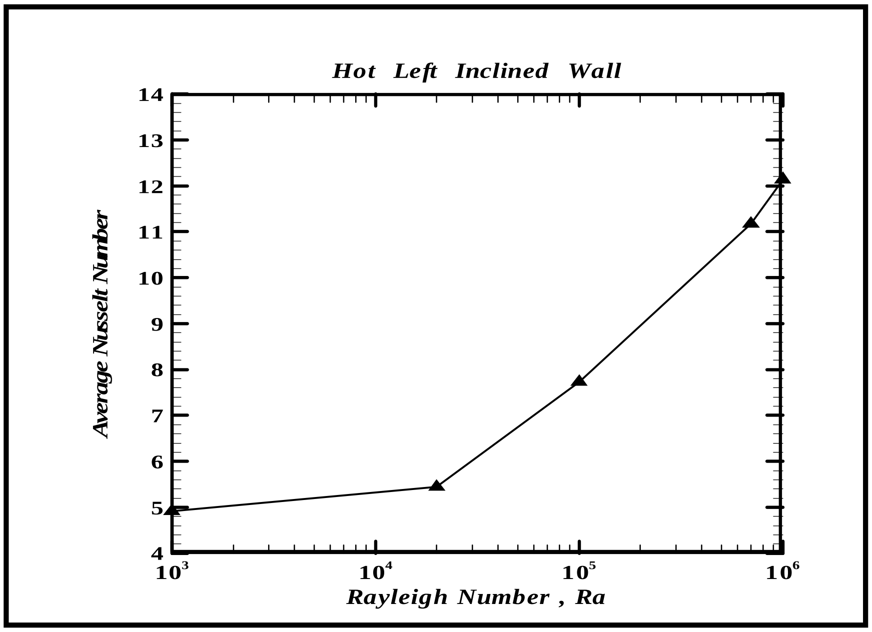


Figure (7) Variation of average Nusselt number along a heated left inclined wall $(\overline{Nu_l})$ with Raleigh number for triangular enclosure with uniformly heated left inclined wall and right cold inclined wall and adiabatic bottom wall at Pr = 0.7.

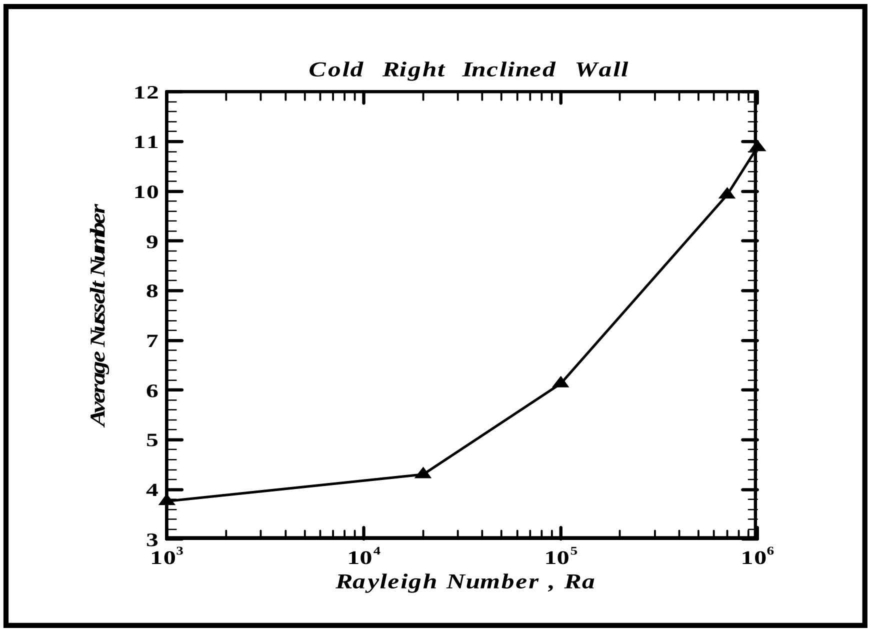


Figure (8) Variation of average Nusselt number along a cold right inclined wall $(\overline{Nu_r})$ with Raleigh number for Isosceles triangular enclosure with uniformly heated left inclined wall, and right cold inclined wall and adiabatic bottom wall at Pr = 0.7.