Complex Discrete Wavelet Transform-Based Image Denoising

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Abstract

Dual tree complex discrete wavelet transform is implemented for denoising as an important image processing application. Two wavelet trees are used, one generating the real part of the wavelet coefficients tree and the other generating the imaginary part tree.

A general computer program computing two dimensional dual tree complex wavelet transform is written using MatLab V.7.0. for a general (NxN) two dimensional signal.

This paper introduces firstly a proposed method of computing one and two-dimensional dual tree complex wavelet transform .The proposed method reduces heavily processing time for decomposition of image keeping or overcoming the quality of reconstructed images. Also, the inverse procedures of all the above transform for multi- dimensional cases verified.

Secondly, many techniques are implemented for denoising of gray scale image. A new threshold method is proposed and compared with the other thresholding methods. For hard thresholding, PSNR gives (13.548) value while the PSNR was increased in the proposed soft thresholding, it gives (14.1734) PSNR value when the noise variance is (20).

Denoising schemes are tested on Peppers noise image to find its effect on denoising application. The noisy version has SNR equals to (11.9373 dB), the denoising image using WT has SNR equals to (17.4661 dB), the denoising image using SWT has SNR equals to (18.1459 dB), the denoising image using WPT has SNR equals to (19.3640 dB), the denoising image using Complex Discrete Wavelet Transform has SNR equals to (21.9138 dB) using hard threshold and has SNR equals to (22.1393 dB) using soft threshold. Matlab V.7.0 is used for simulation.

Keywords: Complex Discrete Wavelet, Image Denoising, Thresholding

تحويل المويجة المركب لازالة التشويش من الصور الرقمية

الخلاصة

تحويل المويجة المركب ذا الهيكل المزدوج تم استخدامه لازالة التشويش كتطبيق مهم في معالجة الصور الرقمية. يستخدم تحويل المويجة المركب ذا الهيكل المزدوج هيكل مزدوج من المرشحات الحقيقية للمويجة احداهما لتوليد الاجزاء الحقيقية لمعاملات المويجة المركبة والاخر لتوليد الاجزاء الخيالية لمعاملات المويجة المركب. تم كتابة برنامج حاسوبي عام لتنفيذ تحويل المويجة المركب ذا الهيكل المزدوج ثنائي الابعاد باستعمال برنامج MatLab يمكن استخدامه لجميع الاشارات ثنائية الابعاد بحجم (NxN).

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يُقدّمُ هذا البحث أولاً طريقة مقترحة لحساب تحويل المويجة المركبة ذا الهيكل المزدوج ذو البعد الواحد وذو البعدين. الطريقةَ المُقتَرَحةَ تُخفضُ كثيراً من زمن المعالجة للصورة المتحللة وتَبقي او تحافظ على تركيب الصورِ المُعَادة. أيضاً التحويلات المعكوسة لكُلّ ما قيل سابقاً من التحويل في الإبعاد المتعددة للحالات تم التحقق منها .

ثانياً، نفذت العديد من التقنيات لرفع التشويش من الصور ذات اللون الرمادي. واستخدمت طريقة جديدة لتحديد حد العتبة وقورنت مع طرق تحديد العتبة الأخرى. بالنسبة لتحديد العتبة الصلب (Hard thresholding)، كانت نسبة الـ(PSNR) هي (13.5483dB) عندما كان توزيع التشويش هو (20). الـ(PSNR) ازدادت هذه النسبة مع تحديد العتبة المقترح (Soft thresholding)، فقد أعطى قيمة (PSNR) مساوية إلى (14.1734dB) بالوقت الذي كان فيه توزيع التشويش هو (20).

البعض مِنْ تقنيات رفع التشويش تم اختبارها على صورةِ الفلفلِ حيث أن الحالة المشوشة لهذه الصورة لها قيمة نسبة الإشارة على الضوضاء (SNR) مساوية الى (11.9373dB). الصورة المخمنة العائدة من رفع التشويش باستخدام التحويل المويجي (WT) كانت قيمة الـ(SNR) مساوية إلى (17.4661dB) , الصورة المخمنة العائدة من رفع التشويش باستخدام ثابتة التحويل المويجي(SWT) كانت قيمة الـ(SNR) مساوية إلى (18.1459dB) , الصورة المخمنة العائدة من رفع التشويش باستخدام حزمة التحويل المويجي(WPT) كانت قيمة الـ(SNR) مساوية إلى (19.3640dB), واخيرا الصورة المخمنة العائدة من رفع التشويش باستخدام تحويل المويجة المركب كانت قيمة الـ(SNR) مساوية إلى (21.9138dB) عند استخدام تحديد العتبة الصلب (HT) وكانت قيمة الـ(SNR) مساوية إلى (21.9138dB) عند استخدام تحديد العتبة المرن (ST).

1. Introduction

Interest in digital image processing methods stems from two principal application areas: improvement of pictorial information for human interpretation; and processing of image data for storage, transmission, and representation for autonomous machine perception [1]. Image processing in some of its applications needs a transformation process to solve problems.

The transformation is a process that translates one object from a given domain to another in order to have some important implicit information, which can be used for its recognition. One of the conventional transformation is the Fourier Transform which usually transforms

He signals from its time domain to the frequency domain [2].

The next form of the Fourier Transform developed to an efficient transform is called the Wavelet Transform (WT).

Denoising of images is an important task in image processing and analysis, and it plays a significant role in modern applications in different fields, including medical imaging and preprocessing for computer vision. Denoising goal is to remove that noise. Plenty of denoising methods exist, originating from various disciplines such as probability theory, statistics, partial differential equations, linear and nonlinear filtering, spectral and multiresolution analysis. All these methods rely on some explicit or implicit assumptions about the true (noise-free) signal in order to separate it properly from the random noise.

In particular, the transform-domain denoising methods typically assume that the true signal can be well approximated by a linear combination of few basis elements. That is, the signal is sparsely represented in the transform domain. Hence, by preserving the few high-magnitude transform coefficients that convey mostly the true-signal energy and discarding the rest which are mainly due to noise, the true signal can be effectively estimated. [3]

2. Discrete Wavelet Transform

The wavelet transform maps the function f(t) in L2(R) to another signal Wf(a,b) in L2(R2) where (a, b) are continuous, and called scaling and shift parameters, respectively. Although short time Fourier transform (STFT) decomposes a signal into a set of equal bandwidth basis functions in the spectrum the wavelet transform provides a decomposition based on constant-Q(equal bandwidth on a logarithmic scale) basis functions with improved multiresolution characteristics in the time frequency plane. Moreover, the wavelet parameters (a,b) are discretized in such a way that the orthogonality is still satisfied and the transform is performed on a grid within the continuous (a, b) plane [4].

The DWT gives a multiscale representation of a signal x(n). The DWT is implemented by iterating the 2- channel analysis filter bank described above. Specifically, the DWT of a signal is obtained by recursively applying the lowpass/highpass frequency decomposition to the lowpass output as illustrated in the diagram. The DWT of the signal x is the collection of subband signals. The inverse DWT is obtained by iteratively applying the synthesis filter bank. DWT has the following advantages [5]:

- Multi-scale signal processing technique.
- Number of significant output samples is very small and hence the extracted features are well characterized.
- Straightforward computation technique.

Although the Discrete Wavelet Transform (DWT) in its maximally decimated form has established an impression, its use for other signal analysis and reconstruction tasks has been hampered by two main disadvantages [comparison]:

- Lack of shift invariance, which means that small shifts in the input signal can cause major variations in the distribution of energy between DWT coefficients at different scales.
- Poor directional selectivity for diagonal features, because the wavelet filters are separable and real.

The 2D DWT is simply the application of the 1DWT repeatedly to first horizontal data of the image, then the vertical data of the image. The discrete wavelet transform is an algorithm for computing the coefficients sj,k and dj,k in the wavelet expansion of a signal.

$$f(x) = \sum_k S_{j,k}\phi_{j,k}(x) + \sum_k d_{j,k} w_{j,k}(x) + \sum_k d_{j-1,k} w_{j-1,k}(x) + \cdots + \sum_k d_{1,k} w_{1,k}(x) \quad \ldots (1)$$

where j is the number of multiresolution components (or scales), and k ranges from 1 to the number of coefficients in the specified component. ϕ is the scaling function and the w is the wavelet function through dilation and translation.

3. Dual-Tree Complex Wavelet Transform

The Dual - Tree Complex Wavelet Transform (DTCWT) has been developed to incorporate the good properties of the Fourier transform in the wavelet transform. As the name implies, two wavelet trees are used, one generating the real part of the complex wavelet coefficients real tree and the other generating the imaginary part of the complex wavelet coefficients imaginary tree [6].

The dual-tree CWT comprises of two parallel wavelet filter bank trees that contain carefully designed filters of different delays that minimize the aliasing effects due to downsampling [7]. The dual-tree CWT of a signal is implemented using two critically-sampled DWTs in parallel on the same data. The transform is two times expansive because for an N-point signal, it gives 2N-DWT coefficients. If the filters in the upper and lower DWTs are the same, then no advantage is gained. So, the filters are designed in a specific way such that the subband signals of the upper DWT can be interpreted as the real part of a complex wavelet transform and the subband signals of the lower DWT can be interpreted as the imaginary part. When designed in this way, the DT CWT is nearly shifting invariant, in contrast to the classic DWT. The structure is illustrated in figure (1). It should be noted that there are no links between the two trees, which makes it easy to implement them in parallel. Also, the filters in the two trees are different, and the filters in the first stage of each tree are different from the filters in all the later stages [6].

The complex coefficients are simply obtained as:

$$d_j^C(k) = d_j^{\Re e}(k) + i\, d_j^{\Im m}(k) \quad \ldots (2)$$

Where d_j^C represent the complex wavelet coefficients, $d_j^{\Re e}$ represent the real part of the complex wavelet coefficients, $d_j^{\Im m}$ the imaginary part of the complex wavelet coefficients, where j is the number of multiresolution components (or scales), and k ranges from 1 to the number of coefficients in the specified component, and the complex wavelet bases functions are given by

$$\psi_{j,k}^C(n) = \psi_{j,k}^{\Re e}(n) + i\, \psi_{j,k}^{\Im m}(n) \ldots (3)$$

The inverse DTCWT is calculated as two normal inverse wavelet transforms, one corresponding to each tree, and the results of each of the two inverse transforms are then averaged to give the reconstructed signal. Again, there is no complex arithmetic needed, since the d_j^C coefficients are split up into $d_j^{\Re e}$ and $d_j^{\Im m}$ before they are used in the corresponding inverse transforms.

4. Filters for the Dual- Two Complex Wavelet Transform

The filterbank structures for both DT-CWTs are identical. Figure (1) shows 1-D analysis filterbanks spanned over three levels. It is evident from the filterbank structure of DT-CWT that it resembles the filterbank structure of standard DWT with twice the complexity. It can be seen as two standard DWT trees operating in parallel. One tree is called as a real tree and other is called as an imaginary tree. Sometimes the real tree will be referred to as tree-a and the imaginary tree as tree-b.

The Dual-tree complex wavelet transform (DTCWT) calculates the complex transform of a signal using two separate DWT decompositions (tree a and tree b). If the filters used in one are specifically designed different from those in the other, it is possible for one DWT to produce the real coefficients and the other the imaginary.

This redundancy of two provides extra information for analysis but at the expense of extra computational power. It provides also approximate shift-invariance (unlike the DWT), yet still allows perfect reconstruction of the signal.

The form of conjugate filters used in 1-D DT-DWT is given as [8]:

$$(h_x + jg_x) \qquad \dots (4)$$

where, h_x is the set of filters $\{h_0, h_1\}$, and gx is the set of filters $\{g_0, g_1\}$ both sets in only x-direction (1-D). The filters h0 and h1 are the real-valued lowpass and highpass filters respectively for real tree. The same is true for g_0 and g_1 for imaginary tree. Though the notation of h_0 and h_1 are use for all level in the real part of analysis tree, h_0 and h_1 of first level are numerically different then the respective filters at all other levels above level-1. The same notion is applied for imaginary tree filters g_0 and g_1.

The design of the filters is particularly important for the transform to occur correctly and the necessary characteristics are:

- The low-pass filters in the two trees must differ by half a sample period.
- Reconstruction filters are the reverse of analysis.
- All filters are from the same orthonormal set.
- Tree (a) filters are the reverse of tree (b) filters.
- Both trees have the same frequency response.

The dual-tree CWT uses length-10 filters. The table of coefficients of the analyzing filters in the first stage (Table 1) and the remaining levels (Table 2) are shown below [7]. The reconstruction filters are obtained by simply reversing the alternate coefficients of the analysis filters.

5. Proposed Computation Method of Dual- Two Complex Wavelet Transform

For computing complex discrete wavelet transform, consider the following real transformation matrix for length-10:

$$\text{Wimag} = \begin{bmatrix} h_0(0) & h_0(1) & h_0(2) & h_0(3) & h_0(4) & h_0(5) & h_0(6) & h_0(7) & h_0(8) & h_0(9) & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & h_0(0) & h_0(1) & h_0(2) & h_0(3) & h_0(4) & h_0(5) & h_0(6) & h_0(7) & h_0(8) & h_0(9) & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ h_0(2) & h_0(3) & h_0(4) & h_0(5) & h_0(6) & h_0(7) & h_0(8) & h_0(9) & 0 & 0 & 0 & 0 & \cdots & h_0(0) & h_0(1) \\ h_1(0) & h_1(1) & h_1(2) & h_1(3) & h_1(4) & h_1(5) & h_1(6) & h_1(7) & h_1(8) & h_1(9) & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & h_1(0) & h_1(1) & h_1(2) & h_1(3) & h_1(4) & h_1(5) & h_1(6) & h_1(7) & h_1(8) & h_1(9) & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ h_1(2) & h_1(3) & h_1(4) & h_1(5) & h_1(6) & h_1(7) & h_1(8) & h_1(9) & 0 & 0 & 0 & 0 & \cdots & h_1(0) & h_1(1) \end{bmatrix} \qquad \dots (5)$$

The corresponding imaginary transformation matrix is

$$\begin{bmatrix} g_0(0) & g_0(1) & g_0(2) & g_0(3) & g_0(4) & g_0(5) & g_0(6) & g_0(7) & g_0(8) & g_0(9) & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & g_0(0) & g_0(1) & g_0(2) & g_0(3) & g_0(4) & g_0(5) & g_0(6) & g_0(7) & g_0(8) & g_0(9) & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ g_0(2) & g_0(3) & g_0(4) & g_0(5) & g_0(6) & g_0(7) & g_0(8) & g_0(9) & 0 & 0 & 0 & 0 & \cdots & g_0(0) & g_0(1) \\ g_1(0) & g_1(1) & g_1(2) & g_1(3) & g_1(4) & g_1(5) & g_1(6) & g_1(7) & g_1(8) & g_1(9) & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & g_1(0) & g_1(1) & g_1(2) & g_1(3) & g_1(4) & g_1(5) & g_1(6) & g_1(7) & g_1(8) & g_1(9) & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ g_1(2) & g_1(3) & g_1(4) & g_1(5) & g_1(6) & g_1(7) & g_1(8) & g_1(9) & 0 & 0 & 0 & 0 & \cdots & g_1(0) & g_1(1) \end{bmatrix} \quad \ldots (6)$$

5.1 Computation Method of Complex Discrete Wavelet Transform for 1-D Signal

To compute a single level discrete wavelet transform for 1-D signal, the next steps should be followed:

a) Checking input dimensions: Input vector should be of length N, where N must be even and N $\geq$ length (analysis filters).
b) Constructing a transformation matrix: Using transformation matrices given in (5) and (6).
c) Transformation of input signal by applying matrix multiplication to the NxN constructed real transformation matrix by the Nx1 input vector.

$$Y_{real} = [W_{real}]_{N\times N} . [X]_{N\times 1}$$

d) Transformation of input signal by applying matrix multiplication to the NxN constructed imaginary transformation matrix by the Nx1 input vector.

$$Y_{imag} = [W_{imag}]_{N\times N} . [X]_{N\times 1}$$

e) Computing the resulting transformatiom signal by taking the average of real and imaginary part from steps c and d.

5.2 Computation of Complex Discrete Wavelet Transform for 2-D Signal

To compute a dual tree CWT transform for 2-D signal the next steps should be followed: (as shown in fig.2 and fig.3)

a) Checking input dimensions: Input matrix should be of length NxN, where N must be even and N >= length (analysis filters) .
b) Constructing a transformation matrix: Using transformation matrices (W) given in (5) and (6).
c) Transformation of input signal by applying matrix multiplication to the NxN constructed real transformation matrix by the NxN input matrix.

$$Y_{real} = [W_{real}]_{N\times N} . [X]_{N\times N}$$

d) Transformation of input signal by applying matrix multiplication to the NxN constructed imaginary transformation matrix by the NxN input matrix.

$$Y_{imag} = [W_{imag}]_{N\times N} . [X]_{N\times N}$$

e) Computeing the resulting transformatiom signal by taking the average of real and imaginary part from steps c and d.

5.3 Computation Method of Inverse Complex Discrete Wavelet Transform

To reconstruct the original signal from the complex discrete wavelet transformed signal, Inverse Discrete Complex Wavelet Transform (IDCWT) should be used. The inverse transformation matrix is the transpose of the transformation matrix as the transform is orthogonal.

5.3.1 Computation of Inverse Complex Discrete Wavelet Transform for 1-D Signal

To compute a single level 1-D inverse Dual Tree Complex Discrete Wavelet Transform, the following steps should be followed:

a) Let Y_{imag} be the Nx1 framelet transformed vector.
b) Construct NxN reconstruction matrix, $T_{real} = Y_{imag}$ T, using transformation matrices given in (5) and (6).
c) Reconstruction of input vector, which can be done by applying matrix multiplication to the NxN reconstruction matrix, T, by the Nx1 framelet transformed vector.

$$X = [T]_{N\times N} . [Y]_{N\times 1}$$

5.3.2 Computation of Inverse Complex Discrete Wavelet Transform for 2-D Signal

To compute a single level 2-D inverse complex discrete wavelet transform, the next steps should be followed: (as shown in fig. 4)

a) Let Y_{real} be the NxN real transformed matrix.
b) Let Y_{imag} be the NxN imaginary transformed matrix.
c) Construct NxN reconstruction matrix, $T=WT$, using transformation matrices given in (5) and (6).
d) Reconstruction columns: By applies matrix multiplication to the NxN reconstruction matrix by the NxN transformed matrix.

$$YY = [T]_{N\times N} . [Y_o]_{N\times N}$$

e) Reconstruction rows: can be done as follows:

i. Transpose the column of reconstructed matrix resulting from step c.

$$Y = [Y\ Y\ ']_{N\times N}$$

ii. Apply matrix multiplication by multiplying the reconstruction matrix with the resultant transpose matrix.

$$X = [T]_{N\times N} . [Y]_{N\times N}$$

6. Experimental Results and Discussion

A general computer program computing a single-level 2-D DT-CWT is written using MatLab V.7.0 for a general (NxN) 2-D signal.

6.1 Hard- and Soft- Thresholding

This section gives the results of hard- and soft- thresholding with DT-CWT shrinkage done using the proposed method for DT-CWT that is discussed in chapter four.

Although denoising by the soft-thresholding is proven to be at least a smooth as the original function and free from spurious oscillations, there is a tradeoff between noise suppression and oversmoothing of image details. Soft-thresholding yields better results than hard-thresholding in terms of MSE. Oversmoothing of soft-thresholding and oscillations caused by hard-thresholding can be seen in figure (5) and figure (6). In this figure, the DT-CWT thresholding image denoising algorithm is tested on Barbara & Cameraman' image, with a white Gaussian noise. Note that the hard-thresholding introduces the spurious oscillations, while soft-thresholding exhibits an oversmoothing to the images.

The objective measures of this algorithm with both hard- and soft-thresholding schemes are compared in table (3) that describes the MSE, SNR and PSNR of a database of images. However, the estimated images obtained from hard-thresholding exhibit typically spurious oscillations and do not have the desired smoothness properties.

6.2 Wavelet and DT-CWT Image Denosing

A comparison is drawn between image denoising using dual-tree complex discrete transform (DT-CWT) with that using wavelet transform (WT), wavelet packet transform (WP), and stationary wavelet transform (SWT).

This comparison study is performed on a database which consists of two gray images. Table (4) compares the results SNR using both wavelet and DT-CWT transform for a Lenna & peppers images as shown in figures (6) and (7) after different SNR of noise on a database of gray images. From table (4), the SNR of DT-CWT is more than the SNR for wavelet transform on all the signal to noise ratio. For example, the SNR(16.5289 dB) to noisy image becomes (21.4893 dB) in WT, (21.9067 dB) in PWT,(23.3835 dB) in SWT and (25.6175 dB) in DT-CWT. The Matlab (wavelet toolbox) is used to perform WT, SWT, and WPT. The WT algorithm is implemented by using 'bestree2', and 'wprec2' functions, PWT algorithm is performed by using 'wpdec2' function, while functions 'swt2' and 'iswt2' are used for performing SWT algorithm.

In figure (7), the RMS error variations value of threshold schemes are plotted, and the RMS of the 2-D wavelet and wavelet packet are shown to be higher than that of the 2-D DT-CWT, this is an evidence for what is mentioned in section 5.1.

It is realised that WT is an important tool for non-stationary signal processing applications. WT has a great potential for singularity detection, denoising and compression and it presents a novel framework of time-scale for analysing and characterising many natural signals with the wealth of time-varying information. With the three major disadvantages of widely used standard DWT, namely; Shift-sensitivity, Poor-directionality, and Lack of Phase-information. These disadvantages severely limit the applications of WTs in certain signal processing applications. It ends with motivation to reduce these disadvantages of WTs through a complex extension known as CWTs.

7. Conclusions

The Dual-Tree Complex Discrete Wavelet Transform has been developed to incorporate the good properties of the Fourier transform in the wavelet transform. As the name implies, two wavelet trees are used, one generating the real part of the complex wavelet coefficients tree and the other generating the imaginary part tree.

Implementation of the Complex Discrete Wavelet Transform with Matlab programming is presented for denoising as an important signal/image processing application.

The following points are concluded:

After implementing the Complex Discrete Wavelet Transform, the average of the resulting real and imaginary parts should be taken.

Soft-thresholding yields better results than hard-thresholding in terms of MSE, SNR, and PSNR as given in Table(3).

Complex Discrete Wavelet Transform technique is a powerful tool in removing signal/image noise.

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Table (1): First Level DT CWT Coefficients [5]

Tree $\Re e$		Tree $\Im m$	
$h_{0f}^{\Re e}$	$h_{1f}^{\Re e}$	$h_{0f}^{\Im m}$	$h_{1f}^{\Im m}$
0	0	0.01122679	0
-0.08838834	-0.01122679	0.01122679	0
0.08838834	0.01122679	-0.08838834	-0.08838834
0.69587998	0.08838834	0.08838834	-0.08838834
0.69587998	0.08838834	0.69587998	0.69587998
0.08838834	-0.69587998	0.69587998	-0.69587998
-0.08838834	0.69587998	0.08838834	0.08838834
-0.01122679	-0.08838834	-0.08838834	0.08838834
-0.01122679	-0.08838834	0	0.01122679
0	0	0	-0.01122679

Table (2): Remaining Level DT CWT Coefficients [5]

Tree $\Re e$		Tree $\Im m$	
$h_{0}^{\Re e}$	$h_{1}^{\Re e}$	$h_{0}^{\Im m}$	$h_{1}^{\Im m}$
0.03516384	0	0	-0.03516384
0	0	0	0
-0.08832942	-0.11430184	-0.11430184	0.08832942
0.23389032	0	0	0.23389032
0.76027237	0.58751830	0.58751830	-0.76027237
0.58751830	-0.76027237	0.76027237	0.58751830
0	0.23389032	0.23389032	0
-0.11430184	0.08832942	-0.08832942	-0.11430184
0	0	0	0
0	-0.03516384	0.03516384	0

Table (3): MSE, SNR& PSNR results of the filtered image with DT-CWT in Hard-Thresholding (HT) and Soft-Thresholding (ST)

Name of image	Noisy image			Denoising by DT-CWT thresholding					
				HT	ST	HT	ST	HT	ST
	MSE	SNR dB	PSNR dB	MSE	MSE	SNR dB	SNR dB	PSNR dB	PSNR dB
Barbara	**179.6238**	**10.7546**	**12.7936**	**57.4199**	**53.859**	**15.4908**	**15.8699**	**27.9731**	**28.0082**
Cameraman	**400.0677**	**18.2707**	**11.0583**	**126.9066**	**97.038**	**23.1484**	**24.4795**	**26.6010**	**26.6088**

Table (4): Comparison between denoising performances using various methods ,Threshold=30.

Name of image	Noisy image	WT	SWT	PWT	DT-CWT
	SNR (dB)	SNR (dB)	*SNR (dB)*	SNR (dB)	SNR (dB)
Lena	16.5289	21.4893	21.9067	23.3835	25.6175
Peppers	11.9373	17.4661	18.1459	19.3640	22.1393

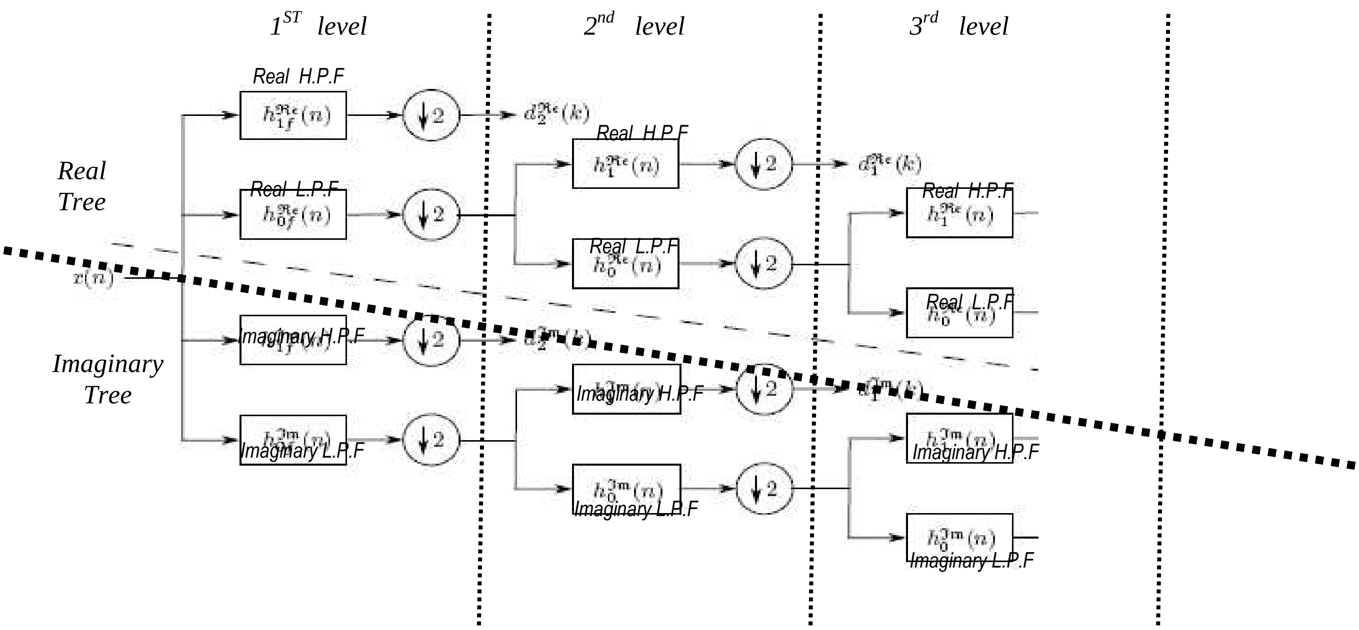


Figure (1): Iterated Filter Bank for the Dual-Tree Complex Wavelet Transform [5].

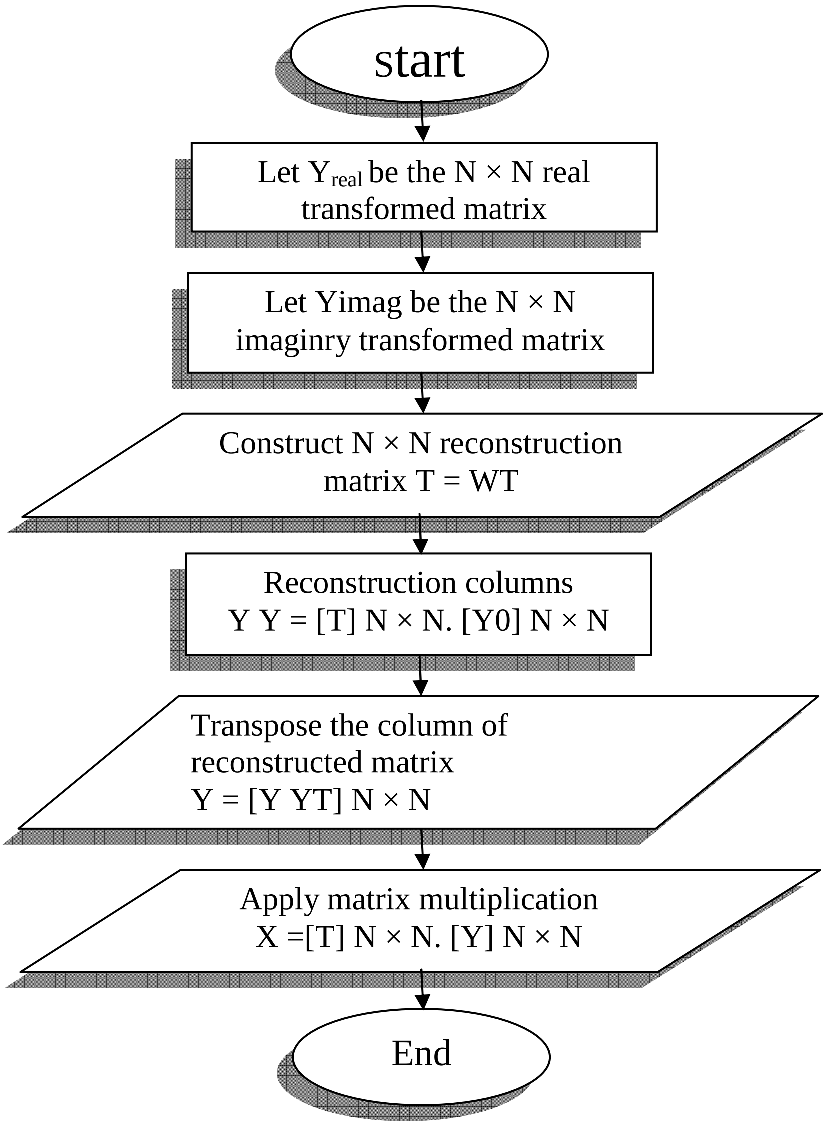


Figure 2: Computation of Complex Discrete Wavelet Transform Flow Chart

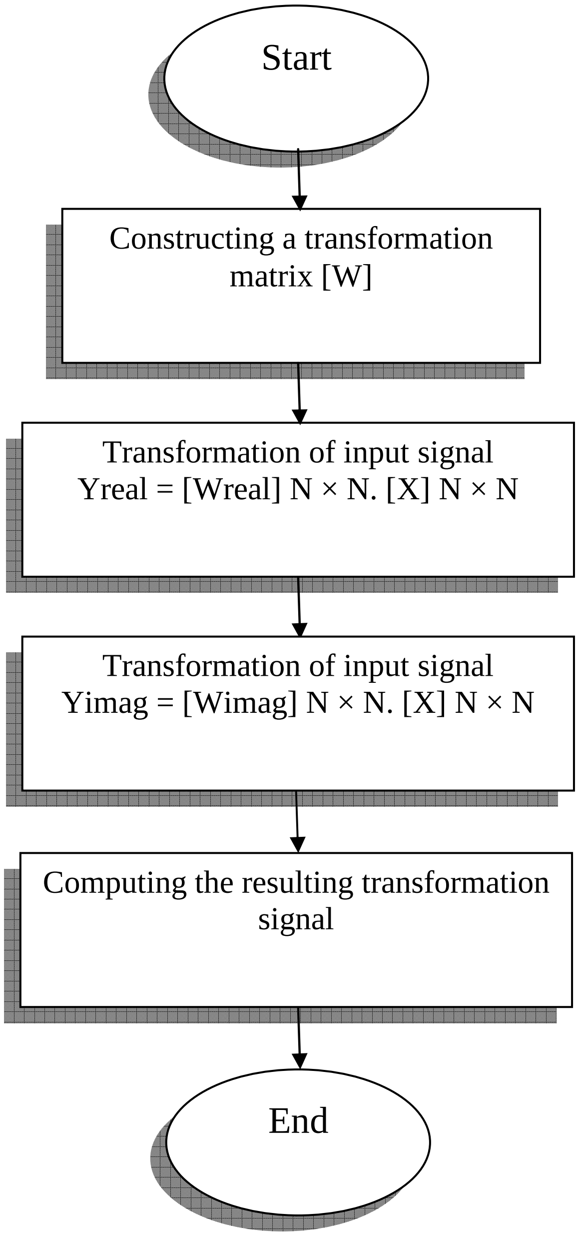


Figure 3: Inverse Complex Wavelet Transform Flow Chart

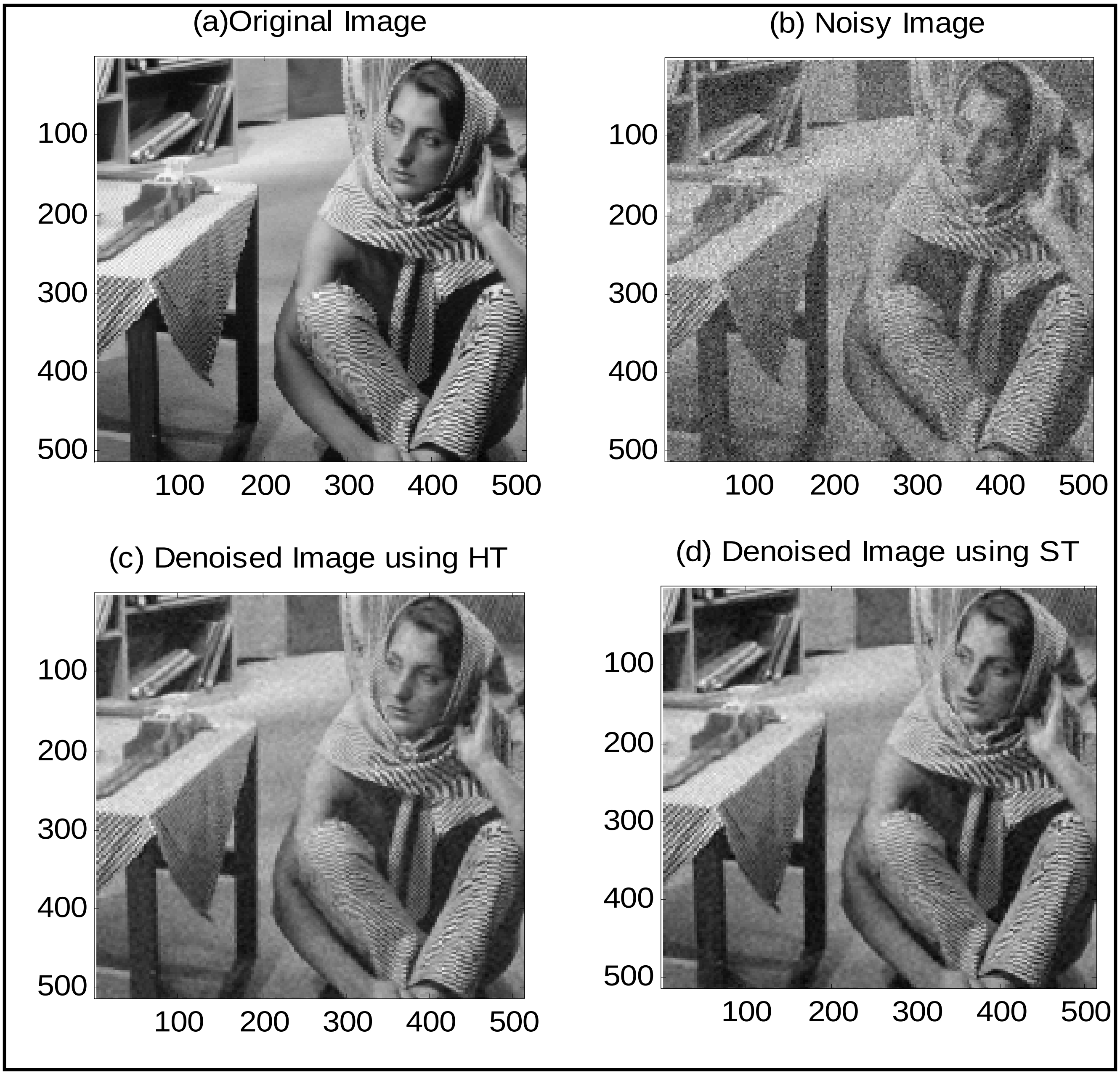


Figure 5: DT-CWT based methods for denoising of 'Barbara' image with a) Original Image with size (512 * 512) , b) Noisy image, c) Denosing image by Hard-Thresholding (HT) method ,d) Denosing image by Soft-Thresholding (ST)method. Threshold =30.

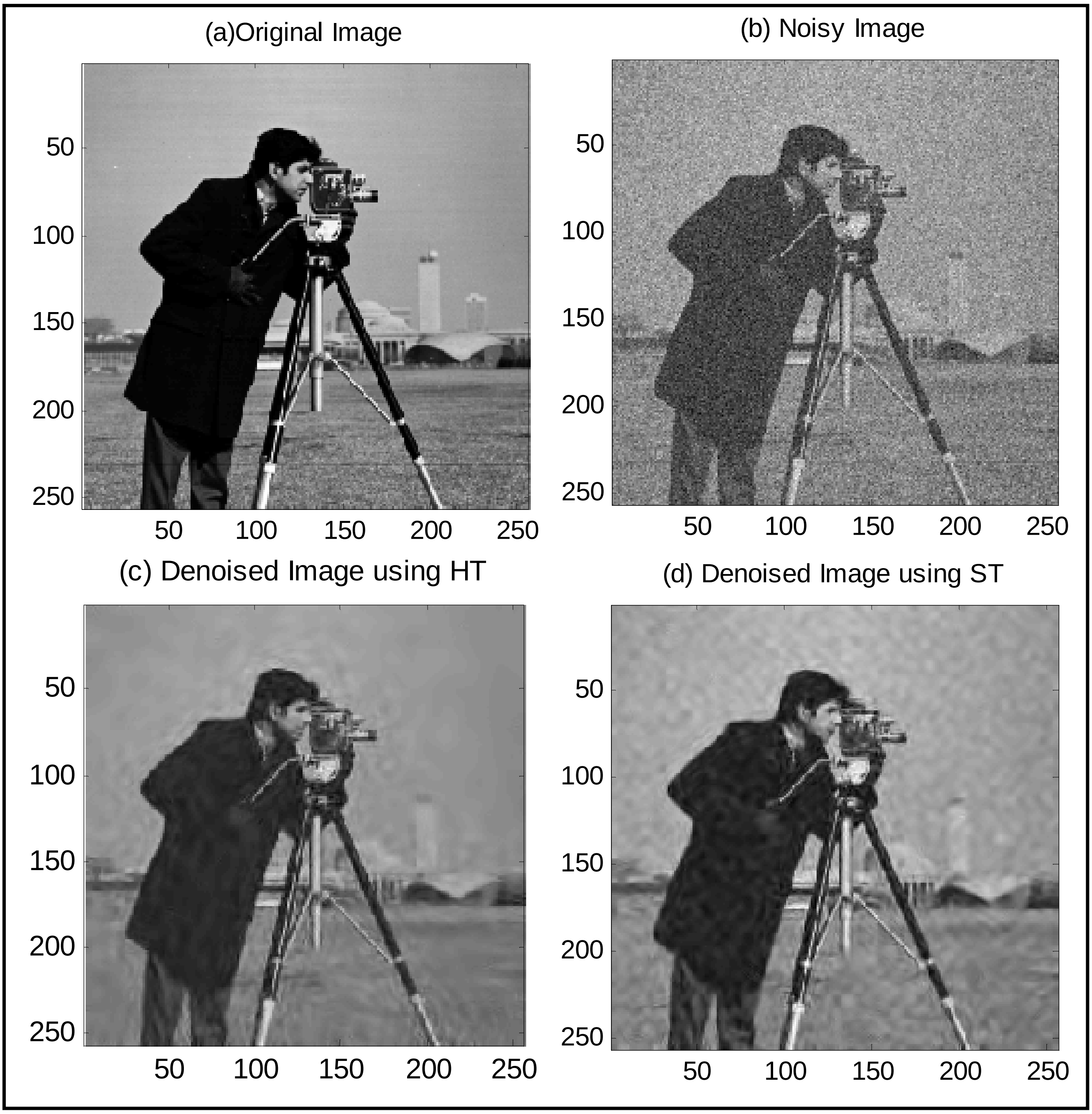


Figure 6: DT-CWT based methods for denoising of 'Cametraman' image with a) Original Image with size (256 * 256) , b) Noisy image, c) Denosing image by Hard-Thresholding (HT) method , d) Denosing image by Soft-Thresholding (ST) method. Threshold =30.

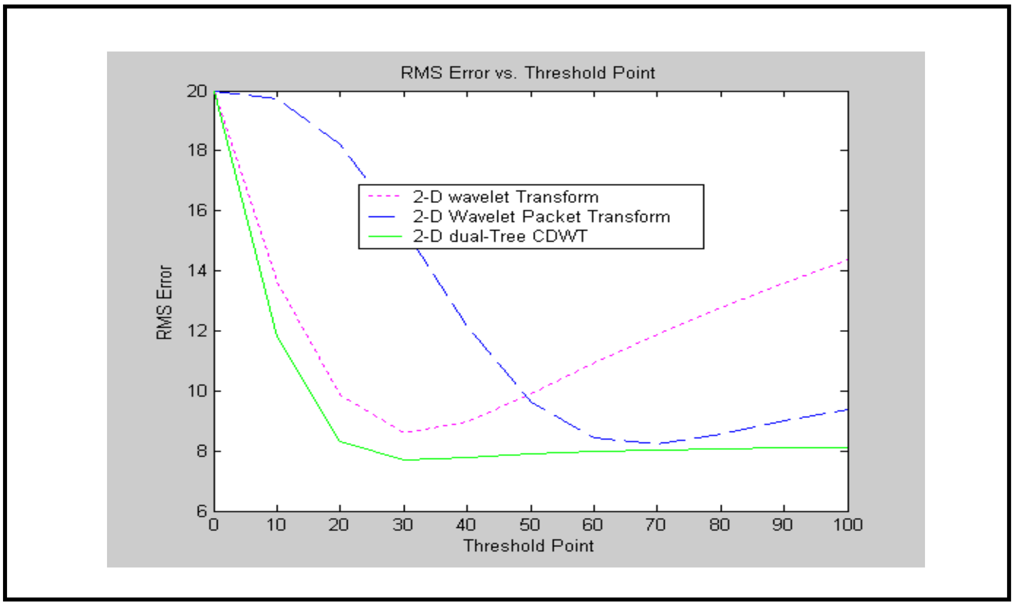
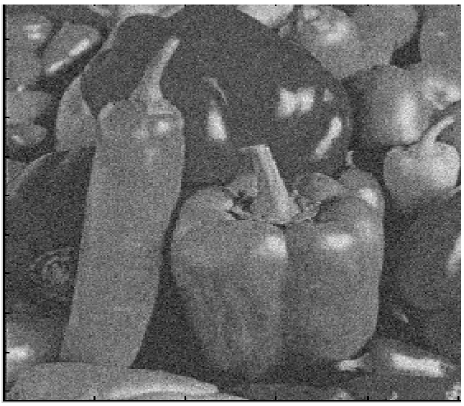


Figure 7: RMS error variations vs. threshold plot for denoised image using DT-CWT, 2-D wavelet transform, and 2-D wavelet packet transform.

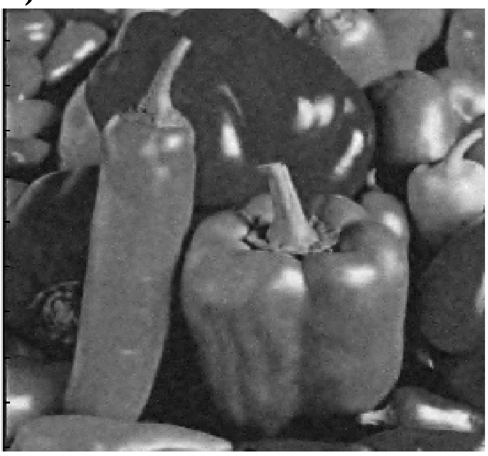
A) **Original images**



B) **Noisy images**

C)WT

D)SWT



E)PWT

F)DT-CW

Figure 8: SNR for various denoising methods for different images methods for denoising of 'Peppers' image, wavelet type = db4. Denosing image by Soft-Thresholding (ST) method. Threshold =30.

A) **Original images** B) **Noisy images**

C)WT **E)SWT**

E)PWT **F)DT-CWT**

Figure 9: SNR for various denoising methods for different images methods for denoising of 'Lenna' image with wavelet type = db9. Denosing image by Soft-Thresholding (ST) method. Threshold =30.