Mathematical Technique for Controlling the Gaps in the Cubic Bezier Curves for Two Dimensional

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Abstract

Designs are generated in the Bezier curves at most contains gaps in form that is drawn up in curve. This state is causing misshaping final form that depended on curves generating.

In this research applies of suggested method is making Bezier curve generating don't contains upon gaps and can controlled to eliminate gaps by using added parameters to control it.

The research applies modified equations by using different shapes in different cases and brought shapes (curves) don't contain the gaps and smoothing curves’.

Keywords: Bezier Curves, Gallier modify equation, Computer graphics, Smoothing curve, Gaps elimination in curve, enhancement drawing algorithms

1. Introduction

Smoothing curves generated in designs are very important, because it overcome the drawn up defects (gaps) that offers in designs. See of the original curve Bezier equation:

\[ P(t) = (1-t)^3P_0 + 3(1-t)^2tP_1 + 3(1-t)t^2P_2 + t^3P_3 \] \hspace{1cm} ...(1)

Where: \( p(t) \) is \( X(t),Y(t) \) of point drawn up of curve and \( t \) is value \( 0 \leq t \leq 1 \) and \( P_1, P_2, P_3, P_4 \) are a four control points that are effecting in curve shape. [Faux, 83],[Gerald, 99], [Watt,00].

In the generating of curve, needed to increase factor to value \( t \) in equation 1 to drawn up curve, in this case the increase factor is effective either a generating gaps or a smoothing curve, for example if increase factor =0.5 therefore curve points generating=3, but if increase factor =0.2 therefore curve points generating=6 and finally if curve points are much more therefore

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2- Gallier Modified Algorithmic [Gallier,00].

The modified De-Castejalu algorithm [Faux, 83], [Gerald, 99]. and Bezier curve is developed by Gallier as form below [A.M.J, 2010], [Jaber, 2005]:

\[ P(t) = (1-\Omega)^3 P_1 + (1-\Omega)^2 \Omega P_2 + (1-\Omega) \Omega^2 P_3 + \Omega^3 P_4 \] ......(2)

Equation (2) called original Gallier modified Bezier curve is dependent on interval \([r, s]\).

\( P_1, P_2, P_3, P_4 \) are control points equation (2)

Where \( \Omega = \frac{(t-r)}{(s-r)} \) and \( 1-\Omega = \frac{(s-t)}{(s-r)} \)

And \( r, s \) are integer number and \( r < s \)

And \( t \) is value \( r \leq t \leq s \)

\[ P(t)=\left(\frac{(s-t)}{(s-r)}\right)^3 P_1+\left(\frac{(s-t)}{(s-r)}\right)^2 \frac{(t-r)}{(s-r)} P_2+\left(\frac{(t-r)}{(s-r)}\right)^2 \frac{(s-t)}{(s-r)} P_3+ \left(\frac{(t-r)}{(s-r)}\right)^3 P_4 \] ......(3)

Equation (3) called Gallier modified Bezier curve is dependent on interval \([r, s]\). [Gallier,00]

But if \( r = 0 \) and \( s = 1 \) in equation (3) then return to equation (1) [original Bezier curve is dependent on interval \([0, 1]\) in De-Castejalu algorithm [Faux ,83], [Gerald ,99].

3. Origin of appearing gaps on Bezier Curves generating

See in section 1 the curve points generate by \( \frac{1}{\text{increase factor}} + 1 \)

where increase factor boundary interval \((0..1]\) \(0<\text{increase factor} \leq 1\)

if increase factor is a big value (reach in 1 'increase factor<1') the number of curve points generate a little number, therefore generate a much number of gaps and lose the smoothing of curve but if increase
factor is small value (reach in 0 'increase factor >0') the number of curve points generate a much number, therefore generate of smoothing curve and eliminated gaps of between curve points
But another problem if control points of Bezier curve are scattered position or long distance, therefore it generates gaps and lose the smoothing curve.

Where in equation 3 enable to control for curve points are generated by using another factors not of the increase factor in classic method and advantage of the Gallier modification equation to solve it. [Gallier 00].

4. Gaps eliminating through Gallier modification equation
the proposal system is advance of Gallier equation "equation 3" because the equation found two parameters r, s where these parameters give controlled to generate for curve points that help to block gaps and smoothing curve and conclusion if difference r, s is big ,therefore the gaps eliminate and otherwise the gaps increasing between curve points.

Look at figure 5 and figure 6 the increase factor is unchanged in two figures and r, s are controlling for smoothing curve and curve points generating.

See Table 1.Properties of the difference value of r ,s if inc. factor =0.001. See Figure 7 show that relationship between smoothing with difference s-r ,and figure 8 show relationship between gaps curve points with difference s-r.

5. Optimal curve Bezier
Optimal curve Bezier is curve if it used equation (1) for original Bezier curve and addition the t value is between 0 to 1 '0<t<1' and can be used in equation (2) (3) to generate optimal curve Bezier if r=0 and s=1 and finally the curve points generated same of number and properties is similar. But another new fact if difference (s-r) =1 therefore generate optimal curve Bezier see table 2.

But difference (s-r) >1 then generate curve Bezier is not optimal curve Bezier that different by number of curve points generated and properties (ex. smoothing gaps see table 1)

See Table 2.Properties of the value of (s-r)=1 in optimal curve Bezier

6. The Proposed Method System
in the propose system advanced of Gailler modification equation "equation 3" see, the properties of table 1 and section 4" that can be eliminated gaps and smoothing curve by advantage of increasing subtraction (s-r) "see, the table 1". Finally, the algorithm in proposed method system as following :

Input: r value, s value: where r<s, 4 control points as (X1,Y1), (X2,Y2), (X3,Y3), (X4,Y4)
Output: Number of curve points and drawing final Bezier curve.

1. Start
2. t=r
3. Generate curve point by using Gallier modify equation “equation 3”

\[ X_{cp} = (\frac{s-t}{s-r})^3 X_1 + (\frac{s-t}{s-r})^2 \left( \frac{t-r}{s-r} \right)^2 X_2 + \left( \frac{t-r}{s-r} \right) X_3 + (\frac{t-r}{s-r})^3 X_4 \]
b. \( Y_{cp} = \left( \frac{s - t}{s - r} \right)^2 Y_1 + \left( \frac{s - t}{s - r} \right)^2 \left( \frac{t - r}{s - r} \right) Y_2 + \left( \frac{t - r}{s - r} \right)^2 Y_3 + \left( \frac{t - r}{s - r} \right)^3 Y_4 \)

3. That parameters \( r, s \) are effecting to time period in generations of Bezier curve because, it generated more curve points to cover this a gaps.

4. The method for controlling the smoothing and gaps of generating the curve which used in the design, effect by the type of the computer used in generating, and different \((s-r)\) used its.

5. controlled points or the curve except smoothing and gaps.

6. If increase factor = 0 therefore generating infinity curve points but Increase factor =1 therefore generating only two points If difference \((s-r)>1\) that it leads to smooth curve and eliminated gaps generated than in \((s-r) = 1\).

7. Parameters \( r, s \) that enable to control to generate curve points but if \((s-r)=1\) therefore generate curve points is same as equation(1) & finally can be controlled without modify or change for increase factor.

8. Can be used in image processing upon computers security domains.

9. This proposes can be done without changing any of the (first control point of Bezier curve, end control point of Bezier curve) in original Bezier curve and Gallier modified Bezier curve if the values \( r=0, s=1 \) OR \((s-r)=1\).

10. This work proposes technique for controlling the smoothing curve and gaps eliminated, depends on Gallier modified cubic Bezier curve.

11. this work is benefit to enhancements of Bezier curves in 3D dimensional.
9. References


[5] [A.M.J,2010] Dr. A. M. J. Abdul-Hussen "Proposed an Algorithm to improvement Performance the Time of Cubic Bezier Curves which are used in Design” published by first conference scientific for computer science 2010. Computer Science Department, University of Technology.


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<th>S</th>
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<th>(s-r)</th>
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Table 2. Properties of the value of \((s-r) = 1\) if inc. factor = 0.001

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Figure 1: Curve smoothing inc. factor = 0.001
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Figure 2: Curve smoothing inc. factor = 0.001 in scattered control points

Figure 3: Curve smoothing inc. factor = 0.005 before scaling Bezier curve
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Figure 4: Curve smoothing inc. factor = 0.005 after scaling Bezier curve

Figure 5: Increase factor =0.01, curves points=100, r=0, s=1, (s-r)=1
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Figure 6: Increase factor = 0.01, curves points = 501, r = 0, s = 5, (s-r) = 5

Figure 7: relationships between smoothing with difference s-r in increase factor = 0.01
Figure 8: show relationship between gaps curve points with difference s-r in increase factor = 0.01

Figure 9: Show top DFD level of proposal work.
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Figure 10 used in image process

Figure 11 how can obtain pixels information