

Multivariate Multisite Model MV.MS. Reg for Water Demand Forecasting

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Abstract

A new multivariate multi site MV.MS.Reg model is developed in this research depended on regression analysis mixed with Auto regressive multisite Matalas model (AMMM) and used for water demand forecasting. This developed model was applied to Kerkuk city as a case study for long term forecasting of water demand for different types such as domestic demand, industrial, commercial and public demand. This was done by dividing the city into four sites and dividing the total water demand in each site into three types of demand (domestic, industrial with commercial and public demand). Each type of water demand in each site was analyzed by multivariate regression base then the cross correlation between this type of demand for the four sites were included in the model using multi site Matalas model. Many explanatory variables were concluded to be most effective factors affecting different types of demands such as monthly temperature, monthly evaporation, number of residential units, number of industrial and commercial units and number of public units which were forecasted successfully using Stochastic weather generation (SWG) method.

Keywords: Multisite Multivariate, Regression, Matalas, SWG, AMMM

للتنبؤ باحتياجات MV.MS.Reg النماذج المتعددة المواقع والمتغيرات المياه

الخلاصة

تم تطوير نموذج رياضي حديث معتمد على دمج أسلوب Multivariate regression مع أسلوب Auto regressive Multi site Matalas (AMMM) لتخمين احتياجات المياه لمدينة كركوك الواقعة شمال مدينة بغداد. لقد اعتمد النموذج على تقسيم الاستهلاك الكلي للمدينة من المياه حسب المواقع. لذا تم تقسيم الاستهلاك الفعلي للمدينة للسنوات السابقة الى اربعة مواقع. بعدها تم تحليل البيانات الخاصة بهذا الاستهلاك في هذه المواقع لغرض ايجاد المعاملات الضرورية لبناء النموذج الرياضي الذي اعتمد على فصل الاستهلاك الكلي للمياه لهذه المواقع الى ثلاثة انواع (منزلي، صناعي وتجاري، عام). لقد تم ربط كل نوع بمجموعة من العوامل المؤثرة على الاستهلاك من خلال الاستفادة من النموذج (Multivariate regression) عن طريق تقاطع الارتباط لأنواع الاستهلاك في المواقع الأربعة باستخدام أسلوب Multisite Matalas model (Auto regressive) AMMM. لقد تبين بان المعدلات الشهرية لدرجات الحرارة والتبخر وعدد الوحدات المخدومة في كل نوع هي العوامل المؤثرة

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على المتطلبات المائية والتي تم ايضا التنبؤ بها بنجاح باستخدام اسلوب (SWG) .stochastic weather generation

Introduction

Forecasts of future demand help managers and municipalities to plan for future and to assess the adequacy of the present resources to meet future demand. Demand forecasting and resource assessments are critical to water resources planners and managers, since the time required to study, plan and build new resources or implement demand management strategies is lengthy. Demand forecasts aid water suppliers in their ability to visualize future water demand and resource management, rather than a predetermined view of future demand, these forecasts provide supply managers with the tools to understand both the quantity of demand and what controls demand on water resources. Regression models were used extensively by many researchers from 1981 till now especially at 2007. John Parsons, et al who used the multivariate regression models for domestic water forecasting for all authorities in England and Wales. (Richard Hern, et al., 2007) presented regression models also for non residential water demand in the Bristol region from April 2007 until April 2020. (Caiado, Jorge, 2007) examined the daily water demand forecasting performance of double seasonal univariate time series models (Exponential Smoothing, ARIMA and GARCH) based on multi-step ahead forecast mean squared errors. They investigated whether combining forecasts from different methods and from different

origins and horizons could improve Forecast accuracy. They used daily data for water consumption in Spain from 1st January 2001 to 30th June 2006. The performance of the Estimated univariate methods were evaluated by computing MSE statistics for multi-step forecasts from 1 to 7 days-ahead.

Multivariate Regression

The most common approach for modeling water demand relies on multiple regression. Multiple regression is an extension of simple regression analysis which enables analysis of relationships between a dependent variable and two or more explanatory variables. The strength of the analysis lies in the fact that a multitude of analysis can be carried out depending on the nature of relationship and trends between the dependent and explanatory variables. Usually the dependent variable is assumed to be a linear function of more than one independent variables.

(David W. Stockburger, 1998)

Multi site Model for Water demand.

One of the aims of this research is to study the correlation between the water demand or use in different sites of the city this can be done by examining the cross correlation between the demand in multisites. The cross correlation takes into account the correlation in both time and space.

Cross correlation and Auto Regressive Multisite Models:

Cross correlation is a standard method of estimating the degree to which two series are correlated. Consider two series x(i) and y(i) where i=0,1,2...N. The cross correlation r at delay or lag k is defined as

$$R_k(x,y) = \frac{\left[\sum_{i=1}^{NK} \sum_{j=1}^{NK} XY_{i+k} - \sum_{i=1}^{NK} X_i \sum_{j=1}^{NK} Y_{i+k} \right]}{\left[\sum_{i=1}^{NK} X_i^2 - \left(\sum_{i=1}^{NK} X_i \right)^2 / NK \right]^{1/2} \left[\sum_{i=1}^{NK} Y_{i+k}^2 - \left(\sum_{i=1}^{NK} Y_{i+k} \right)^2 / NK \right]^{1/2}} \dots(1)$$

Where R k(x,y) is the lag k cross correlation coefficient between the two series X and Y.(Al-Suhaili, 1985).The most commonly used multisites model is the 1st order autoregressive model given by Matalas:

$$\epsilon_t = A\epsilon_{t-1} + B\xi_t \dots\dots\dots(2)$$

$\epsilon_t, \epsilon_{t-1}$ will be vectors with dimension of $m \times 1$ of the dependent stochastic component at time t,t-1 respectively, m represents the number of sites .

(Al-Suhaili, 1985). ξ_t = is the independent stochastic component at time t. Where A,B are ($m \times m$) matrices and could be found by using the equations below:

$$A = M_1(M_0)^{-1} \dots\dots(3)$$

$$BB^T = M_0 - A(M_1)^T \dots\dots(4)$$

And M_0 is the lag zero cross correlation matrix ($m \times m$), M_1 is the lag one cross correlation matrix($m \times m$) .While A can be estimated directly , there is many assumptions or simplifications to find B .One of them was the

assumption of Young Pisano represented by assuming triangle matrix . B also can be estimated by defining a new matrix, $Z = BB^T$ (Justin.T.Schoof.et.al 2003). Then by spectral decomposition, $Z = CLC^T$, where C is the matrix of eigenvectors of BB^T and L has the eigen values of BB^T on the diagonal and zeros elsewhere. Since $BB^T = Z^{1/2}Z^{1/2T} = Z$, $B = Z^{1/2}$. Then by Greene's Theorem, estimates of B can then be computed as $B = CL^{1/2}C^T$ (Justin.T.Schoof, et.al2003).

The Developed Model (MV.Ms .Reg):

An attempt here was done to mix multivariate regression model which was expressed in the previous section with the multisite auto- regressive Matalas model to reach a single model which treat effect of various variables in different sites of Kerkuk city by using the linear form of multivariate regression model as :

$$W_t = b_0 + b_1X_t + b_2y_t + b_3Z_t + \dots + b_kQ_t + \epsilon_t \dots\dots\dots(5)$$

Where W_t is dependent variable (water demand at time t) modeled as a function of the k explanatory-variables (X,Y,Z,.....Q)

$b_0, b_1, b_2, \dots, b_k$ are regression coefficients describe the fixed coefficients that modify the explanatory variables and ϵ_t is called residual (error) term in this equation.

The Error Term ET:

One of the key assumptions is that ϵ_t is uncorrelated series if ϵ_t is not random, then the series is likely contains information that can be used to further improve the forecast

and additional effort is necessary to refine the model. The method adopted was to use auto regressive multi site (Matalas) model to handle the autocorrelations within the e_t term, with the regression models to describe the explanatory relationship. The resulting model mixed with multisite Matalas model The following equation will describe the development of the model in general :

Let i be the number which describes the type of water demand or use then $i= 1,2,3,\dots,n$ and let j be the number of the sites $j=1,2,3,4,\dots,m$. Then the regression equation will be in the following form:

$$(W_{i,j})_t = (a_{0i,j} + a_{1i,j}X_{1i,j} + a_{2i,j}X_{2i,j} + \dots + a_{ki,j}X_{ki,j} + \dots + e_{i,j})_t \quad \dots\dots(6)$$

$W_{i,j}$ =water demand of type i in the site j , $a_0, a_1, a_2, \dots, a_k$ are regression coefficients and X_1, X_2, \dots, X_k are explanatory variables which affect the i type of demand in site j . The dependent stochastic component for $(W_{i,j})_t$ will be in the form of :

$$C_{i,j,t} = (e_{i,j} - T_{i,j} - P_{i,j})_t \quad \dots\dots(7)$$

where $T_{i,j}$ and $P_{i,j}$ are the trend component and periodicity component respectively.

The multisite first order autoregressive model adopted by Matalas will be used for each i type of demand as:

$$C_{i,t} = A_i C_{i,t-1} + B_i \xi_t \quad \dots\dots(8)$$

Where $C_{i,t}$ and $C_{i,t-1}$ = $j \times 1$ vectors of dependent stochastic components at time t , $t-1$ respectively, and ξ_t = $j \times 1$ vectors of independent stochastic components at time t .

A_i, B_i are $j \times j$ matrices where $A_i = M_{i1}(M_{i0})^{-1}$ and $B_i B_i^T = M_{i0} - A_i M_{i1}^T$

M_{i0} and M_{i1} are the lag zero and lag1 cross covariance matrix ($j \times j$) respectively for type i of water demand and their elements could be found using equation 1. Using the above equations in the forecasting steps, the i type of water demand in each site can be found, then the total water demand in each site will be found using the equation below:

$$W_{j,t} = \sum_{i=1}^{i=n} W_{i,j,t} \quad \dots\dots(9)$$

where: $W_{j,t}$ is the total water demand in the site j and the total water demand of the city can be found using equation below:

$$W_t = \sum_{j=1}^{j=m} W_{j,t} \quad \dots\dots(10)$$

Where: W_t is the total water demand in the whole city.

Data Analysis and Results

The city of Kerkuk was divided first into four sites ,this division was depended on the characteristics of each site .The important points that were taken into consideration to adopt this division were :

- a) Residential unit's density in Each site
- b) The presence and density of industrial and commercial areas.
- c) Economic properties of the site (level of individual income, economic state of residential units, and age of buildings).
- d) Distance from the center of city.as shown in Figure (1) and Table (1).The total water demand in each site of the city was separated according to the kind of use into domestic demand ,industrial with

commercial, and public water demand .This means that the number of demand series in this analysis are(12) series of different kinds of water demand.For this analysis of any kind of water demand there were many variables that could be selected as input variables and by using MatLab's functions in the forward stepwise linear regression model for parameter selection to assist in determining water demand many of the explanatory variables were discarded as they did not significantly add to the ability of the model to predict demand. Equations below are the multivariate regression models for each type of demand in each site of the city these models were found after using the stepwise regression tool .

Regression models For Domestic Water Demand:

Site-1

$$Qd = -1.51572 * 10^6 + 0.218015X_1 + 183.803X_2 + 6750.05X_3 \quad \dots(11)$$

Site-2

$$Qd = -479989 + 0.120853X_1 + 153.495X_2 + 2585.61X_3 + 171.341X_4. \quad \dots(12)$$

Site-3

$$Qd = -717256 + 0.123439X_1 + 140.78X_2 + 3293.33X_3 + 206.538X_4 \quad \dots(13)$$

Site-4

$$Qd = 1.30597 * 10^6 + 0.123481 X_1 + 2584 .83 X_3 + 151 .228 X_4 \quad \dots(14)$$

Where: Qd =domestic water demand,X₁ = previous month water demand,X₂ = no of residential units,X₃ = monthly mean temperature,X₄ = monthly mean evaporation mm.

Regression models For Industrial Commercial Water Demand:

Site1

$$Qinc = -4824.38 + 0.208739X_1 + 33.6186X_3 + 221.803X_4 \quad \dots\dots(15)$$

Site2

$$Qinc = -798.828 + 0.1373015X_2 + 36.9637X_3 + 20.7319X_4 + .86373X_5 \quad \dots\dots(16)$$

Site3

$$Qinc = -28989 + 0.165398X_2 + 59.5171X_3 + 446.761X_4 + 47.2375X_5 \quad \dots\dots(17)$$

Site4

$$Qinc = -2839.54 + 0.169755X_2 + 35.8124X_3 + 65.955X_4 + 6.51996X_5 \quad \dots\dots(18)$$

Where :Qinc= The industrial and commercial water demand, X₁ = previous month water demand,X₂ = previous two months water demand, X₃ = no of industrial& commercial units ,X₄ = monthly mean temperature, X₅ = monthly mean evaporation mm.

Regression models For Public Water Demand:

Site1

$$Qp = 215688 - 0.181036X_2 + 79.8247X_4 \dots(19)$$

Site2

$$Qp = 16816 + 0.133542 X_1 + 44.6634 X_3 + 12.7554 X_4 \dots(20)$$

Site3

$$Qp = 65233.8 + 0.780953X_1 + 6.98285X_5 \dots(21)$$

Site4

$$Qp = 36704.4 + 0.164215X_2 + 3.84139X_4 + 0.396153X_5 \dots(22)$$

Where:Qp=The public water demand, X₁ = previous month water demand ,X₂ = previous two months water demand ,X₃ = no of public units ,X₄ = monthly mean temperature, X₅ = monthly mean evaporation mm.

Results of Domestic or Residential Water Use:

The residuals component of domestic water use for each site can be found from the difference between the actual and theoretical values of the demand.This must be checked to detect trend if any exists .Table(2) shows the results of trend tests on the residual components of domestic water demand for the four sites by using the t-test and finding the linear and non linear correlation coefficients.

The results indicate the absence of trend but by calculating the serial correlation coefficient for these series there were an obvious periodicity which were extracted by suitable method

The Model Parameters

The Auto regressive multisite Matalas parameters for domestic use were estimated and shown below :

$$M0 = \begin{Bmatrix} 1.0000 & 0.0676 & 0.0730 & 0.1026 \\ 0.0676 & 1.0000 & 0.9370 & 0.9211 \\ 0.0730 & 0.9370 & 1.0000 & 0.8517 \\ 0.1026 & 0.9211 & 0.8517 & 1.0000 \end{Bmatrix}$$

$$M1 = \begin{Bmatrix} 0.5155 & -0.1433 & -0.1582 & -0.1027 \\ 0.0761 & 0.3506 & 0.4080 & 0.2866 \\ 0.0334 & 0.2742 & 0.3292 & 0.2111 \\ 0.0811 & 0.2596 & 0.3121 & 0.3045 \end{Bmatrix}$$

$$A = \begin{Bmatrix} 0.5285 & 0.0086 & -0.2346 & 0.0350 \\ 0.0549 & -0.0504 & 0.6277 & -0.2073 \\ 0.0190 & -0.0427 & 0.5688 & -0.2359 \\ 0.0391 & -0.7391 & 0.6046 & 0.4663 \end{Bmatrix}$$

$$B = \begin{Bmatrix} 0.8294 & 0.0421 & 0.0542 & 0.0521 \\ 0.0421 & 0.6273 & 0.4469 & 0.4709 \\ 0.0542 & 0.4469 & 0.7252 & 0.3808 \\ 0.0521 & 0.4709 & 0.3808 & 0.6990 \end{Bmatrix}$$

The independent stochastic components from the four series were extracted by using the same above parameters. The statistical properties of these independent stochastic components for the four sites of the domestic water demand are shown in table(3).

Generation of data

3 series of normally distributed random numbers were generated with length of (10) years this means that the forecasting period will be for 10 years this is was done for all kinds of water demand. By referring to the final models of multivariate regression it seems that the most important variables that have effect on the water use are the climatic factors and the number of residential or industrial with commercial or public units therefore these variables need to be forecasted for the same period which were forecasted using the stochastic weather generation .The number of units (residential or industrial with commercial or public)were increasing linearly .By using the forecasted values of climatic factors and other effective variables in forecasting the domestic water demand as in the final model for each site ,three sets of forecasted domestic water demand for each site were found and plotted as shown in Figures(2-5)

Results Industrial with Commercial Water Demand

As was done with domestic water demand the same way in the analysis was done with this kind of use for the four sites of Kerkuk city after multivariate regression analysis was done ,the residual component was tested for trend as shown in table (4). Table (4) indicates the absences of any kind of trend .

Model Parameters

The Autoregressive-multisite Matalas-parameters for

industrial&commercial use were estimated and shown below :

$$\begin{matrix}
 M0= \\
 \left. \begin{matrix} 1.0000 & 0.0712 & 0.1486 & 0.2072 \\ 0.0712 & 1.0000 & 0.7211 & 0.8279 \\ 0.1486 & 0.7211 & 1.0000 & 0.9385 \\ 0.2072 & 0.8279 & 0.9385 & 1.0000 \end{matrix} \right\} \\
 M1= \\
 \left. \begin{matrix} 0.5686 & -0.0483 & 0.0100 & 0.0605 \\ 0.3266 & 0.4982 & 0.2241 & 0.3524 \\ 0.3630 & 0.3164 & 0.2768 & 0.3145 \\ 0.4330 & 0.3484 & 0.2576 & 0.3447 \end{matrix} \right\} \\
 A= \\
 \left. \begin{matrix} 0.5487 & -0.1854 & -0.2682 & 0.352 \\ 0.3138 & 0.6751 & -0.4572 & 0.1575 \\ 0.3500 & 0.3123 & 0.1270 & -0.1358 \\ 0.3939 & 0.2779 & -0.2710 & 0.2874 \end{matrix} \right\} \\
 B= \\
 \left. \begin{matrix} 0.8109 & -0.0525 & -0.0114 & 0.0028 \\ -0.0525 & 0.6703 & 0.2326 & 0.3193 \\ -0.0114 & 0.2326 & 0.7179 & 0.4606 \\ 0.0028 & 0.3193 & 0.4606 & 0.6239 \end{matrix} \right\}
 \end{matrix}$$

The independent stochastic components from the four series were extracted as was mentioned before in forecasting by using the same above parameters.The statistical properties of these independent stochastic components for the industrial&commercial water demand of the four sites are shown in the Table(5) while the forecasted indusrial&commercial demand are shown in figures(6-9)

Results Public Water Demand

After multivariate regression analysis was done to the public demand the residual components of this kind of water demand from the four sites were tested for trend as shown in table (6) .

Model Parameters

The Auto regressive multisite Matalas parameters for public water demand were estimated and shown below :

$$\begin{aligned}
 &M0= \left\{ \begin{array}{cccc} 1.0000 & 0.6082 & 0.1955 & 0.3896 \\ 0.6082 & 1.0000 & 0.0474 & 0.2717 \\ 0.1955 & 0.0474 & 1.0000 & 0.1769 \\ 0.3896 & 0.2717 & 0.1769 & 1.0000 \end{array} \right\} \\
 &M1= \left\{ \begin{array}{cccc} 0.6145 & 0.4158 & 0.2827 & 0.1280 \\ 0.3466 & 0.6554 & -0.0099 & 0.0630 \\ 0.2294 & 0.1372 & 0.4599 & 0.0376 \\ 0.2853 & 0.1513 & -0.0148 & 0.2519 \end{array} \right\} \\
 &A= \left\{ \begin{array}{cccc} -0.5792 & 0.0974 & 0.1928 & -0.1582 \\ -0.0384 & 0.7099 & -0.0162 & -0.1121 \\ 0.1547 & 0.0535 & 0.4477 & -0.1164 \\ 0.2683 & -0.0561 & -0.0964 & 0.1797 \end{array} \right\} \\
 &B= \left\{ \begin{array}{cccc} 0.6994 & 0.2437 & -0.0266 & 0.1415 \\ 0.2437 & 0.6956 & -0.0211 & 0.1057 \\ -0.0266 & -0.0211 & 0.8636 & 0.0934 \\ 0.1415 & 0.1057 & 0.0934 & 0.9194 \end{array} \right\}
 \end{aligned}$$

The independent stochastic components from the four series were extracted by using same above parameters ,the statistical properties of these independent stochastic components for the four sites of public water demand are shown in Table(7).Four series which represent the public water demand for the four sites are shown in figures(10-13).

The statistical tests on the cumulated demand for each site and for the different types of demand indicated to the success of the applied model to the four sites of the city.

Stochastic weather generation of the Climatic variablesOne of the most important recommended manners by many researchers for generating climate data for future is

the stochastic modeling climatic data as was recommended by (R.Srikanthan&T.A.Mcmahon2001) who indicated that a special characteristic that must be preserved in stochastic modeling climate data is the cross correlation between variables and pointed to the ability of generating annual climate data for a single site by using multi-site type model.
 $X_t = AX_{t-1} + BC_t$,Where X_t $=(p*1)$ vectors of standardized annual climate data for year t
 $C_t = (p*1)$ vector of independent random deviates with zero mean and unit variance .

A,B $= (p*p)$ matrices of constant coefficients to preserve the cross correlations .P = number of climate variables . The coefficient matrices can be obtained as was mentioned before. In this study the climate data that were observed from Kerkuk weather office were all monthly mean values of temperature,-evaporation ,humidity, wind speed sun shine duration ,rainfall.Dependingonthe recommendations of previously mentioned works for choosing variables that having most proper simulation between each other sun shine duration was selected to use it with temperature and evaporation in stochastic weather generation due to high correlation between the three variables.

Conclusions

1.) Separating water demand in different kinds of use gave a clear picture of each kind of water use and gave more accurate and more successful model in forecasting .

2).By using multivariate regression-multisite MV.MS.Reg model in forecasting the monthly value of temperature , monthly evaporation value and number of serviced units in each kind of water use were found to be the most effective factors affecting the water demand.

3)This developed model can be considered as a multivariate multi site model which investigate the cross correlation of different variables in many sites in both space and time.

References

- [1]AlSuhaili.Rafa.H.Shaker, 1985 "Stochastic Analysis Of Daily Stream Flow Of Tigris River "Thesis submitted to College Of Engineering ,Irrigation and Drainage Department ,University Of Baghdad.
- [2]Ciado Jorge 2007"Forecasting Water consumption in Spain Using Univariate Time Series Models"research= report submittedto Department of Economics andManagement,University Lisbl .<http://www.esce.ips.pt/docents/jcaido/papers/Forecasting-SICo2007.pdf>.
- [3]DavidW.Stockburger.1998 "Multivariate Stochastic :Concepts,Models Applications Hand book published at 1998 ,University of Missouri State .
- [4]Justin T. Schoof,Scott M.Robeson .2003 "Seaspnal and Spatial Variations Of Cross-Correlation Matrices Used By Stochastic Weather Generation" Clim Res journa Climate research vol .24:95-102 Atmospheric Science Program ,Department Of,Bloomington,Indiana
- . Geography,Indiana University ,Bloomington,Indiana
- [5]Richard Hern Mathieu Parson,Yues Gueron 2007 "Non Residential demand For Water In Bristol Water Region" A report for Bristol Water NERA Economic consulting. <http://www.bristol.co.uk/pdf/environment/w2008/5/...../>.
- [6]R.Srikanthin and T.A .MC Mahon ,2001"Stochastic generation Of Annual Monthly and Daily Climate Data 'Research report , Hydrology and Earth System Sciences vol5(4) ,653-670.
- [7]JustinT.Schoof,Scott M.Robeson .2003 "Seaspnal and Spatial Variations Of Cross-Correlation Matrices Used By Stochastic WeatherGeneration"ClimRes journal Climate research vol 24:95-102 Atmospheric Science Program ,Department Of Geography,Indiana University.

Table (1) Characteristics of selected four sites of kerkuk city

Site No	Recently no of residential units (2007)	State of Buildings	No of industrial& commercial units (2007)	No of schools & Colleges & (2007)	No of public units (2007)	Average income/capita	Remarks
1	24966	New buildings High class	1506 Industries commerce with high density	60	263 High public services	High	
2	16091	Medium	203 low density	38	25	Medium&low	
3 old city	19104	Very old buildings	3512 High density	223	111 Medium public services	Low	High population density
4	15100	Old buildings	538	14	17 Low service	Low&medium income	High population density

Table (2) Trend analysis for domestic residuals.

Site no	T	Rlinear	Rnonlinear
1	0.3384	0.1668	0.7603
2	0.0936	0.04	0.0797
3	0.0281	0.11	0.014
4	1.6	0.69	0.6

Table (3) The Statistical properties of the independent stochastic Components of domestic demand

Site no	Mean	Std	Cs	Ck
S1	0.0146	0.9519	0.3047	4.627
S2	-0.0368	1.0449	-1.0986	8.95
S3	-0.001	0.9057	1.9498	13.344
S4	0.0838	0.9324	0.1826	2.24

Table (4) the results of trend analysis for industrial & commercial demand

Site no	T	Rlinear	Rnonlinear
1	0.0982	0.049	0.125
2	0.8462	0.38	0.399
3	0.8701	0.398	0.2316
4	1.31	0.54	0.59

Table (5) the statistical properties of independent stochastic components of industrial & commercial demand

Site no	Mean	Std	Cs	Ck
S1	0.0172	0.9794	0.66	3.7
S2	-0.0391	1.0355	0.13131	3.51
S3	0.0096	0.9812	-0.739	5.45
S4	0.0223	1.0036	0.2652	3.48

Table (6) trend analysis results for public water demand

Site no	T	Rlinear	Rnonlinear
1	0.847	0.39	0.96
2	1.2	0.6751	0.75
3	1.719	0.9177	0.86
4	1.14	0.55	0.49

Table (7) The Statistical properties of independent stochastic residuals of public demand

Site no	Mean	Std	Cs	Ck
S1	0.0633	1.0252	0.27	2.7
S2	0.015	0.9679	2.2	16
S3	-0.0049	0.9102	0.18	7.1
S4	0.0169	0.9716	0.27	2.5

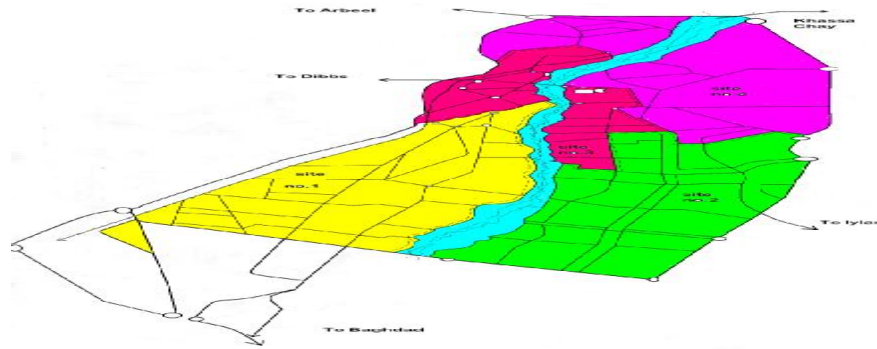


Figure (1) the selected four sites of Kerkuk.

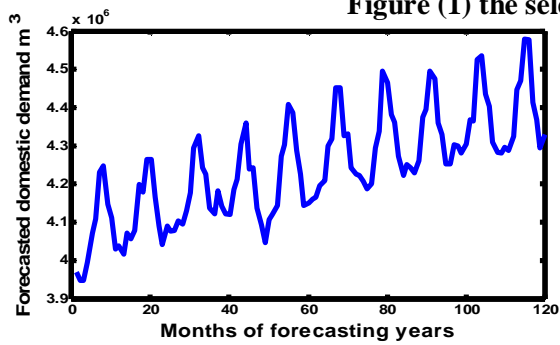


Figure (2) Forecasted domestic demand for Site1.

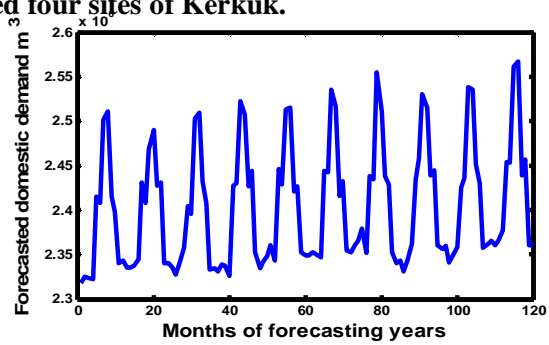


Figure (3) Forecasted domestic demand for Site2

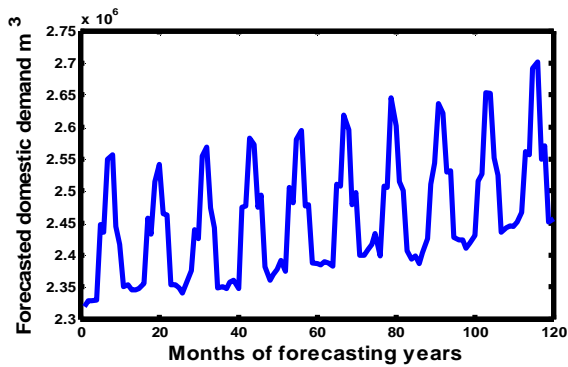


Figure (4) Forecasted domestic demand for Site3.

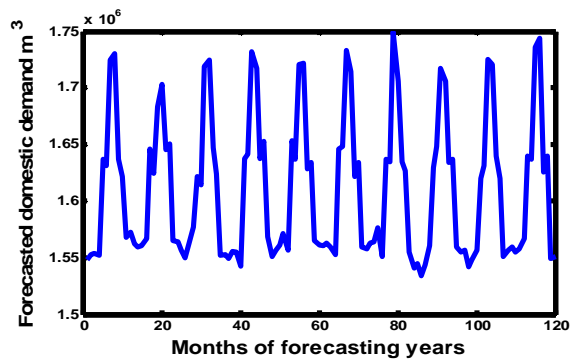


Figure (5) Forecasted domestic demand for Site4.

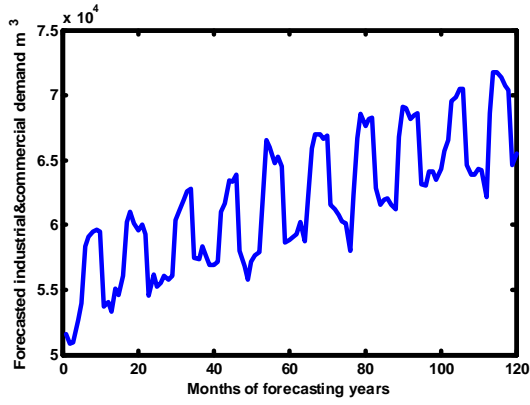


Figure (6) Forecasted demand for Site1

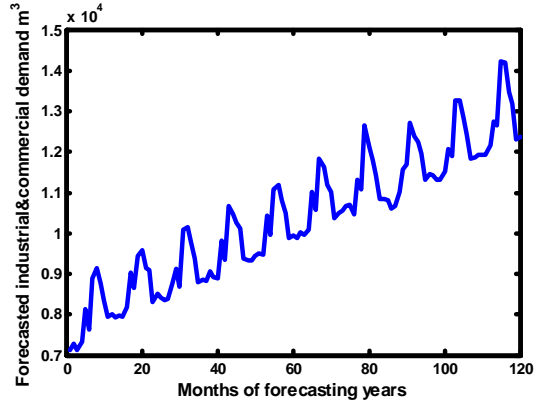


Figure (7) Forecasted demand for Site2

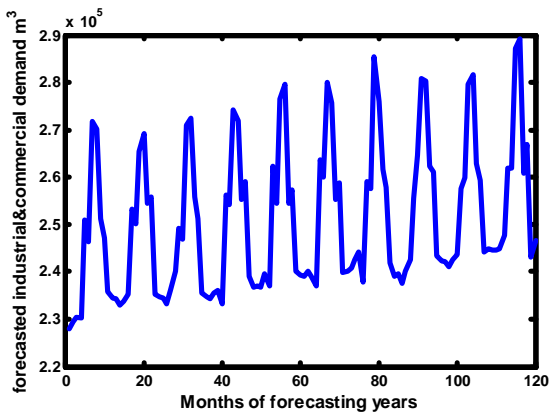


Figure (8) Forecasted industrial & commercial Demand for Site3.

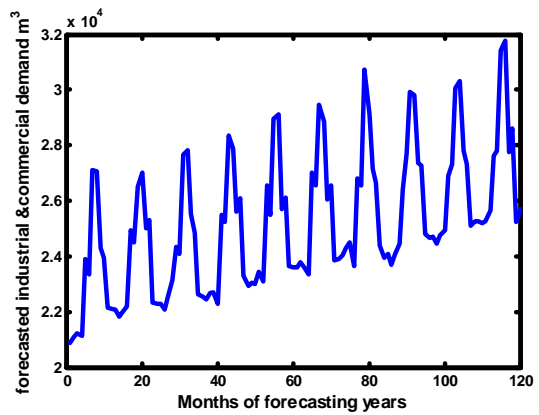


Figure (9) Forecasted demand for Site4.

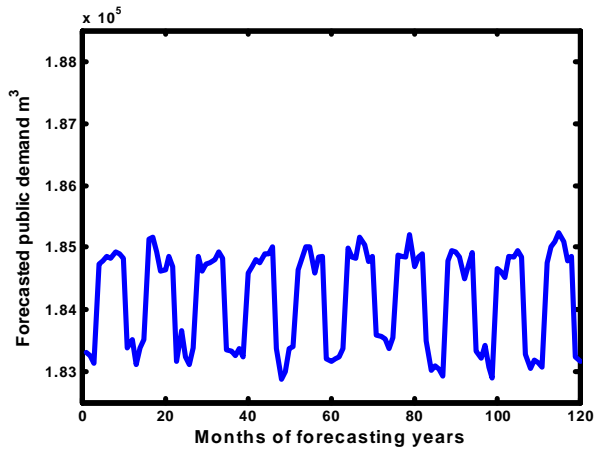


Figure (10) Forecasted public demand for Site1.

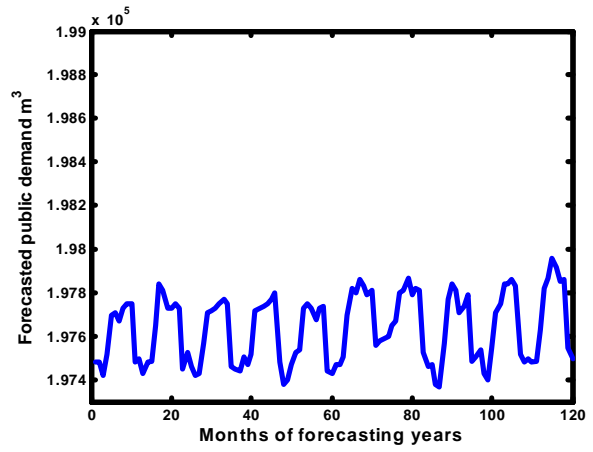


Figure (11) Forecasted public demand for Site2.

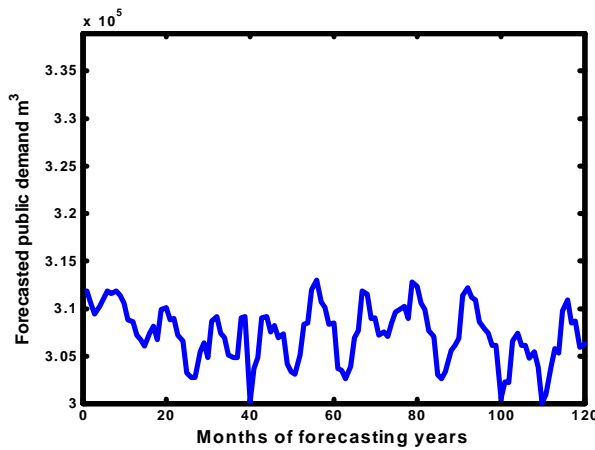


Figure (12) Forecasted public demand for Site3.

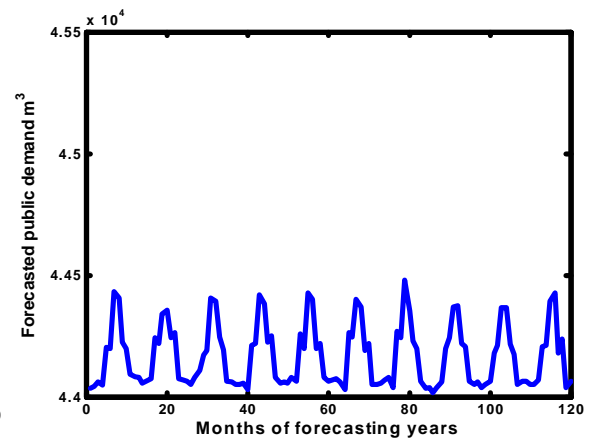


Figure (13) Forecasted public demand for Site 4.