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Flexural Rigidity of Slender RC Columns

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Abstract

166320 hypothetical reinforced concrete (RC) columns, each with a different combination of variables, were used to investigate the major variables that affect the flexural rigidity (EI) of slender RC columns. Using linear regression analysis, new EI expression was statistically developed for 131 slender RC columns. These columns were experimentally tested and available in the literature. This proposed EI expression were introduced into the ACI 318M-05 Code column design procedure to make comparisons between 150 column experimental data with theoretical estimates of the nominal strength using theoretical other methods. These estimates include, in addition to the proposed EI expression, other calculations from the literature.

Keywords: building codes; columns; flexural strength; reinforced concrete; rigidity; slenderness ratio; statistical; structural design.

معامل الجساءة للأعمدة الخرسانية النحيفة

الخلاصة

تم استحداث 166320 عموداً افتراضيا من الخرسانة المسلحة وحللت إنشائيا وتم استخدام النتائج إحصائياً لدراسة العوامل الرئيسية المؤثرة على معامل الجساءة للأعمدة الخرسانية النحيفة. باستخدام التحليل الارتدادي الخطي لـــ 131 عمود خرساني مفحوصة مختبرياً ومتوفرة في المصادر، تم استحداث معادلة جديدة لمعامل جساءة العمود الخرساني. تم مقارنة نتائج الفحص لهذه المعادلة مع عدد من المعادلات المقترحة لبعض الباحثين بالإضافة إلى الكودين العالميين الأمريكي (ACI 318M-05) والبريطاني (BS 8110-97) مع نتائج فحص 150 عمود مختبري. تم إنشاء مخططات تصميمية لهذه المعادلة المقترحة.

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1-Introduction:

Recently slender buildings and slender building components have become more common. At the same time, the development of concrete technology has made possible the production of high strength concrete (HSC) which is mainly used for construction of reinforced concrete (RC) compression members.(3) Thus, there is an increased need for research on RC columns-especially HSC and slender ones. The ACI 318M-05 Code(1) permits the moment magnifier approach for design and analysis of slender RC columns, ACI 318M-05 Chapter 10, Clauses (10.10, 11, 12 and 13), Eqs. 10-11 and 10-12, (Eqs. 1 and 2 in this research). In contrast, the British Standard (BS 8110-97)(4) permits the additional moment approach for design and analysis of slender RC columns, BS 8110-97 (Eq. 32 of this Standards)*. However, these expressions are quite

* $e_{add} = \beta_a\, k\, h$
where $\beta_a = (\ell_e/b')^2 / 2000$ and
$k = (P_o\text{-}P_u)/(P_o\text{-}P_b) \le 1$

approximate when compared with the values derived from load-moment-curvature relationships as indicated in the Commentary of reference (1). A statistical analysis of the ratios of theoretical EI to EI used by the ACI Code for all slender columns studied as part of this investigation confirmed that the variations in the ACI 318M-05 and the BS 8110-97 EI expressions are high. The understanding of slender column behavior has been greatly enhanced during the past 30 to 35 years,(10) and analytical procedures have become available to accurately model the slender RC column strength and stiffness. However, these procedures are generally too complex to be efficiently used in normal calculations in design offices. As a result, several computer studies were conducted to develop EI design equations for slender columns.(2, 3 & 9) However, these studies did not consider the full range of variables that affect the flexural rigidity of slender concrete columns. This work was undertaken to determine the influence of a full range of variables on the effective flexural rigidity of slender tied RC columns. 166320 hypothetical slender square columns, each with a different combination of specified values of variables, were used to determine the major factors that affect the column rigidity of the slender column, Table (1). The EI expressions were then statistically developed using 131 experimentally tested columns available in the literature for use in slender column designs. The studied columns were bent in symmetrical single curvature, in nonsway frames. These were subjected to short-time loads. The moment magnifier approach specified in the ACI Building Code was developed for this type of column. The effects of different end restraints, loading conditions, and lateral supports are accounted for in the ACI Building Code through the use of effective length factor k, equivalent uniform moment diagram factor C_m, and sustained load factor β_d.

The studied columns are graphically represented in Fig. 1. They are similar to those investigated earlier by MacGregor et al.(7) These columns were chosen because the errors in k, C_m and β_d factors would not affect the accuracy of the EI expressions derived in the later part of this paper. Design charts for calculating the effective rigidity factor were presented.

Research Significance

Based on evaluations of the parameters that affect the flexural rigidity of slender RC columns, EI expression is proposed for slender column designs. The ACI 318M-05

* $e_{add} = \beta_a\, k\, h$
where $\beta_a = (\ell_e/b')^2 / 2000$ and
$k = (P_o\text{-}P_u)/(P_o\text{-}P_b) \le 1$

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Code's expressions of the flexural rigidity (Eq. 1) and (Eq. 2) use constant values for calculating the effective flexural rigidity EI for the slender column. This may led to inaccuracies in calculating the effective flexural rigidity (EI).[3]

$$EI = 0.2\, E_c\, I_g + E_s\, I_{se} / (1 + \beta_d) \quad (1)$$

$$EI = 0.4\, E_c\, I_g / (1 + \beta_d) \quad (2)$$

Eqs. (1 and 2) have a significant difference between their values to predict the flexural rigidity EI for the slender column. Eq. (1) is more conservative than Eq. (2) for small values of compressive concrete strength with low quantity of steel ratios. The increasing use of HSC in highrise buildings has increased the need for more accurate strength evaluation of slender RC columns.

A modified version of this expression will take the form:

$$EI = \alpha\, E_c\, I_g + E_s\, I_{se} \quad (3)$$

In which α is a dimensionless reduction factor (effective rigidity factor) which depends on a number of variables that affect the stiffness of slender columns.

Rearranging Eq. (3), the value of α can be expressed as:

$$\alpha = (EI - E_s\, I_{se}) / E_c\, I_g \quad (4)$$

The bending moment relationship at midheight of the column is:[9]

$$M_c = M_{end}\, sec\left(\frac{\pi}{2}\sqrt{P_u/P_c}\right) \quad (5)$$

In which M_c is the bending moment capacity of a cross section which includes second order effects and M_{end} is the applied end moment calculated by a conventional elastic frame analysis.

Eq. (5) can be solved for P_c as:

$$P_c = \frac{\pi^2 P_u}{4\left[sec^{-1}\left(\frac{M_c}{M_{end}}\right)\right]^2} \quad (6)$$

In which $P_c = \frac{\pi^2 EI}{\ell^2}$ (7)

Equating Eqs. (6 and 7) yields:

$$EI = \frac{P_u \ell^2}{4\left[sec^{-1}\left(\frac{M_c}{M_{end}}\right)\right]^2} \quad (8)$$

Analysis of cross section strength

The strength of a RC cross section is represented by an axial load-bending moment interaction curve. In this work, a number of moment-curvature diagrams were generated for various levels of axial load. The maximum moment obtained from the moment-curvature diagram for each axial load level defines one point on the cross-sectional capacity interaction curve. M_c could then be calculated easily for the desired end eccentricity ratio e/h.

To obtain the cross sectional capacity, various expressions have been proposed to describe the stress-strain relationship of concrete. The most significant parameters affecting the shape of the stress-strain curve of confined concrete for all section shapes were the ultimate compressive strength of the concrete, volumetric ratio and the yield strength of the confining reinforcement.[14] Of these, the one suggested by Collins et al [mentioned in Ref (12)] shows a considerable promise, Fig. (2). Accordingly, the concrete stress f_c is related to the strain ε_c by the expression:

$$f_c = k_3\, f'_c\, \frac{\varepsilon_c}{\varepsilon'_c} \cdot \frac{n}{n-1+\left(\frac{\varepsilon_c}{\varepsilon'_c}\right)^{nk_3}} \quad (9)$$

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where f'_c is the cylinder compressive strength and k_3 is the reduction factor to relate f'_c to the in situ concrete strength. Based on test data, Collins et al have recommended that:

$$k_3 = 0.6 + 10 / f'_c \leq 0.85 \quad (9\ a)$$

$$n = 0.8 + f'_c / 17 \quad (9\ b)$$

$$k_1 = 0.67 + f'_c / 62 \quad \text{when } \varepsilon_c / \varepsilon'_c > 1.0 \quad (9\ c)$$

$$k_1 = 1.0 \quad \text{when } \varepsilon_c / \varepsilon'_c \leq 1.0 \quad (3\ d)$$

$$\varepsilon'_c = (f'_c / E_c)\, n / (n-1) \quad (9\ e)$$

and

$$E_c = 3320\sqrt{f'_c} + 6900 \quad (9\ f)$$

The modulus of elasticity of concrete was taken as the function of the peak compressive strength ($E_c = 4700\sqrt{f'_c}$) instead of Collins et al proposal of Eq. (9 f).[2] Noting that Eqs. (9) are expressed in terms of MPa. In addition, the effect of tensile stresses of concrete has been ignored. An elastic perfectly plastic stress-strain curve was assumed for the reinforcing steel. The specified values of reinforcing steel yield strength and modulus of elasticity listed in Table (1) were used for computing the cross-sectional capacities. These assumptions for concrete compressive strength and steel reinforcing bars were used by many researchers.[13]

To investigate whether these assumptions of the theoretical model Eqs. (9) and reinforcing steel strength behavior have accurate results, tests results of 19 experimental short columns (listed in ref. 2) have been compared with the theoretical results obtained by analyzing these short columns applying the models for concrete strength and steel reinforcement.

The theoretical strength-to-test strength ratio ranged from 0.89 to 1.11 with a mean value of 0.999 and a coefficient of variation (COV) that equals 5 %.

Effect of slenderness on column strength

For a slender column bent in single curvature mode under equal eccentricities at both ends, Fig. (3), a second-order parabola has been suggested to represent the shape function of the curvature line between the mid-height and the ends of the column.[10] Quast[9] proposed an expression (Eq. 10 a) to compute the lateral deflection at midheight of the RC slender column which is a conservative approximation of the results obtained from a numerical integration technique with maximum errors of the order of 6 %.

$$\Delta_m = \mathbf{l}^2 (\text{Ø}_m + 0.25\, \text{Ø}_e) / 10 \quad (10\ a)$$

where

$$\text{Ø}_e = P.e / (EI)_{end} \quad (10\ b)$$

$$\text{Ø}_m = P.(e + \Delta_m) / (EI)_{mid} \quad (10\ c)$$

A trial and error solution is required to solve Eq. (10 a). For a given level of axial load P, α is assumed equal to unity at the first iteration.

Substituting Eqs. (10 b & c) into Eq. (10 a):

$$\Delta_m = P\, \mathbf{l}^2 [(e + \Delta_m) / EI_{mid} + 0.25\, e / EI_{end}] / 10 \quad ...(11)$$

Iteration processes will be stopped when Δ_m converged. Using Eq. (4), α can be obtained. The axial load P is

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also decreased as (α *P*) is introduced for the next iteration.

The externally applied end moment was recorded against the curvature at the midheight. The maximum moment from this diagram and the corresponding axial load define one point on the axial load-end moment M_{end} interaction curve of the slender column. A series of these points for different axial load levels defines the entire interaction curve that includes the effect of slenderness in the column strength. M_{end} could then be calculated easily for a desired end eccentricity ratio (e/h).

To check the accuracy of the theoretical strength model for slender columns, the partiality and variability were computed from test data available in the literature. The ratios of calculated to test strengths for 150 available slender column tests[3] ranged from 0.5 to 2 with a mean value of 0.924 and COV of 29.79 %.

Hypothetical Slender RC Columns

In order to investigate the major factors that affect EI of the slender RC column, it is important to study all factors that affect RC columns including material and geometric properties. Table (1) lists the factors which were used for generated hypostatical slender RC columns.

Pin ended columns with equal load eccentricities on both ends acting in the same plane were considered. The analysis used can be divided into two parts: (a) the strength of cross section and (b) the effect of slenderness on column strength.

Simulation of Stiffness Data for Columns Studied

Since the dimensional tolerances in RC cross sections are independent of the size, the deviations in actual strength of a slender column tend to become more significant as the cross section size decreases. This makes the columns with smaller cross sections more critical.[9] A (300 × 300 mm) cross section was chosen in the present study because this size represents the smallest column cross section usually employed in building construction.[10]

166320 columns were used, with each column having a different combination of the specified properties of variables. The specified concrete strengths f'_c and reinforcing steel yield strengths f_y used in this study [listed in Table (1)] represent the usual ranges of these variables employed by the construction industry.[5]

The slenderness ratios ℓ/h selected were intended to approximate the range of ℓ/h for columns in nonsway frames designed according to ACI 318M-05 Clauses 10-11.[1] Eleven end eccentricity ratios e/h ranging from 0.05 to 1.0 were used, as indicated in Table (1). It is noted that the usual e/h values for columns in concrete buildings varies from 0.1 to 0.65.[11] Finally, the longitudinal reinforcement ratios ρ_s and steel arrangements for the column cross sections studied are listed Table (1). The steel ratios used cover the range of ρ_s commonly employed for concrete buildings.[5]

The short time theoretical *EI* for each of the columns studied was computed from Eq. (8) using the interaction diagrams for the cross section and slender column capacities described above. The effective rigidity factor α was then computed for each column from Eq. (4) using the theoretical *EI*. Finally, the simulated column stiffness data were statistically analyzed for examining the current ACI column

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stiffness equations for developing the design equations for *EI* expression proposed later.

Development of proposed design expression for short time *EI*

The effective flexural rigidity of a slender column is strongly affected by cracking along its length and inelastic actions in the concrete and reinforcing steel. The flexural rigidity *EI*, therefore, is a complex function of a number of variables and does not lend itself to the derivation of a unique and simple analytical equation. Multiple linear regression analyses of the simulated theoretical stiffness data were conducted to evaluate *EI* expressions. The linear regression was chosen as a method of analysis since the objective was to develop accurate simple equations for *EI*. The variables used in this regression analysis are listed in Table (1). These variables were considered important because many researches established the effects of these variables on strength and behavior of slender columns.[9] Axial load index $[1 - (P_u/P_o)^2]$ or $[1 - P_u/P_o]$ was suggested by Wood and Shaw,[10] in which the second form was taken as a simplification of the first form.[9] The variables were considered as dependent variables.

Hence, the best statistical results for variables will indicate that these variables can be considered as major variables that affect the flexural rigidity of the slender column, and it will be used for a particular regression analysis for the experimental RC slender columns available to select the best expression for *EI*. Table (3) shows the results obtained from statistical results for 166320 slender columns depending on selected variables.

Multiple linear regression analysis of the simulated theoretical stiffness data (α-values) was conducted and the resulting *EI* expression was developed for each combination of variables listed in Table (3). The prediction accuracy of a regression *EI* equation was based on the standard error[8] (*S.E*), a measure of sampling variability, and the multiple correlation coefficient[8] (r_c), which is an index of the relative strength of the relationship.

Proposed Formula

Based on the results obtained from the linear regression analysis for the 166320 hypothetical RC slender columns, it is found that the parameters ℓ/h, e/h and P/P_o have a significant effect on the flexural rigidity (*EI*) of the slender columns, Table (3). In which Po is the axial compression column strength under pure axial. Another linear regression analysis has been carried out for the available experimentally tested RC slender columns with regard to e/h, ℓ/h and P/P_o and many other variables [such as the steel ratio (ρ), the concrete cover index (γ) and the material property ratio ($\rho\, f_y / f'_c$)] as independent variables with exact effective flexural rigidity factor (α) as a dependent variable. Table (4) indicates that the best results for expressing (α) will be obtained depending on e/h, ℓ/h and $[1-(P/P_o)^2]$ values.

Table (4) shows that the produced variable combinations of case 4 has the highest Multiple Correlation Coefficient (r_c) and the lowest Standard Error (*S.E*) values among the regression results. The corresponding regression expressions obtained from this analysis are:

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$\alpha = 0.3804 - 0.0113\ \ell/h - 1.2916\ e/h + 0.4458\ [1 - P/P_o)^2]$ (12 a)

in which:

$EI_{effective} = \alpha\ E_c\ I_g + E_s\ I_{se}$ (12 b)

Eq. (12 a) shows that the increase in e/h ratio decreases the flexural rigidity *EI* of a slender column. This is expected because a larger e/h value is associated with more cracking of the column cross section. In addition, the increase in the axial load value will decrease the flexural rigidity *EI* of a slender column. The expression also indicates that a decrease in the *EI* value occurs as ℓ/h ratio is increased. This is also expected since the slenderness ratio of the column is proportional to the critical column load.

The effective flexural rigidity factor (α) values computed from Eq. (12 a) are calculated for all experimentally tested columns against the corresponding theoretical values. The values of ($\alpha_{proposed}$ / α_{exact}) have COV of 40 % with mean value of 1.2. It may be noted that the analysis has been conducted for experimentally tested columns with f'_c values ranging between 16.27 and 98 MPa, ℓ/h values ranging between 4 and 27.95, e/h values ranging between 0.0 and 0.4 and P/P_o ranging between 0.12 and 1.09.

For practical purposes, it is important to establish upper and lower limits for Eq. (12) to prevent unrealistic values of α. Inspections of the previous work for effective flexural rigidity of columns under different loading conditions, it is suitable to simplify Eq. (12 a) to:

$$\alpha = 0.38 - 0.011\ \ell/h - 1.3\ e/h + 0.45\ (1 - P/P_o)^2$$
$$0.1 \le \alpha \le 0.85$$ **(13)**

Comparisons with Test Results

The experimental ultimate loads of the selected RC columns have been compared with the nominal load capacity predicted by the methods considered in this work that which available in Ref. (3). For the 150 RC columns available in literature (19 of these columns are short), the specified cylinder compressive strength of concrete f'_c ranged between 16.3 and 98 MPa and the eccentricity (e/h) ranged between 0 and 1.3, the slenderness ratio ($k\ell_u$/h) ranged between 8.4 and 26 [or (ℓ_u/h) is ranged between 4 and 28].

It is important to note that this work assumes all safety factors, strength reduction factors, and stability reduction factors for concrete and steel reinforcement have values of unity for comparison purposes.

For each selected method, the 150 RC experimentally tested columns are analyzed by applying the ultimate extreme compressive strain (ε_{cu}) that equals 0.0025, 0.003, 0.0035 and 0.004 to cover the conservative limits of the obtained results. The modulus of elasticity of concrete used is the ACI 318M-05 {$E_c = 4700\sqrt{f'_c}$ 4700 (in MPa)}.

Table (5) summarizes the selected analytical methods used to compute the nominal strength of the tested columns.

Figs. 4 and to 5 represent the correlation between the calculated and the test nominal strength for the tested 150 RC columns using the different methods considered herein. For all obtained results, as expected, all considered methods tend to lead to unconservative predictions with increasing the ultimate compressive strain of

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concrete ε_{cu}.

The stress intensity factor, Eq. (14) and the ratio of the depth of the stress block to the depth of neutral axis, Eq. (15) suggested by Ibrahim et al. [(6)] are introduced for the method proposed in this work to obtain better results of the column capacity.[(3)]

$$\alpha_1 = 0.85 - (f'_c / 800) \geq 0.725 \qquad (14)$$

$$\beta_1 = 0.95 - (f'_c / 400) \geq 0.700 \qquad (15)$$

for the values of ε_{cu}, Fig. (5) shows that the proposed method leads to lowest COV. Fig. (6) is selected to review the (COV % / COV % $_{max.}$) and the mean values, using the proposed method for all suggested ranges of ultimate concrete compressive strain, ($0.0025 \leq \varepsilon_{cu} \leq 0.004$).

Fig. (6) shows that the use of an ultimate compressive concrete strain of (ε_{cu} = 0.003) for the method proposed in this work will give the best results among all other values of strain levels.

Design Charts

Most of the structural design codes apply design charts to carry out ready-made quick design of a given RC element. RC columns are the most critical members in concrete structural design applications. For this purpose, the equation proposed in this work for the effective flexural rigidity factor (α) [Eq. (13)] is graphically presented in design charts. Design charts are suggested to predict the column effective flexural rigidity factor and to make a general view of this equation in addition to facilitate the use of the proposed equation. These charts are represent (α) with respect to e/h and P/P_o for ℓ/h values ranges between 5 and 30, Figs (7 a to 7 g).

Conclusions

This study has been conducted to investigate existing code methods and some suggested methods by researchers related to analysis and design of RC columns. The study involved 150 experimentally tested rectangular NSC and HSC columns available in the literature subjected to short-term concentric and eccentric loading. The selected specimens cover short columns and slender columns that buckle in nonsway and sway modes. Also 166320 hypothetical rectangular RC slender columns, each with a different combinations of specified values of variables were used to generate the stiffness data to investigate the major variables that affect the load carrying capacity and the behavior of the columns. A new proposed expression for the flexural rigidity (*EI*) of a RC column was statistically derived. With strength reduction factors set to unity, comparisons were made between test results of the nominal strength for the experimentally tested columns with the results obtained using the considered methods and the method proposed in this work.

Five methods are considered in this work to predict the nominal capacity of the RC columns. These are the universal Codes ACI 318M-05 Code and BS 8110-97 Standards and some methods suggested by different researchers in addition to the method proposed in this work. From observations and studies of results presented in this research, the following conclusions are drawn:

1. The ACI 318M-05 Code provisions underestimate the column capacity. The ultimate compressive strain of concrete of 0.003, used in this code, led to a mean value of P_{cal} / P_{test} that equals 0.83 with corresponding COV that

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equals 30.2 percent.

2. Introducing Rangan and Mirza methods for calculating the effective flexural rigidity of the column into the ACI 318M-05 Code provisions will result in mean value of 0.9 and COV equal to 29 percent for all values of assumed ultimate concrete compressive strain. The Chen et al method resulted in a higher COV and conservative predictions of the nominal strength of the RC columns, (COV = 36 percent, mean value equals 0.8).
3. The method proposed in this work has better prediction of the nominal capacity of the RC columns resulting in mean value that equals 1.0 with the smallest COV value among all other considered methods which is equal to 23.6 percent.
4. Using the proposed design method has proved to be insensitive to the value of concrete ultimate compressive strain, ε_{cu}. The COV values were 23.7, 23.6, 23.7 and 23.8 percent respectively for ε_{cu} values of 0.0025, 0.0030, 0.0035 and 0.0040.

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Notations

e	eccentricity of compressive loading on a column.
e / h	eccentricity ratio.
E_c	moduli of elasticity of concrete.
EI	flexural rigidity of the column.
E_s	moduli of elasticity of reinforcing steel.
f'_c	specified compressive strength of cylinder concrete.
f_y	specified yield strength of reinforcement.
I_g	moments of inertia of gross concrete cross section taken about centroidal axis of cross section.
I_{se}	moments of inertia of steel reinforcement taken about centroidal axis of cross section.
ℓ	height of the column.
ℓ / h	slenderness ratio.
M_c	factored moment to be used for design of compression member.
P_c	critical load, Euler load, the axial force that is sufficient to keep the member in such a slightly bent form.
P_o	nominal RC cross sectional column capacity under pure axial Load.
P_u	factored axial load at given eccentricity.
α	a dimensionless reduction factor (effective rigidity factor) which depends on a number of variables that affect the stiffness of slender columns.
α_1	the intensity of stress at the extreme fiber of stress block.
β_1	the ratio of the depth of the stress block to the depth of neutral axis.
β_d	creep factor, (a) for nonsway frames, β_d is the ratio of the maximum factored axial sus-

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tained load to the maximum factored axial load associated with the same load combination; (b) for sway frames, β_d is the ratio of the maximum factored sustained shear within a story to the maximum factored shear in that story.

Δ_m lateral deflection at mid-height of the column.

ε strain.

$Ø_e$ curvature at column ends.

$Ø_m$ curvature at mid-height of the column.

Table (1)-Specified properties of columns studied

Properties	Specified Values	No. of Specified Values
f'_c (MPa)	20; 30; 40; 50; 60; 70; 80	7
f_y (MPa)	150; 200; 300; 400	4
ℓ / h	8; 10; 20; 30; 40	5
e / h	0.05; 0.1; 0.2; 0.3; ….;1.0	11
ρ_t	Table (2) for combinations of steel ratios and bars arrangements	12
P / P_o	0.1; 0.2;…..; 0.9	9
Total Number = 7 × 4 × 5 × 11 × 12 × 9 = 166320 columns.		

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Table (2)-Longitudinal reinforcement details of selected hypothetical columns

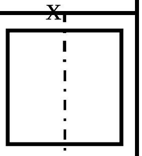
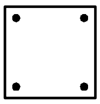
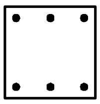
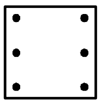
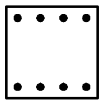
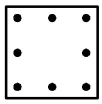
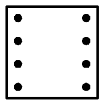
Steel Ratio ρ_t %, b = h = 300 mm.						
Bar Size (mm)	**No. of Bars**					
	4	**6**	**6**	**8**	**8**	**8**
16	**0.89**	**1.34**	**1.34**	**---**	**---**	**---**
20	**---**	**---**	**---**	**2.79**	**2.79**	**2.79**
22	**---**	**---**	**---**	**3.38**	**3.38**	**3.38**
25	**---**	**---**	**---**	**4.36**	**4.36**	**4.36**
 x-x is the axis of bending	Arrangement of Longitudinal Reinforcement					
						

Table (3) Variable Combinations used for regression analyses

Combination No.	Variables considered in the linear regression analysis for the 166320 RC slender columns					Standard error (*S.E*)	Multiple correlation coefficient (r_c)
	e/h	$1-(P_u/P_o)^2$	$1-P_u/P_o$	ℓ/h	$\rho f_y / f'_c$		
1	x		x	x	x	0.086	0.64
2	x		x		x	0.070	0.54
3	x	x				0.080	0.67
4			x		x	0.058	0.50
5	x	x		x		0.053	0.78
6	x			x		0.056	0.80

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Table (4) Statistical results obtained using a linear regression analysis for the tested 131 slender columns ($\alpha_{cal.}$)

Case	Inter-cept	Factors					r_c	S.E
		ℓ/h	e/h	γ^*	1-(P/P$_o$)	1-(P/P$_o$)^2		
1	0.2908	x	x	x		x	0.510	0.143
		-0.011	-1.553	0.150		0.650		
2	0.3941	x	x				0.377	0.165
		-0.002	-0.500					
3	0.3941	x	x		x		0.595	0.143
		-0.011	-1.583		0.640			
4	0.3804	x	x			x	0.621	0.080
		-0.011	-1.292			0.446		
5	0.4480	x	x	x			0.605	0.142
		-0.005	-0.502	-0.001				

* γ = concrete cover index = (h – 2d′) / d

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Table (5) Summary of methods used for analyzing the 150 RC experimentally tested columns

No.	Method	Column Cross Section		Slender Column-Magnification Method
		α_1	β_1	
1	ACI 318M-05(1)	0.85	$0.65 \leq 0.85-(f'_c - 28)/140 \leq 0.85$	$\alpha = 0.2$
2	BS 8110-97(4)	0.85*	0.9	Ref (4)
3	Rangan(13)	0.85	$0.65 \leq 0.85-(f'_c - 28)/140 \leq 0.85$	$\alpha_{rangan} = 0.6 + (e_b / 8 e) \leq 1.0$
4	Chen et al.(16)	0.85	$0.65 \leq 0.85-(f'_c - 28)/140 \leq 0.85$	$EI_{effective} = M_n / (\alpha_{Chen}\, Ø_{yc})$ $\alpha_{Chen} = 2 + 0.2\ (e/h)$, $Ø_{yc} = (0.7 + 2.8\ \xi) \times 10^{-3} + f_y / E_s$
5	Mirza(10)	0.85	$0.65 \leq 0.85-(f'_c - 28)/140 \leq 0.85$	$\alpha_{Mirza} = [0.27 + 0.003\ (\ell/h) - 0.3\ (e/h)] \geq 0$
6	Proposed	Eq. 14	Eq. 15	Eq. (13)

* The BS Standards depends on the cube concrete compressive strength. The cylinder concrete compressive strength is equals 0.788 of the cube strength. For the cube $\alpha_1 = 0.67$.

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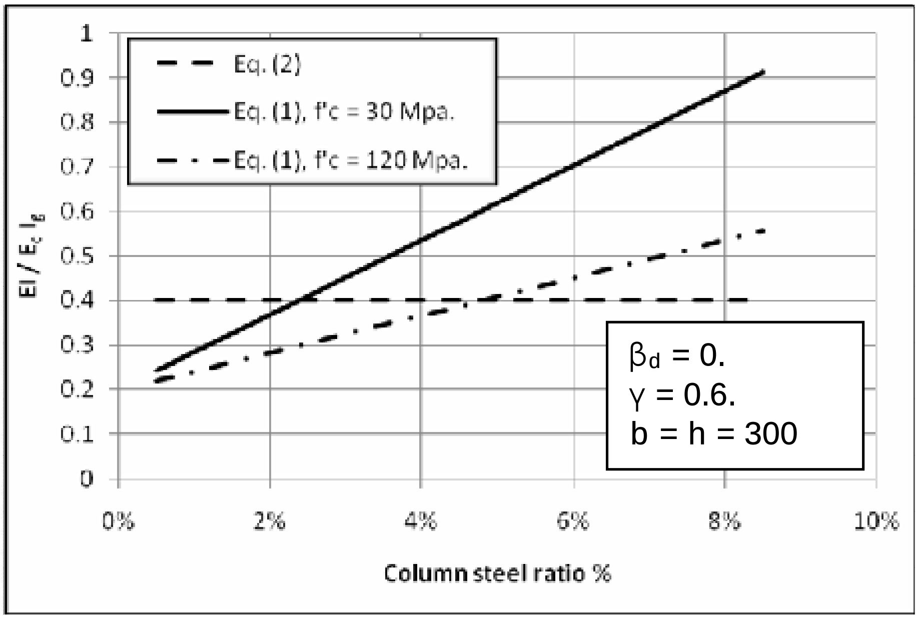

Figure (1) Effect of f'_c (MPa) and column steel ratio on the *EI* of slender RC columns for Eqs. (1 and 2).

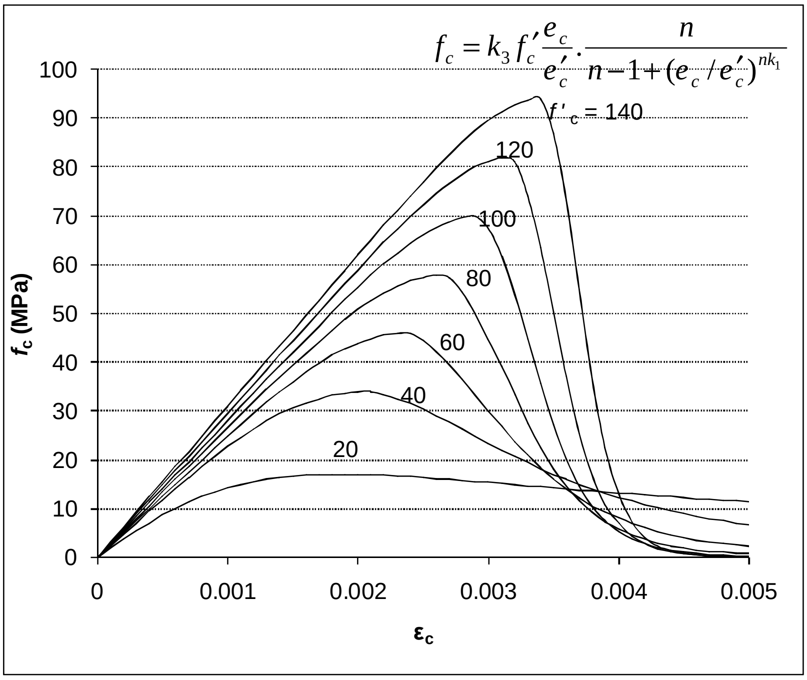

Figure (2) Typical compressive stress-strain curves for normal density concrete proposed by Collins et al.[15]

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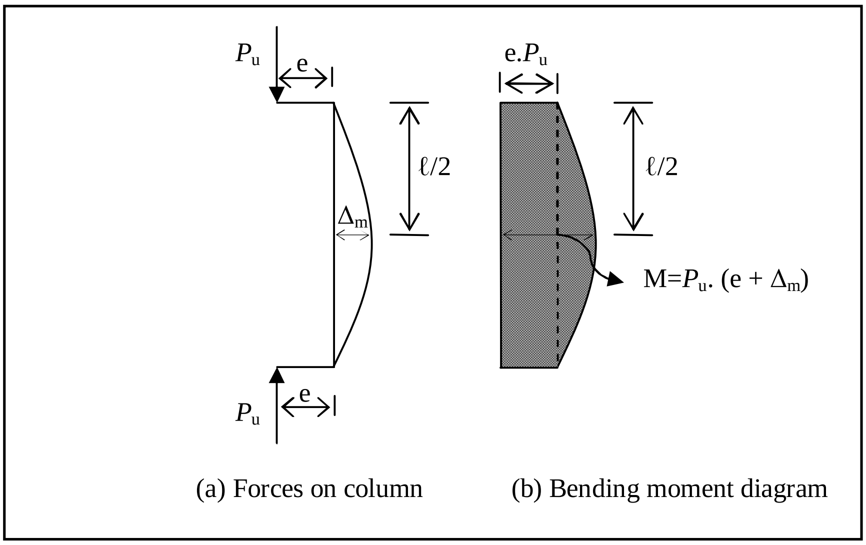

Figure (3) Type of columns studied

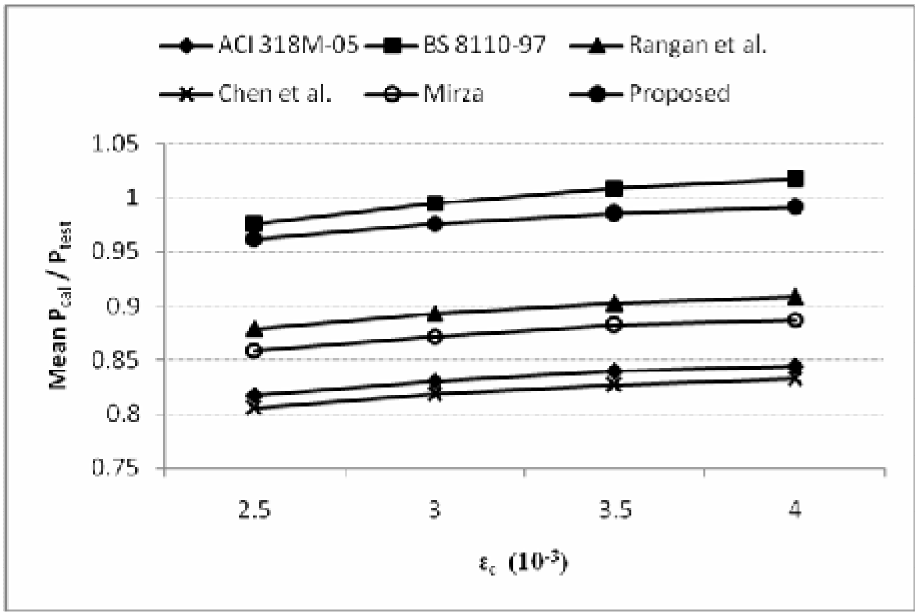

Figure (4) Mean values of P_{cal}/P_{test} obtained for the 150 RC experimentally tested columns analyzed by the considered methods.

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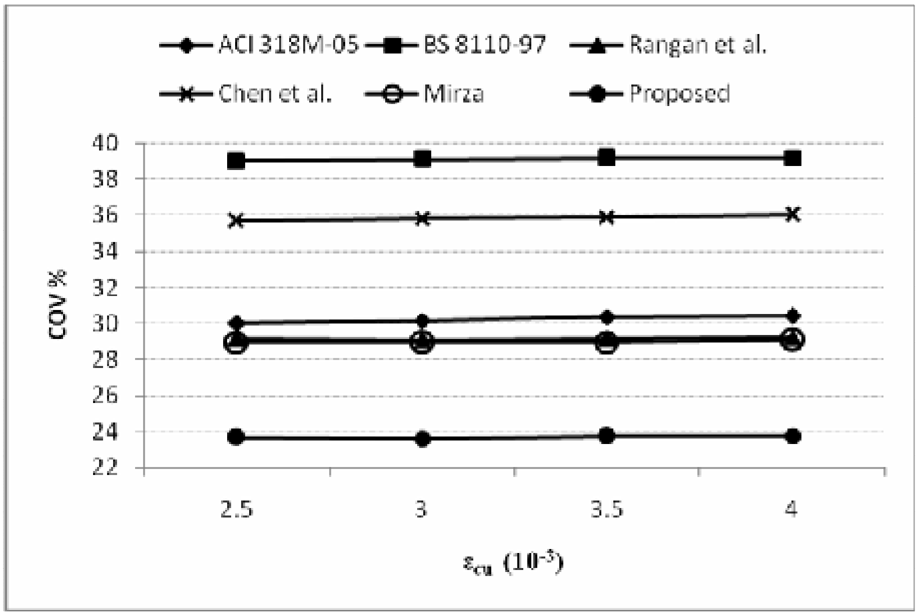

Figure (5) COV values of P_{cal}/P_{test} obtained for the 150 RC experimentally tested columns analyzed by the considered methods.

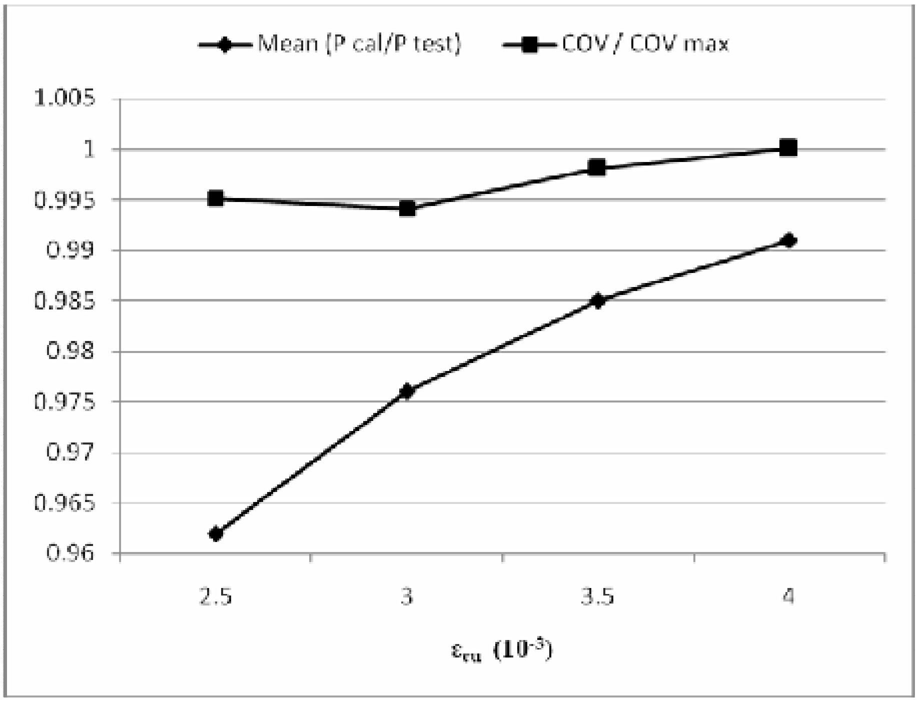

Figure (6) COV values of P_{cal}/P_{test} obtained for the 150 RC experimentally tested columns analyzed by the proposed method.

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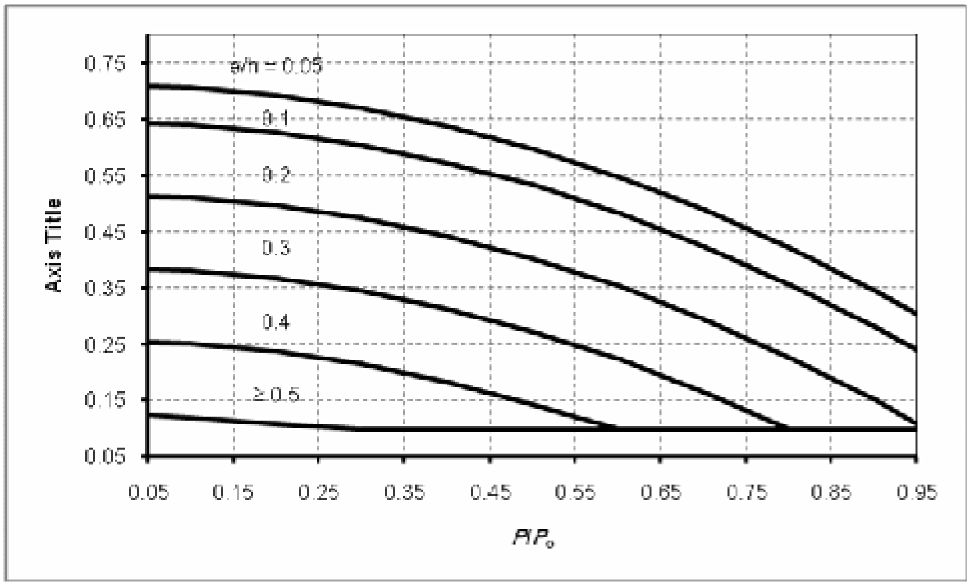

Figure (7 a) Effective flexural rigidity factor (α) for ℓ/h = 5.

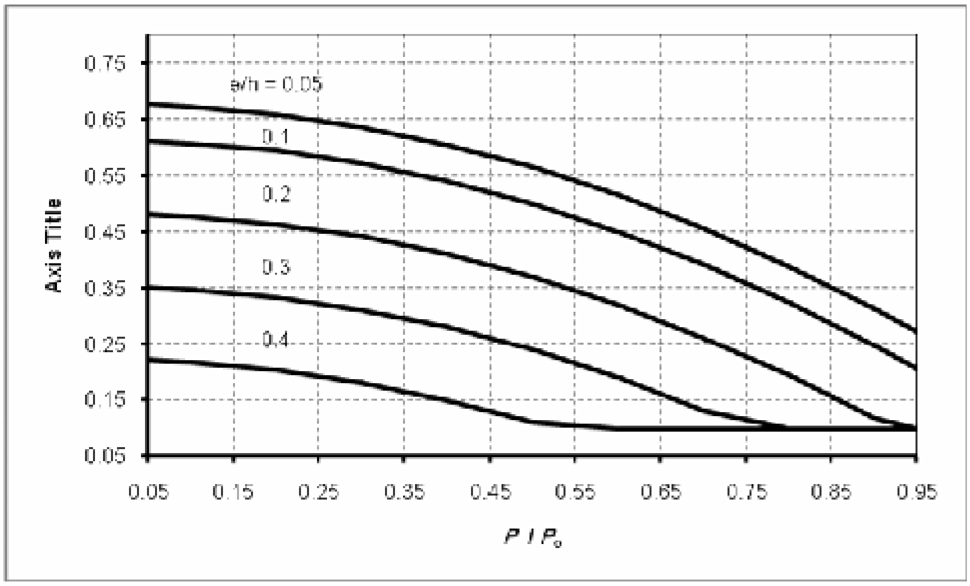

Figure (7 b) Effective flexural rigidity factor (α) for ℓ/h = 8.

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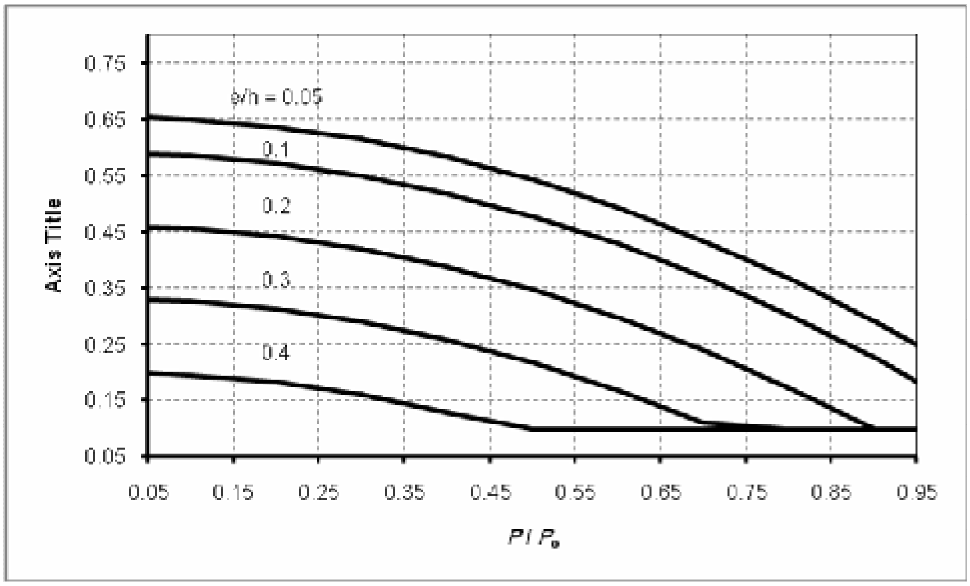

Figure (7 c) Effective flexural rigidity factor (α) for ℓ/h = 10.

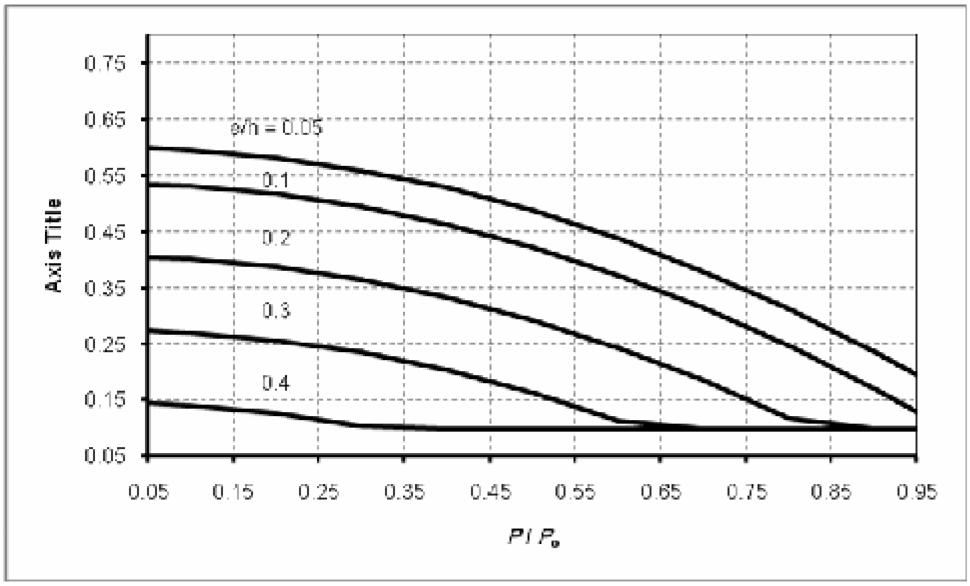

Figure (7 d) Effective flexural rigidity factor (α) for ℓ/h = 15.

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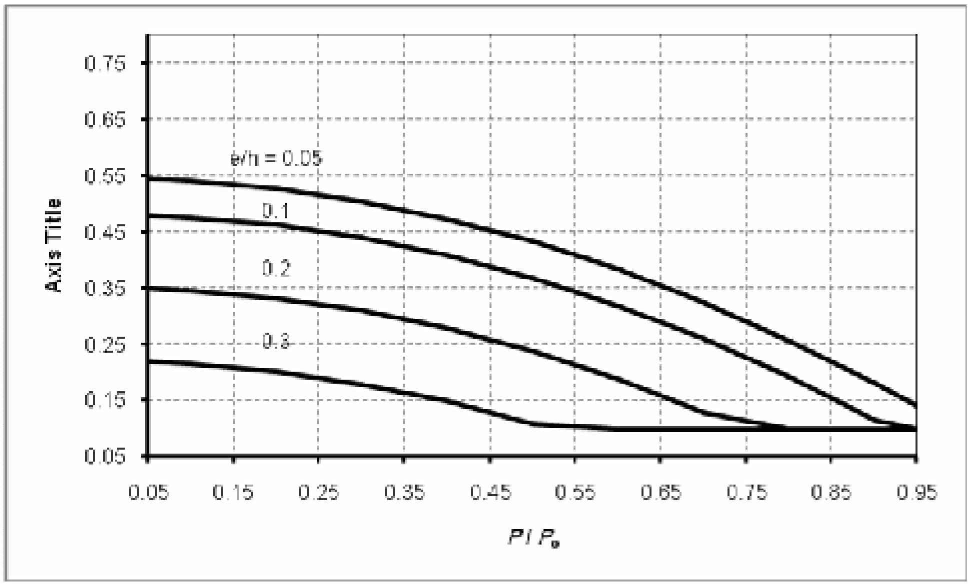

Figure (7 e) Effective flexural rigidity factor (α) for ℓ/h = 20.

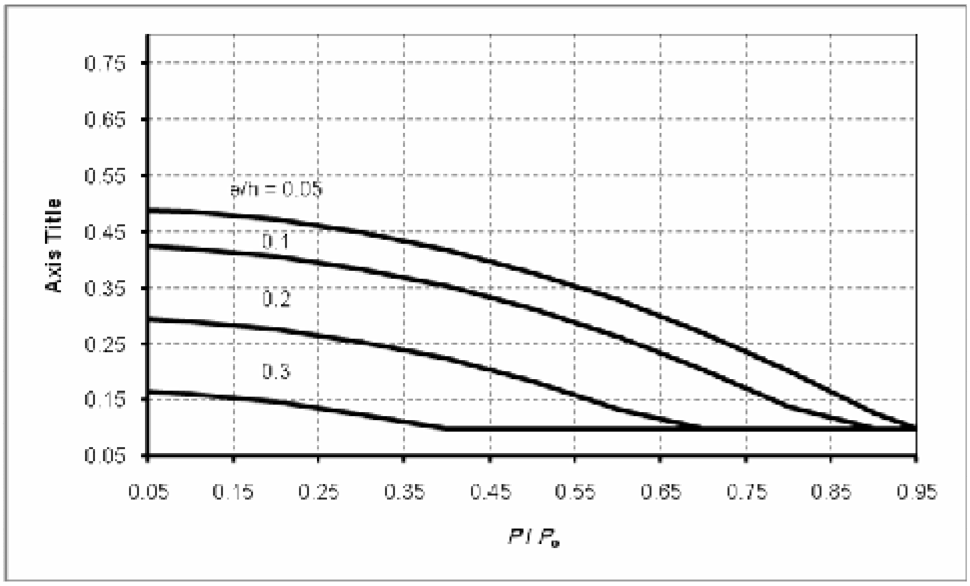

Figure Fig. (7 f) Effective flexural rigidity factor (α) for ℓ/h = 25.

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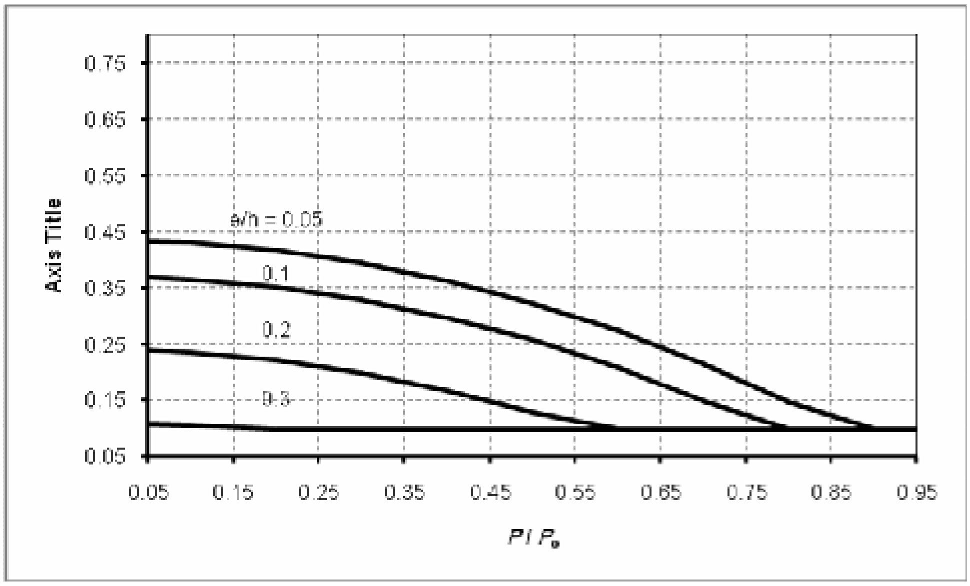

Figure (7 g) Effective flexural rigidity factor (α) for ℓ/h = 30.

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