Kinematic Coupling Analysis of Autorotation Flying Body

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Received on: 1/11/2009
Accepted on: 2/12/2010

Abstract
The kinematic coupling dynamic stability has been analyzed. The Laplace transformation and the coefficient matrix determinant are used to find the rolling stability characteristic equation. The effect of parameters is investigated with different values of roll rate \( p_0 \). It is found that the kinematics coupling or autorotation is critical at flying regime of low \( C_{nq} \) and high \( C_{mq} \). The results can be used as real design requirements for further configuration improvements of the airplane.

Introduction
The inertial coupling has been generally tamed as a potential problem in modern fighter aircraft. Even the most austere of these are equipped with stability augmentation systems that can be provide the required feedbacks to minimize excursions in rapid rolls[1]. During the rolling maneuvers large angles of sideslip may occur as a result of kinematics coupling [2]. The vertical tail may produce large yawing moment that acts in the direction of roll. In such a case, it may not be possible to stop the flying body from Rolling, although the lateral control is held against the roll direction. This is known as autorotation rolling. In this situation positive “G” would facilitate recovery [3]. As the angle of attack is increased to a positive value, kinematics coupling will be result in a moment that opposes the original direction of roll, thus alleviating the tendency for autorotation rolling [4]. The divergence experienced during rolling manufacture is complex because it involves not only inertia properties, but aerodynamic as well, [4]. Coupling results when a disturbance about one aircraft axis causes a disturbance created by an elevator deflection during straight and level flight, [5]. The resulting motion is

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2013

https://doi.org/10.30684/etj.29.10.13
2412-0758/University of Technology-Iraq, Baghdad, Iraq
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restricted to pitching motion and no disturbance occurs in yaw or roll. An example of couple motion is the disturbance created by a rudder deflection [6]. The ensuing motion will be some combination of both yawing and rolling motion [7]. Although all lateral disturbance motion are coupled, the only motion that ever results in coupling problems large enough to threaten the structural integrity of the aircraft is coupling as a result of rolling motion, [8].

T-38 jet plane was taken as case study (Figure (1), Table (1) [9].

Mathematical Analysis of Rolling Divergence

The overall equation of motion, [2].

\[ \sum \dot{x} = \frac{m}{\rho \beta} \dot{\beta} \left[ -c_{\beta} \dot{\beta} - \frac{1}{2} c_{\beta \gamma} \dot{\gamma} - \frac{1}{2} c_{\beta \alpha} \dot{\alpha} - c_{\beta \delta} \dot{\delta} - c_{\beta v} \dot{v} \right] \]

\[ + \left( \frac{1}{\rho \beta} - \frac{1}{\rho \alpha} \right) \dot{c}_{\beta \alpha} \dot{\alpha} + \frac{1}{\rho \alpha} \dot{c}_{\beta \delta} \dot{\delta} + \frac{1}{\rho \beta} \dot{c}_{\beta \gamma} \dot{\gamma} \]

\[ = \alpha \dot{x} - (\omega \beta + \phi \gamma) \]  

\[ \text{(1)} \]

\[ \sum \dot{y} = \frac{2m}{\rho \alpha} \dot{\alpha} \left[ -c_{\alpha} \dot{\alpha} - 2c_{\alpha \delta} \dot{\delta} - c_{\alpha \gamma} \dot{\gamma} - c_{\alpha \beta} \dot{\beta} \right] \]

\[ + \frac{2m}{\rho \beta} \dot{c}_{\alpha \beta} \dot{\beta} + \frac{2m}{\rho \gamma} \dot{c}_{\alpha \gamma} \dot{\gamma} - c_{\alpha \delta} \dot{\delta} = \omega \beta \]  

\[ \text{(2)} \]

\[ \sum \dot{z} = -2c_{\beta \delta} \left[ -c_{\delta} \dot{\delta} - c_{\delta \alpha} \dot{\alpha} - c_{\delta \beta} \dot{\beta} \right] \]

\[ - c_{\delta \beta} \dot{\beta} - c_{\delta \gamma} \dot{\gamma} \]

\[ = \phi \dot{z} + (\omega \beta - \phi \gamma) \]  

\[ \text{(3)} \]

Rolling Velocity

\[ \frac{2\dot{u}}{\dot{\beta}} = \dot{\beta} - \sin \theta \dot{\psi} \]  

\[ \text{(5)} \]

Yawing Velocity

\[ \frac{2\dot{u}}{\dot{\beta}} = \cos \theta \dot{\psi} \]  

\[ \text{(6)} \]

The approach for solving the autorotation rolling equations was derived based on some necessary assumption to fit into the present analysis of autorotation rolling [1].

1. Velocity remains constant during the roll maneuver.
2. The rate roll rate is constant; \( \dot{\beta} = 0, u = u_0 \).
3. V, W, Q, R are small therefore their products are negligible.
4. Engine gyroscopic effect is negligible.
5. Rudder and elevator are fixed in their initial trim position.
6. Aerodynamic coefficients are negligible with the exception of \( c_{m\alpha}, c_{m\dot{q}}, c_{n\delta} \), and \( c_{n\beta} \).
7. Small angle assumption on \( \alpha \) and \( \beta \).

When these assumptions are applied to the six equations of Motion the following results are obtained
The determinant must be expanded to solve for the characteristic equation

\[ \begin{vmatrix}
-\frac{1}{2} p_{\alpha\alpha} & -\frac{1}{2} p_{\alpha\beta} & -\frac{1}{2} p_{\alpha q} & 0 \\
-\frac{1}{2} p_{\beta\alpha} & -\frac{1}{2} p_{\beta\beta} & -\frac{1}{2} p_{\beta q} & 0 \\
0 & 0 & \frac{1}{2} p_{\alpha\alpha} & 0 \\
0 & 0 & 0 & \frac{1}{2} p_{\beta\beta}
\end{vmatrix} \]

The determinant must be expanded to solve for the characteristic equation

\[ \begin{vmatrix}
I_{xx} & 0 & 0 & 0 \\
0 & I_{xx} & 0 & 0 \\
0 & 0 & I_{xx} & 0 \\
0 & 0 & 0 & I_{xx}
\end{vmatrix} \]

Note that there are four equations in four unknowns \((\alpha, \beta, \phi, \text{and } r)\).
... (20)

Results and Discussion

All the parameters exits in autorotation characteristic equation are selected as effective parameters, which may be tested with different roll rate \( [\dot{\theta}_\alpha] \). Wing mean chord line \( [\bar{c}] \) and wing span \( [b] \) have negative effect toward autorotation stability because any increment in these parameters decrease the directional stability \( (C_{m_d}) \). (2) and Fig (3).

Autorotation stability is much better at low altitude due to lift increase, Fig (4). Any change in moment of inertia in x, y plane \( (I_{xx}, I_{yy}) \) has limited effect on kinematics coupling dynamic stability (Fig (5), Fig (6)), but any change in moment of inertia in z-plane has great effect on roll coupling (Fig (7)). Stability Derivatives have different behavior because each derivative is depending on the way of its generation. \( C_{m_d} \) represents the longitudinal stability so it has limited positive effect toward autorotation stability, Fig (8). \( C_{m_d} \) presents the damping in pitch. It has great negative effect toward autorotation stability, Fig (9). \( C_{n_d} \) represents damping in yaw. It has limited negative effect toward autorotation stability, but it should be maintain below zero \( (C_{n_d} < 0) \) to keep the B coefficient of characteristic equation greater than zero in order to avoid autorotation Fig (10) and \( C_{n_d} \) represents the directional stability and it has great positive effect toward autorotation stability and should be kept larger than zero, since this parameter determine the C coefficient which it should be positive to avoid the autorotation. Fig (11).

Conclusions

1. Vertical Tail stability design more important than wing-body and horizontal tail the for kinematic coupling.
2. Autorotation stability was found much better at low altitude
3. Decrease weight distribution in Z-plane and increase weight distribution in Y-plane one of best solution of kinematic coupling of autorotation.
4. The most serve cases naturally should be expected in the flight regime of low \( C_{n_d} \) and high \( C_{m_d} \).
5. It can be notice that \( \dot{\theta}_\alpha = 20 \text{ deg/sec} \) was quite reasonable for optimum stability.

References

### List of symbols

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Definition</th>
<th>Units</th>
</tr>
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<tbody>
<tr>
<td>$b$</td>
<td>Wing Span</td>
<td>ft</td>
</tr>
<tr>
<td>$c$</td>
<td>Wing Chord Line</td>
<td>ft</td>
</tr>
<tr>
<td>$S$</td>
<td>Wing Area</td>
<td>ft$^2$</td>
</tr>
<tr>
<td>$G$</td>
<td>Ground Force</td>
<td>slug*ft$^2$/sec$^2$</td>
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<td>$m$</td>
<td>mass</td>
<td>slug</td>
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<td>$q$</td>
<td>Dynamic Pressure</td>
<td>slug/ft*sec$^2$</td>
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<tr>
<td>$u$</td>
<td>Air Speed</td>
<td>ft/sec</td>
</tr>
<tr>
<td>$u_i$</td>
<td>Initial Air Speed</td>
<td>ft/sec</td>
</tr>
<tr>
<td>$p_o$</td>
<td>Roll Rate</td>
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<tr>
<td>$I_{xx}$</td>
<td>$X$–Axis moment of Inertia</td>
<td>slug*ft$^2$</td>
</tr>
<tr>
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<td>$Y$–Axis moment of Inertia</td>
<td>slug*ft$^2$</td>
</tr>
<tr>
<td>$I_{zz}$</td>
<td>$Z$–Axis moment of Inertia</td>
<td>slug*ft$^2$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Air Density</td>
<td>slug/ft$^3$</td>
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<tr>
<td>$c_{mp}$</td>
<td>Damping in pitch</td>
<td>1/radian</td>
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<tr>
<td>$c_{m_0}$</td>
<td>Static longitudinal stability</td>
<td>1/ rad</td>
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<td>$\alpha_i$</td>
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<td>$C_X$</td>
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<tr>
<td>$C_T$</td>
<td>Coefficient of thrust force</td>
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<tr>
<td>$C_T\mu$</td>
<td>Coefficient of Thrust in $X$–Axis</td>
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### Aerodynamic Data

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<th>Wing Span (ft)</th>
<th>Wing Area ($ft^2$)</th>
<th>Wing Mean Chord (ft)</th>
<th>Aspect Ratio</th>
<th>Wing Sweep Angle</th>
<th>Taper Ratio</th>
<th>Airfoil Section</th>
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<td>25.3</td>
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<td>7.73</td>
<td>3.75</td>
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### Stability Derivatives

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<thead>
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<th>$C_{m_{alpha}}$</th>
<th>$C_{m_{alpha}}$</th>
<th>$C_{n_{alpha}}$</th>
<th>$C_{n_{beta}}$</th>
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<tr>
<td>-0.16/rad</td>
<td>-8.4/rad</td>
<td>-0.54/rad</td>
<td>+0.28/rad</td>
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### Other Data

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<thead>
<tr>
<th>$I_{xx}$ (Sl-ft²)</th>
<th>$I_{yy}$ (Sl-ft²)</th>
<th>$I_{zz}$ (Sl-ft²)</th>
<th>$I_{xz}$ (Sl-ft²)</th>
<th>Max Speed (M)</th>
<th>Weight(lbf) &amp; mass(slug)</th>
<th>Density (M=0.8 Alt=20000 ft (Sl/ft³))</th>
<th>Density (M=0.8 Alt=20000 ft(ft/sec))</th>
<th>Engine Type</th>
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<td>1.63</td>
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<td>0.001267</td>
<td>831</td>
<td>J85-GE_5 Turbojet</td>
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</table>

Figure (1): Views of Supersonic Aircraft T-38 Taylon (Case Study)
Figure (2) Effect of Wing Mean Chord on Aircraft Autorotation

Figure (3) Effect of Wing Span on Aircraft Autorotation
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Figure (4) Effect of Wing Area on Aircraft Autorotation

Figure (5) Effect of Wing Span on Aircraft Autorotation
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Figure (6) Effect of Air Density on Aircraft Autorotation

Figure (7) Effect of X-Axis moment of Inertia on Aircraft Autorotation
Figure (8) Effect of Y-Axis moment of Inertia on Aircraft Autorotation

Figure (9) Effect of Z-Axis moment of Inertia on Aircraft Autorotation
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Figure (10) Effect of Static Longitudinal Stability on Autorotation

Figure (11) Effect of Damping in Pitch on Autorotation
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Figure (12) Effect of Damping in Yaw on Autorotation

Figure (13) Effect of Directional stability on Autorotation