Two Layer Optimum Design of Low Dispersion Optical Fiber (1500-1600 nm) Using Evolutionary Synthesis.

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Abstract

A mathematical procedure for optimization depends on the evolutionary algorithm has been proposed to estimate the optimum design of single mode optical fiber upon finding the core and cladding material and core radius. The results of the optimum design achieved minimum total dispersion and minimum dispersion slope. The operating wavelengths are chosen in the range (1500-1600) nm. The optimization using evolutionary algorithm can explore all the possible solutions and then choosing the best solution that has maximum merit function.

Keywords: Optical Fiber, Low Dispersion Optical Fiber

Introduction

Optical fiber designing depend on the way of mathematical optimization that can be used to obtain the best design with less time. Recently, the genetic and evolutionary algorithm used in optimization of optical fiber [1-6]. The idea that the evolution mechanisms can be used to implement algorithms searching the solution of physical and/or engineering systems goes back to 1950s/1960s. In the following years, a number of actual applications have been proposed. An evolutionary strategy allows evolving a population of candidate solutions to physical/engineery problems using calculation operators directly inspired by genetic laws and Darwinian natural selection [7]. As an example, in 1973 an evolutionary method was used to optimize the design in the field of aerodynamics. Evolutionary computation approach is sometimes distinguished by genetic algorithm. More precisely, in the former case the solutions are represented as a finite states machines and their state-transition randomly evolves towards the best
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Diagrams/solution. In the latter case, i.e. the genetic algorithm like that proposed by Holland [7], is not written for a specific problem: it is a method directly originated by the abstraction of biological evolution laws thus permitting the solution of a wide class of problems. Nowadays, the genetic algorithm and the evolutionary strategies are not well distinguished. During the last years, the two approaches have been largely merged by the spreading of their applications. Different frequencies (or spectral components) of the pulse travel at slightly different group velocities, a phenomenon referred to as group-velocity dispersion. When an optical pulse in a single mode fiber is launched into an optical fiber, the pulse may spread outside its timing window due to dispersion, which causes pulse overlapping between adjacent timing windows and limits the transmission data rate.

1) Dispersion in Single-Mode Fiber

The major sources of dispersion are material (or chromatic) dispersion and waveguide dispersion [8, 9]. In a single mode fiber there is a point for zero dispersion where the material and waveguide dispersion cancel each other at a definite wavelength. Most of current long distance fiber networks operate at wavelengths ranging from (850 to 1600 nm). However, there are many reasons to work in 1550 nm band. These reasons are; the fiber attenuation is much lower in 1550 nm band. EDFA operates in this band and WDM system requires a large amplified bandwidth, and this means that we have to use 1550 nm window and optical amplifier. With a single mode fiber (conventional type), the dispersion is about 17 ps/km.nm, it is very comparatively large, however, by using lasers with a very narrow line width we can reduce the dispersion effect.

1-1) Material (Chromatic) Dispersion

This is caused by the fact that the refractive index of the glass we are using varies slightly with the wavelength. Some wavelengths therefore have higher group velocities and so travel faster than others. Since every pulse consists of a range of wavelengths it will spread out to some degree during its travel.

All optical signals consist of a range of wavelengths. This range may be only a fraction of a nanometer wide but there is always a range involved. Typically optical pulses used in communications systems range from about 0.2 nm wide to 5 nm wide for systems using single-mode fiber (with lasers).

The material dispersion $D_m$ is defined by [9]:

$$D_m = -\frac{\lambda}{c} \frac{d^2 n}{d\lambda^2} \quad \ldots (1)$$

where $n = n_1$ or $n_2$ for the core or cladding respectively. The refractive index variation for fused or doped silica is given by Sellmeier equation:

$$n^2 = 1 + \frac{A\lambda^2}{\lambda^2 - a^2} + \frac{B\lambda^2}{\lambda^2 - b^2} + \frac{C\lambda^2}{\lambda^2 - c^2} \quad \ldots (2)$$

Both $(A,B,C,a,b,c)$ are constants depending on the chemical construction of the material.

1-2) Waveguide Dispersion

The shape (profile) of the fiber has a very significant effect on the group velocity. This is because the electric and magnetic fields that constitute the pulse of light extend outside of the core into the cladding. The amount that the fields overlap between core
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and cladding depends strongly on the wavelength. The longer the wavelength the further the electromagnetic wave extends into the cladding. The refractive index experienced by the wave is an average of the refractive indices of core and cladding depending on the relative proportion of the wave that travels there. Thus since a greater proportion of the wave at shorter wavelengths is confined within the core, the shorter wavelengths “see” a higher refractive index than do longer wavelengths, because of the refractive index of the core is higher than that of the cladding. Therefore shorter wavelengths tend to travel more slowly than longer ones. Thus signals are dispersed (because every signal consists of a range of wavelengths).

The waveguide dispersion parameter \( D_w \) which may be expressed as [9]:

\[
D_w = -\frac{n_i^2 - n_2^2}{\lambda} \frac{d^2 V_b}{d V^2} \quad \ldots \quad (3)
\]

Where are:

\[
b = \frac{k_2^2}{n_i^2 - n_2^2} \quad \ldots \quad (4)
\]

\[
V = \frac{2\pi a}{\lambda} \sqrt{n_i^2 - n_2^2} \quad \ldots \quad (5)
\]

So that the total effected dispersion is \( D_T \):

\[
D_T = D_m + D_w
\]

3) Single mode optical fiber equations

If the electromagnetic wave propagating along a cylindrical fiber in the z-axis direction, the optical fiber equations can be written as [10]:

\[
\begin{bmatrix}
\frac{\partial j_i}{\partial x} + (j_i + k_i)(j_i - k_i) & \frac{\partial j_i}{\partial x} & \frac{\partial k_i}{\partial x} \\
\frac{\partial j_i}{\partial x} & \frac{\partial j_i}{\partial x} + (j_i + k_i)(j_i - k_i) & \frac{\partial j_i}{\partial x} + (j_i + k_i)(j_i - k_i) \\
\frac{\partial j_i}{\partial x} & \frac{\partial j_i}{\partial x} & \frac{\partial j_i}{\partial x} + (j_i + k_i)(j_i - k_i)
\end{bmatrix} = 0
\]

\[
\ldots \quad (6)
\]

Where:

\[
u^2 = k_2^2 n_i^2 - \beta^2 \quad \ldots \quad (7)
\]

\[
w^2 = \beta^2 - k_2^2 n_2^2 \quad \ldots \quad (8)
\]

\( J_\nu (\alpha a) \) is the Bessel function of the first kind of order \( \nu \).

\( K_\nu (\alpha a) \) is the modified Bessel function of the order \( \nu \).

Evaluation of the above determinant yields the following Eigen values equation for \( \beta \):

\[
\begin{bmatrix}
\frac{\partial j_i}{\partial x} & \frac{\partial j_i}{\partial x} & \frac{\partial j_i}{\partial x} \\
\frac{\partial j_i}{\partial x} & \frac{\partial j_i}{\partial x} & \frac{\partial j_i}{\partial x} \\
\frac{\partial j_i}{\partial x} & \frac{\partial j_i}{\partial x} & \frac{\partial j_i}{\partial x}
\end{bmatrix} = 0
\]

\[
\ldots \quad (9)
\]

For single mode fiber \( \nu=0 \), and using the Bessel function relations:

\[
j_i'(x) = \frac{v}{x} j_i(x) - j_{i+1}(x) \quad \ldots \quad (10)
\]

\[
k_i'(x) = \frac{v}{x} k_i(x) - k_{i+1}(x) \quad \ldots \quad (11)
\]

and:

\[
V = k_0 a \sqrt{n_i^2 - n_2^2} \quad \ldots \quad (12)
\]

\[
b = \frac{n_i^2 - n_2^2}{n_i^2 - n_2^2} \quad \ldots \quad (13)
\]

Yields the final equation for single mode optical fiber:

\[
\sqrt{1-b^2 j_0(V\sqrt{1-b})} - j_0(V\sqrt{1-b}) - \frac{\sqrt{b} K_0(V\sqrt{b})}{K_0(V\sqrt{b})} = 0 \ldots \quad (14)
\]
4) Minimization Using Evolutionary Algorithm

The choice of fitness function is important in the optimization process, because this function is the connection between the physical problem and the optimization technique.

There are in literature many fitness functions for optimization of optical filters. The most used function is root mean square error, which is used in this paper. The error between calculated dispersion and target dispersion is calculated using the following equation [11]:

$$MF_1 = \sum_{i=1}^{m} \frac{(DT_i - DT_0)^2}{m} = \text{min.value} \ ... (15)$$

$$MF_2 = \sum_{i=1}^{m} \frac{(ST_i - ST_0)^2}{m} = \text{min.value} \ ... (16)$$

$$MF_3 = \sum_{i=1}^{m} \frac{(E_i)^2}{m} = \text{max.value} \ ... (17)$$

$$MF = \frac{MF_3}{MF_1 + \lambda MF_2} = \text{max.value} \ ... (18)$$

Where \((DT_i , DT_0)\) are the designing (resulting) total dispersion and the target total dispersion, respectively. \((ST_i , ST_0)\) are the designing total dispersion slope and the target total dispersion slope, respectively. \((E_i)\) is the percentage wave energy transmitted in the core relative to incident wave energy. \(m\) is the number of wavelengths points through the whole spectrum.

Thus the basic purpose of the minimization is to obtain maximum value for the total merit function \(MF\) that can achieve minimum total dispersion , minimum dispersion slope and maximum transmitted energy via the core.

The evolutionary algorithms [12] are adaptive methods which may be used for optimization problems. It works with a population of individuals, each representing a possible solution to a given problem. Each individual is assigned a fitness score according to how good a solution to the problem it is. The highly fit individuals are given opportunities to reproduce by cross-over and mutation operation with other individuals in the population. This produces new individuals as offspring, which share some features taken from each parent. The least fit members of the population are less likely to get selected for reproduction, and so excluded.

A whole new population of possible solutions is thus produced by selecting the best individuals from the current generation, and mating them to produce a new set of individuals. This new generation contains a higher proportion of the characteristics possessed by the good members of the previous generation. In this way, over many generations, good characteristics are spread throughout the population. By favoring the mating of the more fit individuals, the most promising areas of the search space are explored. If the evolutionary algorithm has been designed well, the population will converge to an optimal solution of the problem.

In this paper, two single mode optical fibers consists of two layers (core and cladding) are designed using evolutionary methods. The basic purpose of the design is to obtain the materials used as core and cladding, and the radius of the core. The results must verified minimum dispersion, minimum dispersion slope and maximum power transmitted in the core region. Both material and waveguide dispersion are considered in designing operation.
The evolutionary algorithm introduces two fibers with small dispersion and dispersion slope in the wavelength range (1500-1600 nm). The proposed method can set zero dispersion wavelengths with high accuracy. The final results are two different designs, with different materials and core radius.

In our research each chromosome consists of three genes, the first one represents the first material (core material), the second represents the second material (cladding material) and the third gene represents the core radius. Each materials will be defined by its refractive index as a function of wavelength using Sellmeier equation.

5) The Programming Code

An programming code (EVOFIBER) in FORTRAN language are used to evaluate the basic equations (12-18) and determine the materials of the core and cladding that can achieved minimum total dispersion along specific or wide range of wavelengths. Also this code calculate the core radius, the material and waveguide dispersion, the dispersion slope and the percentage of the energy transferred in both core and cladding relative to incident wave energy.

Results and Discussion

An optimization method is proposed to design a fiber structure by using Evolutionary algorithm. The parameters used as degrees of freedom in the optimization in this case include optical (refractive indices of the core and cladding) and geometrical dimension of the core.

The core or cladding materials are defined by Sellmeier formula. These materials are tabulated in Appendix (A) with some information about the composition of the doping materials.

The program (EVOFIBER) will check randomly all core and cladding materials and core radius which verify the boundary conditions. The best two designs that give minimum total dispersion along the whole wavelength range (1500-1600 nm) are tabulated in Table (1). The first design consists of material number (23) as core and material (21) as cladding material (Appendix A). The second design consists of material number (2) as core material and material (3) as cladding material.

The total dispersion of design (1) is ranging between (-0.4 to 0.06 ps/km/nm), the zero dispersion wavelength is at (1542.77 nm) and the core radius is 1.282 µm. The dispersion slope for this design is ranging between (0.012 to 0.0062 ps/km/nm²). The width of total dispersion is (0.83 ps/km/nm) for the whole wavelength range. These results are concluded in Table (2) and Figures (1 and 2).

The total dispersion of design (2) is ranging between (-2.196 to 2.348 ps/km/nm), the zero dispersion wavelength is (1545.94 nm) and the core radius is 3.014 µm. The dispersion slope is ranging between (0.042 to 0.050 ps/km/nm²). The width of total dispersion is (4.54 ps/km/nm) for the whole wavelength range. These results are concluded in Table (3) and Figures (3 and 4).

These results indicate that the design (1) is much better than design (2) since the width of the total dispersion of design (1) is less than the one of design (2), moreover the dispersion slope are smaller.

Figures (5, 6) show the total dispersion for different core radius. One can conclude that the total dispersion is very sensitive to change in core radius especially in design (1),
while this sensitivity is less for design (2). The selection of core radius is very critical to achieve a specified total dispersion for the single mode fiber, and hence the manufacturing tolerance or the deformation of the fiber core radius during installation. The evolutionary algorithm is considered as a powerful method for exploring the whole possible solutions comparing with other optimization methods.

References
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Table (1) The designing results

<table>
<thead>
<tr>
<th>Core material</th>
<th>Cladding material</th>
<th>Core Radius µm</th>
<th>Zero Dispersion wavelength nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design (1)</td>
<td>23</td>
<td>21</td>
<td>1.282</td>
</tr>
<tr>
<td>Design (2)</td>
<td>2</td>
<td>3</td>
<td>3.014</td>
</tr>
</tbody>
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Table (2) The results of program (EVOFIBER) for design (1)

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>Core Ref. index</th>
<th>Cladding Ref. index</th>
<th>W.G. Dispersion ps/km/nm</th>
<th>Material Dispersion ps/km/nm</th>
<th>Total Dispersion ps/km/nm</th>
<th>Dispersion Slope ps/km/nm²</th>
<th>Core Power %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1500</td>
<td>1.50793659</td>
<td>1.43924761</td>
<td>-25.89636</td>
<td>25.49444</td>
<td>-0.40192</td>
<td>0.01201</td>
<td>83.16</td>
</tr>
<tr>
<td>1510</td>
<td>1.50825787</td>
<td>1.43910944</td>
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<td>26.50615</td>
<td>-0.29931</td>
<td>0.00943</td>
<td>82.91</td>
</tr>
<tr>
<td>1520</td>
<td>1.50812173</td>
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<td>-0.10792</td>
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<td>1540</td>
<td>1.50784755</td>
<td>1.43869174</td>
<td>-29.53094</td>
<td>29.50757</td>
<td>-0.02337</td>
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<td>82.13</td>
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<tr>
<td>1550</td>
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<td>-30.43747</td>
<td>30.49833</td>
<td>0.06086</td>
<td>0.00843</td>
<td>81.86</td>
</tr>
<tr>
<td>1560</td>
<td>1.50757110</td>
<td>1.43841052</td>
<td>-31.43516</td>
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<tr>
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<tr>
<td>1590</td>
<td>1.50715196</td>
<td>1.43798399</td>
<td>-34.06124</td>
<td>34.42418</td>
<td>0.36295</td>
<td>0.00627</td>
<td>80.79</td>
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<tr>
<td>1600</td>
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<td>1.43784058</td>
<td>-34.96256</td>
<td>35.39838</td>
<td>0.43582</td>
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<td>80.52</td>
</tr>
</tbody>
</table>

Table (3) The results of program (EVOFIBER) for design (2)

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>Core Ref. index</th>
<th>Cladding Ref. index</th>
<th>W.G. Dispersion ps/km/nm</th>
<th>Material Dispersion ps/km/nm</th>
<th>Total Dispersion ps/km/nm</th>
<th>Dispersion Slope ps/km/nm²</th>
<th>Core Power %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1500.00</td>
<td>1.46646321</td>
<td>1.45591492</td>
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<td>4.60430</td>
<td>-2.19615</td>
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<tr>
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<td>7.83674</td>
<td>0.18564</td>
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<tr>
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<td>-7.82013</td>
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<td>76.39</td>
</tr>
</tbody>
</table>
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Figure (1) Dispersion for design (1) versus wavelength

Figure (2): Dispersion slope for design (1) versus wavelength

Figure (3): Dispersion for design (2) versus wavelength

Figure (4): Dispersion slope for design (2) versus wavelength
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Figure (5) Total Dispersion versus core radius at the whole wavelength range for Design (1).

Figure (6) Total Dispersion versus core radius at the whole wavelength range for Design (2).
The Sellmeier coefficient of the refractive indices of the choosing materials.

<table>
<thead>
<tr>
<th>No</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>Composition</th>
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<tbody>
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<td>0.080679030</td>
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</tr>
<tr>
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<tr>
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<td>0.7925634</td>
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</tr>
<tr>
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