Deflection Estimation of Un-Symmetric Isotropic Cam with Three Circular-Arc Contact Profiles

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ABSTRACT
In this paper the principal objectives is to design a suitable profile that produces minimum value of jerk and contact stress keeping the acceleration within a limit especially in high-speed machine. Many works in the experimental part are done on the synthesis of cam profile in accuracy and system flexibility on the output follower motion; but there is a lack in the analytical part. The analytical formulation has been done with classical plate theory of un-symmetric cam with three circular-arc contact profiles using the equation of circular plate solution due to the distributed load comes from the perpendicular contact harmonic motion of the follower. The cam used in the paper can be found in cutting and metal forming tools, heavy duty of marine engine, and fast manufacturing equipment. The aim of the present paper is to calculate the maximum deflection on cam boundaries varying with \( r \) and \( \theta \) coordinates between beginning and ending of contact follower loadings. The results were classified into mathematical model and finite element using software ANSYS.

Keywords: Cam Profile, Contact Loading, Circular Plate, Finite element, Un-symmetric Cam.

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Nomenclatures:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Normal Letters</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Major distance of Hertzian contact axis</td>
<td>m</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>Minor distance of Hertzian contact axis</td>
<td>m</td>
<td></td>
</tr>
<tr>
<td>a₁</td>
<td>Major distance of ellipse axis</td>
<td>m</td>
<td></td>
</tr>
<tr>
<td>b₁</td>
<td>Minor distance of ellipse axis</td>
<td>m</td>
<td></td>
</tr>
<tr>
<td>a₂</td>
<td>Difference radius of curvatures between ellipse and semi-circle centers</td>
<td>m</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>Particular solution constant</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>Bending rigidity</td>
<td>N, m</td>
<td></td>
</tr>
<tr>
<td>E₁, E₂</td>
<td>Modulus of elasticity for both cam and follower</td>
<td>N/m²</td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>Length of simply-supported beam</td>
<td>m</td>
<td></td>
</tr>
<tr>
<td>L₄</td>
<td>Difference length between two points of contact</td>
<td>m</td>
<td></td>
</tr>
<tr>
<td>m, n</td>
<td>Functions of the geometry of the contact surfaces</td>
<td></td>
<td></td>
</tr>
<tr>
<td>m₃</td>
<td>Single trigonometric of Fourier series</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M₁, M₂, M₃</td>
<td>Circular plate bending and twist moment</td>
<td>N.m</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Greek Letters</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>σ_y</td>
<td>Yield Stress</td>
<td>N/m²</td>
<td></td>
</tr>
<tr>
<td>π</td>
<td>Constant (3.1416)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ν₁, ν₂</td>
<td>Poisson's ratio for both cam and follower</td>
<td>Degree</td>
<td></td>
</tr>
<tr>
<td>α</td>
<td>Angle depend upon the functions geometry m, n</td>
<td>Degree</td>
<td></td>
</tr>
<tr>
<td>θ₁, θ₂</td>
<td>Angles of the beginning and the ending for both flanks and noses</td>
<td>Rad, Rad</td>
<td></td>
</tr>
</tbody>
</table>

INTRODUCTION

Shape optimization of a two-dimensional cam profile that produces minimum values of jerk and contact stress is designed in a heavily constrained environment keeping the peak values of acceleration within a
ANALYTICAL PROCEDURE:

Higher values of acceleration and jerk of the cam-follower driven system are never desirable as these affect smoothness of operation of the system and generate force that induces high contact stress on the cam surface. In this study of general contact loading case, assuming elastic and isotropic material behavior, Hertz showed that the intensity of pressure between the contacting surfaces could be represented by the elliptical (or, rather, semi-ellipsoid) construction shown in Fig. 1, [10]. Maximum contact pressure \( P_0 \) for shakedown of the contact region is:

\[
P_0 = 0.6 \cdot \sigma_y
\]

And the total contact load is given by the volume of the semi-ellipsoid, [10]:

\[
P = \frac{2\pi a b P_0}{3}
\]

Where:

\[
a = m \cdot \left[ \frac{3P \Delta}{4A_4} \right]^{1/3}, \quad b = n \cdot \left[ \frac{3P \Delta}{4A_4} \right]^{1/3}
\]

And:

\[
A_4 = \frac{1}{2} \left[ \frac{1}{R_1} + \frac{1}{R_1'} + \frac{1}{R_2} + \frac{1}{R_2'} \right]
\]

\[
\Delta = \frac{1}{E_1} \left( 1 - \nu_1^2 \right) + \frac{1}{E_2} \left( 1 - \nu_2^2 \right)
\]

For flat-sided \( R_1 \) will be the wheel radius and \( R_1' \) will be infinite. Similarly for railway lines with head radius \( R_2 \) the value of \( R_2' \) will be infinite to produce the flat length of radii. Also:

\[
\psi = 90^\circ, \text{ because the contact load is perpendicular to the cam profile.}
\]

i.e;
\[ \alpha = \cos^{-1} \left( \frac{B_1}{A_1} \right) \]

Also:

\[ B_1 = \frac{1}{2} \left( \left( \frac{1}{R_1} - \frac{1}{R_2} \right)^2 + \left( \frac{1}{R_2} - \frac{1}{R_1} \right)^2 \right) + 2 \left( \frac{1}{R_2} - \frac{1}{R_1} \right) * \left( \frac{1}{R_1} - \frac{1}{R_2} \right) * \cos (2 * \psi) \]

\[ \text{(5)} \]

According to the value of angle (\( \alpha \)), it can be found the value of n, m from Table (1), [11]:

**Table (1) The value of angle (\( \alpha \)) against the values of m, and n.**

<table>
<thead>
<tr>
<th>( \alpha ) (degree)</th>
<th>20</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
<th>55</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>3.778</td>
<td>2.731</td>
<td>2.397</td>
<td>2.136</td>
<td>1.926</td>
<td>1.754</td>
<td>1.611</td>
<td>1.486</td>
</tr>
<tr>
<td>n</td>
<td>0.408</td>
<td>0.493</td>
<td>0.530</td>
<td>0.567</td>
<td>0.604</td>
<td>0.641</td>
<td>0.678</td>
<td>0.717</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \alpha ) (degree)</th>
<th>65</th>
<th>70</th>
<th>75</th>
<th>80</th>
<th>85</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>1.378</td>
<td>1.284</td>
<td>1.202</td>
<td>1.128</td>
<td>1.061</td>
<td>1</td>
</tr>
<tr>
<td>n</td>
<td>0.759</td>
<td>0.802</td>
<td>0.846</td>
<td>0.893</td>
<td>0.944</td>
<td>1</td>
</tr>
</tbody>
</table>

From the circular plate equation as a function of \((r, \text{and } \theta)\) coordinates is,[12]:

\[
\left( \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} \right) * M_r + \left( -\frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right) * M_\theta \\
+ \left( -\frac{2}{r} \frac{\partial^2}{\partial r \partial \theta} - \frac{2}{r^2} \frac{\partial}{\partial \theta} \right) * M_{r\theta} + q = 0. \text{[12]}
\]

\[
\text{…..(6)}
\]

Where:

For isotropic plate, the bending and twist moments are, [12]:

\[ M_r = -D * \left[ \frac{\partial^2 w}{\partial r^2} + \nu * \left( \frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} \right) \right] \]

\[ M_\theta = -D * \left[ \frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} \right] + \nu * \frac{\partial^2 w}{\partial r^2} \]

\[ M_{r\theta} = D * (1 - \nu) * \left[ \frac{1}{r} \frac{\partial^2 w}{\partial r \partial \theta} - \frac{1}{r^2} \frac{\partial w}{\partial \theta} \right] \]

The value of first term of eq. (6) is:
\[
\left( \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} \right) M_r = -D \left( \frac{\partial^4 w}{\partial r^4} \frac{1}{r^2} + \frac{2}{r} \frac{\partial^3 w}{\partial r^3} \frac{1}{r^2} + \frac{2 + 2\nu}{r^4} \frac{\partial^2 w}{\partial r^2} \frac{1}{r} + \frac{2 + 2\nu}{r^4} \frac{\partial^3 w}{\partial r^2} \frac{1}{r} + \frac{2 + 2\nu}{r^4} \frac{\partial^4 w}{\partial r^2} \frac{1}{r^2} \right)
\]

And the value of second term of eq. (6) is:

\[
\left(-\frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2}\right) M_0 = -D \left( -\frac{1}{r^2} \frac{\partial^2 w}{\partial r^2} + \frac{1}{r^3} \frac{\partial w}{\partial r} + \frac{2}{r^4} \frac{\partial^3 w}{\partial r^3} + \frac{2 + \nu}{r^4} \frac{\partial^2 w}{\partial r^2} \frac{1}{r} + \frac{1}{r^4} \frac{\partial^4 w}{\partial r^4} \right)
\]

Also the value of third term of eq. (6) is:

\[
\left(-\frac{2}{r^2} \frac{\partial^2}{\partial r \partial \theta} - \frac{2}{r^2} \frac{\partial^2}{\partial \theta^2}\right) M_{10} = D \left( -\frac{2}{r^2} \frac{\partial^2 w}{\partial r^2} + \frac{2}{r^3} \frac{\partial w}{\partial r} + \frac{2}{r^4} \frac{\partial^3 w}{\partial r^3} + \frac{2 + \nu}{r^4} \frac{\partial^2 w}{\partial r^2} \frac{1}{r} + \frac{1}{r^4} \frac{\partial^4 w}{\partial r^4} \right)
\]

It can be put the three terms derived above in eq. (6) to obtain:

\[
\frac{\partial^4 w}{\partial r^4} + \frac{2}{r^2} \frac{\partial^3 w}{\partial r^3} + \frac{1}{r^3} \frac{\partial^2 w}{\partial r^2} + \frac{2 + \nu}{r^4} \frac{\partial^2 w}{\partial r^2} + \frac{4}{r^3} \frac{\partial^3 w}{\partial r^3} + \frac{1}{r^4} \frac{\partial^4 w}{\partial r^4} = \frac{q}{D}
\]

\[\ldots(7)\]

The homogenous solution of eq. (7) is, [13]:

\[
w(r, \theta) = \sum_{m=1}^{\infty} \left[ R_m \sin (m \theta) + R_m \cos (m \theta) \right]
\]

Put the homogenous solution in eq. (7) to obtain:

\[
w(r, \theta) = \sum_{m=1}^{\infty} \left[ \left( A_m r^m + B_m r^{-m} + C_m r^{m+2} + D_m r^{-m-2} \right) \sin (m \theta) + \left( A_m' r^m + B_m' r^{-m} + C_m' r^{m+2} + D_m' r^{-m-2} \right) \cos (m \theta) \right]
\]

For un-symmetric cam the deflection, slope, and the moment must be infinite at the center of the plate, then the homogenous will become:

\[
w(r, \theta) = \sum_{m=1}^{\infty} \left[ \left( A_m r^m + C_m r^{m+2} \right) \sin (m \theta) + \left( A_m' r^m + C_m' r^{m+2} \right) \cos (m \theta) \right]
\]

Apply the infinite series on the homogenous solution as below:

\[
(r^m \sin(m \theta)) \sin(m \theta) = (1 + r + r^2 + r^3 + r^4 + r^5 + \cdots) \sin(m \theta) - \frac{(m \theta)^3}{3!} + \frac{(m \theta)^5}{5!} + \cdots = n \theta - \frac{(m \theta)^3}{3!} + \frac{(m \theta)^5}{5!} + \cdots
\]

It can be taken into account the un-symmetric terms (odd functions) because the un-symmetric isotropic cam and ignore the symmetric terms (even functions) to obtain:
\[(r^m \sin (m \cdot \theta)) = (r \cdot m \cdot \theta - \frac{(r \cdot m \cdot \theta)^3}{3!} + \frac{(r \cdot m \cdot \theta)^5}{5!}) = \sin (r \cdot m \cdot \theta)\]

It can be taken the first mode, \(m = 1\):

\[(r^m \sin (m \cdot \theta)) = \sin (r \cdot \theta)\]

Also the same procedure for the second term:

\[(r^{m+2} \sin (m \cdot \theta)) = \sin (r \cdot \theta)\]

As before:

\[(r^m \cos (m \cdot \theta)) = (1 + r + r^2 + r^3 + r^4 + \cdots) \cdot (1 - \frac{(m\cdot\theta)^2}{2!} + \frac{(m\cdot\theta)^4}{4!} - \cdots) =
\]
\[1 - \frac{(m\cdot\theta)^2}{2!} + \frac{(m\cdot\theta)^4}{4!} - \frac{(m\cdot\theta)^6}{6!} + \frac{(m\cdot\theta)^8}{8!} - \frac{(m\cdot\theta)^{10}}{10!} + \frac{(m\cdot\theta)^{12}}{12!} - \frac{(m\cdot\theta)^{14}}{14!} + \frac{(m\cdot\theta)^{16}}{16!} - \cdots\]

As above it can be taken into account the un-symmetric terms (odd functions) because the un-symmetric isotropic cam and ignore the symmetric terms (even functions) to obtain:

\[(r^m \cos (m \cdot \theta)) = (1 - \frac{(r \cdot m \cdot \theta)^2}{2!} + \frac{(r \cdot m \cdot \theta)^4}{4!}) = \cos (r \cdot m \cdot \theta)\]

It can be taken the first mode \(m = 1\):

\[(r^m \cos (m \cdot \theta)) = \cos (r \cdot \theta)\]

Also the same procedure for the second term:

\[(r^{m+2} \cos (m \cdot \theta)) = \cos (r \cdot \theta)\]

The homogenous solution of eq. (7) will become:

\[w(r, \theta) = A \cdot \sin (r \cdot \theta) + B \cdot \cos (r \cdot \theta), \quad [13] \]

(8)

Where:

\(A\) and \(B\) are constants.

It can be derived the homogenous solution (1, 2, 3, 4) times with respect to \(r\) and \(\theta\) to obtain:

\[\frac{\partial w}{\partial r} = A \cdot \theta \cdot \cos (r \cdot \theta) - B \cdot \theta \cdot \sin (r \cdot \theta)\]

\[\frac{\partial^2 w}{\partial r^2} = -A \cdot \theta^2 \cdot \sin (r \cdot \theta) - B \cdot \theta^2 \cdot \cos (r \cdot \theta)\]

\[\frac{\partial^3 w}{\partial r^3} = -A \cdot \theta^3 \cdot \cos (r \cdot \theta) + B \cdot \theta^3 \cdot \sin (r \cdot \theta)\]
\[ \frac{\partial^4 w}{\partial r^4} = A \cdot \theta^4 \cdot \sin(r \cdot \theta) + B \cdot \theta^4 \cdot \cos(r \cdot \theta) \]

\[ \frac{\partial^4 w}{\partial r^2 \partial \theta^2} = -A \cdot \left( -\theta^2 \cdot r^2 \cdot \sin(r \cdot \theta) + 4 \cdot \theta \cdot r \cdot \cos(r \cdot \theta) + 2 \cdot \sin(r \cdot \theta) \right) - B \cdot \left( -\theta^2 \cdot r^2 \cdot \cos(r \cdot \theta) - 4 \cdot \theta \cdot r \cdot \sin(r \cdot \theta) + 2 \cdot \cos(r \cdot \theta) \right) \]

\[ \frac{\partial^3 w}{\partial r \partial \theta^2} = A \cdot \left( -\theta \cdot r^2 \cdot \sin(r \cdot \theta) - 2 \cdot r \cdot \cos(r \cdot \theta) \right) - B \cdot \left( \theta \cdot r \cdot \cos(r \cdot \theta) \right) \]

\[ \frac{\partial^4 w}{\partial \theta^4} = -A \cdot r^2 \cdot \sin(r \cdot \theta) - B \cdot r^2 \cdot \cos(r \cdot \theta) \]

\[ \frac{\partial^4 w}{\partial \theta^6} = A \cdot r^4 \cdot \sin(r \cdot \theta) + B \cdot r^4 \cdot \cos(r \cdot \theta) \]

After putting the above derivatives in eq. (7) and after simplification to obtain:

\[ w(r, \theta) = \left( \frac{1}{2 \cdot 0^3 \cdot r^3 + 6 \cdot 0^2 \cdot 0 \cdot 0} \right) \cdot A \cdot \cos(r \cdot \theta) - \left( \frac{1}{2 \cdot 0^3 \cdot r^3 + 6 \cdot 0^2 \cdot 0 \cdot 0} \right) \cdot B \cdot \sin(r \cdot \theta) \]

And the particular solution is, [13]:

\[ w(r, \theta) = \sum_{m=1,3,5}^{\infty} \left( \frac{C \cdot r^4 \cdot \sin(m \cdot \theta)}{C \cdot r^4 \cdot \sin(m \cdot \theta)} \right) \]  

(9)

Applying the infinite series on the particular solution as below:

\[ r^4 \cdot \sin(m \cdot \theta) = \left( 1 + r + r^2 + r^3 + r^4 + \cdots \right) \cdot \left( \frac{(m \cdot \theta)^3}{3!} + \frac{(m \cdot \theta)^5}{5!} \right) = m \cdot \theta - \frac{(m \cdot \theta)^3}{3!} + \frac{(m \cdot \theta)^5}{5!} + r \cdot m \cdot \theta - r \cdot \frac{(m \cdot \theta)^3}{3!} + r \cdot \frac{(m \cdot \theta)^5}{5!} \]

Ignoring the higher order terms of the above series and applying the symmetric and un-symmetric terms:

\[ r^4 \cdot \sin(m \cdot \theta) = r \cdot m \cdot \theta \]

It can be taken the first mode, \( m = 1 \):

The particular solution of eq. (7) will become:

\[ w(r, \theta) = C \cdot r \cdot \theta \]

(11)

Substituting eq. (11) into plate equation eq. (7) and finding the value of constant (C):

\[ C = \frac{q \cdot r^3}{\theta \cdot D} \]

\[ w(r, \theta) = \frac{q \cdot r^4}{D} \]

(12)
The complementary solution of deflection is as below:

\[ w(r, \theta) = w(r, \theta)_H + w(r, \theta)_P \]

\[ w(r, \theta) = \left( \frac{2\theta^3 r^2 + 6r^2 \theta^2 - \theta}{(4\theta^3 r^3 + 2\theta^2 r^2 + 2r^2 \theta^2 - 4r^2 + r^3)} \right) * A \cdot \cos(r \cdot \theta) + \left( \frac{2\theta^3 r^2 + 6r^2 \theta^2 - \theta}{(4\theta^3 r^3 + 2\theta^2 r^2 + 2r^2 \theta^2 - 4r^2 + r^3)} \right) * B \cdot \sin(r \cdot \theta) + \frac{q r^4}{D} \]  

(13)

Applying the boundary conditions to eq. (13), the constants (A and B) can be obtained:

At  \[ r = r_1 = 2.5 \text{ cm}, \theta = \theta_1, w(r, \theta) = 0 \]

At  \[ r = r_1 = 2.5 \text{ cm}, \theta = \theta_2, w(r, \theta) = 0 \]

Where: \( \theta_1 \) and \( \theta_2 \) vary along each flank and nose profile.

Then:

\[ A = \frac{C_1 + C_3 \sec(r_1 \cdot \theta_2) \cdot \cos(r_1 \cdot \theta_1) \cdot \tan(r_1 \cdot \theta_2) - C_2 \cdot C_3 \cdot \tan(r_1 \cdot \theta_2) - C_3 \cdot \sec(r_1 \cdot \theta_2)}{C_1 \cdot C_2 \cdot \tan(r_1 \cdot \theta_2) \cdot \cos(r_1 \cdot \theta_1) \cdot \sin(r_1 \cdot \theta_1)} \]

\[ B = \frac{C_1 \cdot C_3 \cdot \sec(r_1 \cdot \theta_2) \cdot \cos(r_1 \cdot \theta_1) - C_2 \cdot C_3}{C_1 \cdot C_2 \cdot \tan(r_1 \cdot \theta_2) \cdot \cos(r_1 \cdot \theta_1) - \sin(r_1 \cdot \theta_1)} \]

Where:

\[ C_1 = \frac{2\theta^3 r^2 + 6r^2 \theta^2 - \theta_1}{\theta^3 r^3 + 2\theta^2 r^2 + 2r^2 \theta^2 - 4r^2 + r^3} \]

\[ C_2 = \frac{2\theta^3 r^2 + 6r^2 \theta^2 - \theta_2}{\theta^3 r^3 + 2\theta^2 r^2 + 2r^2 \theta^2 - 4r^2 + r^3} \]

\[ C_3 = \frac{q r^4}{D} \]

\[ \therefore \quad w(r, \theta) = \left( \frac{2\theta^3 r^2 + 6r^2 \theta^2 - \theta}{(4\theta^3 r^3 + 2\theta^2 r^2 + 2r^2 \theta^2 - 4r^2 + r^3)} \right) * A \cdot \cos(r \cdot \theta) + \left( \frac{2\theta^3 r^2 + 6r^2 \theta^2 - \theta}{(4\theta^3 r^3 + 2\theta^2 r^2 + 2r^2 \theta^2 - 4r^2 + r^3)} \right) * B \cdot \sin(r \cdot \theta) + \frac{q r^4}{D} \]

\[ \cos(r \cdot \theta) = \left( \frac{2\theta^3 r^2 + 6r^2 \theta^2 - \theta}{(4\theta^3 r^3 + 2\theta^2 r^2 + 2r^2 \theta^2 - 4r^2 + r^3)} \right) * \left[ C_1 \cdot C_2 \cdot \sec(r_1 \cdot \theta_2) \cdot \cos(r_1 \cdot \theta_1) - C_2 \cdot C_3 \right] + \frac{q r^4}{D} \]

\[ \sin(r \cdot \theta) = \left( \frac{2\theta^3 r^2 + 6r^2 \theta^2 - \theta}{(4\theta^3 r^3 + 2\theta^2 r^2 + 2r^2 \theta^2 - 4r^2 + r^3)} \right) * \left[ C_1 \cdot C_2 \cdot \sec(r_1 \cdot \theta_2) \cdot \cos(r_1 \cdot \theta_1) - C_2 \cdot C_3 \right] + \frac{q r^4}{D} \]

\[ \ldots (14) \]

It can be assumed that the two points of contact load are as the simply-supported beam subjected to distributed load (\( P_0 \)) per unit length of point loading on the projected area (a \* b), then:
Deflection Estimation of Un-Symmetric Isotropic Cam with Three Circular-Arc Contact Profiles

\[ q = P_o \times \frac{2\pi}{3} \times \frac{L^2}{8} \times \frac{1}{L_1} \] ........(15)

Where:

\( L \): is the length of simply-supported beam.

\( L_2 \): is the difference length between two points of contact.

In this study the cam profile boundary contains from three flanks and two noses with its dimensions, [10], as illustrated in Fig. (1); but it can be applied the equation of circular plate on (flank no.2 and a part of nose no.1) and (flank no.3 and a part of nose no.2) to obtain the value of deflection. Also the deflection of (flank no.1, a part of nose no.1, and a part of nose no. 2) can be found using the superposition theorem with the cam profile points as indicated in Fig. (2) By applying the elliptic and the semi-circle plate equations as below, [13]:

The elliptic equation is:

\[ w(x, y) = \frac{q \times a_1^2 \times b_1^2}{8 \times D} + \frac{\left(\frac{x^2}{a_1^2} + \frac{y^2}{b_1^2} - 1\right)^2}{(3 \times a_1^4 + 3 \times b_1^4 + 2 \times a_1^2 \times b_1^2)} \] ........(16)

Where: \( a_1 \) and \( b_1 \) is the major and minor distance axis of ellipse.

And the semi-circle equation is:

\[ w(r, \theta) = \sum_{m_1=1,3,5}^{\infty} \left[ \frac{4 \times q \times r^4}{\pi \times m_1 \times (16 - m_1^2) \times (4 - m_1^2) \times D} + A_{1m_1} \times r^{m_1} + A_{3m_1} \times r^{m_1+2} \right] \times \sin (m_1 \times \theta) \] ........(17)

Where:

\[ A_{1m_1} = \frac{-2 \times q \times (m_1 + 1) \times a_2^{4-m_1}}{\pi \times m_1 \times (16 - m_1^2) \times (4 - m_1^2) \times D} \]

\[ A_{3m_1} = \frac{2 \times q \times (m_1 - 1) \times a_2^{2-m_1}}{\pi \times m_1 \times (16 - m_1^2) \times (4 - m_1^2) \times D} \]

Figure (1) Cam Profile Specifications.
Figure (2) The Points of Cam Profile with superposition theorem.

Numerical procedure:
For comparison, a static analysis was carried out with ANSYS Program software. The linear elastic isotropic model is used to investigate the deflection on cam profile boundaries. For this problem, the (SHELL 99) element is used. This element is used for the two-dimensional modeling of shell structure and is defined by eight nodes having six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z axes. The mesh generation of cam profile can be shown in Fig. (3).

Figure (3) The mesh generation of cam un-symmetric cam profile.
Figure (4) shows the deflection of cam profile against angle of contact for nose no. 1, flank no.1, and nose no. 2. The deflection of cam profile increases with the increasing of the angle of contact. The values of deflection of flank no. 1 vary randomly because the curve degree of flank no.1 is very high than nose no. 1 and nose no. 2.

Figure (5) shows the deflection of cam profile against angle of contact for flank no. 2 and nose no. 1. The deflection of cam profile vary sinosoidaly with the angle of contact for flank no. 2 and nose no. 1 because the contact loading vary sinosoidaly at cam profile boundary. The percentage error between of deflection
obtained from the analytical results and ANSYS results is very high than Fig. (4) because the difference in length between two points of contact ($L_a$) is not accurate.

![Graph](image.png)

Figure (6) Deflection of cam profile against angle of contact for nose no. 2 and flank no. 3.

Fig. (6) shows the deflection of cam profile against angle of contact for nose no. 2 and flank no. 3. The values of deflection decrease with the increasing of the angle of contact because the contact loading decreases with the increasing of the angle of contact.

Table (2) The values of deflection varies with point's number of nose no. 1, flank no.1, and nose no. 2 for the un-symmetric pressure angle from the beginning and the ending points of contact.

<table>
<thead>
<tr>
<th>Points Number</th>
<th>Analytical Results (m)</th>
<th>ANSYS Results (m)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.00120034</td>
<td>0.0012824</td>
<td>6.3989 %</td>
</tr>
<tr>
<td>4</td>
<td>0.0014627</td>
<td>0.0013535</td>
<td>7.4656 %</td>
</tr>
<tr>
<td>5</td>
<td>0.0013924</td>
<td>0.0014197</td>
<td>1.9229 %</td>
</tr>
<tr>
<td>6</td>
<td>0.0015784</td>
<td>0.0014434</td>
<td>8.5529 %</td>
</tr>
<tr>
<td>7</td>
<td>0.0015447</td>
<td>0.0014636</td>
<td>5.2502 %</td>
</tr>
<tr>
<td>8</td>
<td>0.00140015</td>
<td>0.0014526</td>
<td>3.6107 %</td>
</tr>
<tr>
<td>9</td>
<td>0.00149682</td>
<td>0.0015132</td>
<td>1.0824 %</td>
</tr>
<tr>
<td>10</td>
<td>0.00161237</td>
<td>0.0015879</td>
<td>1.5176 %</td>
</tr>
</tbody>
</table>
Deflection Estimation of Un-Symmetric Isotropic Cam with Three Circular-Arc Contact Profiles

<table>
<thead>
<tr>
<th>Points Number</th>
<th>Analytical Results (m)</th>
<th>ANSYS Results (m)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>0.00160498</td>
<td>0.0017259</td>
<td>7.0062 %</td>
</tr>
<tr>
<td>12</td>
<td>0.00187767</td>
<td>0.0019148</td>
<td>1.9391 %</td>
</tr>
<tr>
<td>13</td>
<td>0.002625078</td>
<td>0.0024764</td>
<td>5.6637 %</td>
</tr>
<tr>
<td>14</td>
<td>0.00245408</td>
<td>0.0026312</td>
<td>6.7315 %</td>
</tr>
<tr>
<td>15</td>
<td>0.003052388</td>
<td>0.0028545</td>
<td>6.4830 %</td>
</tr>
<tr>
<td>16</td>
<td>0.003525277</td>
<td>0.0037995</td>
<td>7.2173 %</td>
</tr>
<tr>
<td>17</td>
<td>0.003883302</td>
<td>0.0042693</td>
<td>9.0412 %</td>
</tr>
<tr>
<td>18</td>
<td>0.005356948</td>
<td>0.0049005</td>
<td>8.5206 %</td>
</tr>
<tr>
<td>19</td>
<td>0.005874945</td>
<td>0.0053147</td>
<td>9.5361 %</td>
</tr>
<tr>
<td>20</td>
<td>0.005239998</td>
<td>0.0056304</td>
<td>6.9338 %</td>
</tr>
<tr>
<td>21</td>
<td>0.005647505</td>
<td>0.0057659</td>
<td>2.0533 %</td>
</tr>
<tr>
<td>22</td>
<td>0.0059227</td>
<td>0.0054491</td>
<td>7.9963 %</td>
</tr>
</tbody>
</table>

Table (2) shows the values of deflection varies with point's number of nose no. 1, flank no.1, and nose no. 2 for the un-symmetric pressure angle from the beginning and the ending points of contact. The deflection of cam boundary profile increased transiently with the increasing of point's number on cam boundary because varying the radius of curvature at these points from the point of beginning at nose no.1 to the point of ending at nose no. 2.

Table (3) The values of deflection varies with point's number of flank no. 2 and nose no. 1 of the un-symmetric pressure angle from the beginning and the ending points of contact.

<table>
<thead>
<tr>
<th>Points Number</th>
<th>Analytical Results (m)</th>
<th>ANSYS Results (m)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>68</td>
<td>0.0010844</td>
<td>0.00093793</td>
<td>13.507 %</td>
</tr>
<tr>
<td>69</td>
<td>0.00115413</td>
<td>0.0010993</td>
<td>4.7507 %</td>
</tr>
<tr>
<td>70</td>
<td>0.00115213</td>
<td>0.0012491</td>
<td>7.7631 %</td>
</tr>
<tr>
<td>71</td>
<td>0.00123124</td>
<td>0.0012932</td>
<td>4.7912 %</td>
</tr>
<tr>
<td>72</td>
<td>0.00123323</td>
<td>0.0013015</td>
<td>5.2454 %</td>
</tr>
<tr>
<td>73</td>
<td>0.00132137</td>
<td>0.0013883</td>
<td>4.821 %</td>
</tr>
</tbody>
</table>
Table (3) shows the values of deflection varies with point's number of flank no. 2 and nose no. 1 of the un-symmetric pressure angle from the beginning and the ending points of contact. The deflection of cam boundary profile increased transiently with the increasing of point's numbers on cam boundary and then the deflection can be decreased because varying the radius of curvature at these points from the point of beginning at flank no. 2 to the point of ending at nose no. 1. The percentage errors is very high in some locations because the contact beginning of the un-symmetric isotropic cam.

Table (4) The values of deflection varies with point's number of nose no. 2 and flank no. 3 of the un-symmetric pressure angle from the beginning and the ending points of contact.

<table>
<thead>
<tr>
<th>Points Number</th>
<th>Analytical Results (m)</th>
<th>ANSYS Results (m)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>0.01025422</td>
<td>0.0098552</td>
<td>3.8912 %</td>
</tr>
<tr>
<td>23</td>
<td>0.0102533</td>
<td>0.010417</td>
<td>1.5714 %</td>
</tr>
<tr>
<td>24</td>
<td>0.0108133</td>
<td>0.010394</td>
<td>3.8776 %</td>
</tr>
<tr>
<td>25</td>
<td>0.0100656</td>
<td>0.010599</td>
<td>5.0325 %</td>
</tr>
<tr>
<td>26</td>
<td>0.009128903</td>
<td>0.0094546</td>
<td>3.4448 %</td>
</tr>
<tr>
<td>27</td>
<td>0.0101918</td>
<td>0.0092871</td>
<td>8.8767 %</td>
</tr>
<tr>
<td>28</td>
<td>0.0089525</td>
<td>0.0090685</td>
<td>1.2791 %</td>
</tr>
<tr>
<td>29</td>
<td>0.00804704</td>
<td>0.0084081</td>
<td>4.2942 %</td>
</tr>
<tr>
<td>30</td>
<td>0.00701183</td>
<td>0.007521</td>
<td>6.7699 %</td>
</tr>
<tr>
<td>31</td>
<td>0.0077291</td>
<td>0.0071413</td>
<td>7.605 %</td>
</tr>
<tr>
<td>32</td>
<td>0.006876</td>
<td>0.006573</td>
<td>4.4066 %</td>
</tr>
</tbody>
</table>
Table (4) shows the values of deflection varies with point's number of nose no. 2 and flank no. 3 of the un-symmetric pressure angle from the beginning and the ending points of contact. The deflection of cam boundary profile decreased transiently with the increasing of point's number on cam boundary because varying the radius of curvature at these points from the point of beginning at nose no. 2 to the point of ending at flank no. 3.

CONCLUSIONS

1. The deflection of cam profile was increased because the radius of curvature and angle of contact were increased.
2. The contact loading is approximately constant for each nose and flank of cam profile because the radius of curvature is constant.
3. The maximum deflection will occur at the nose no. 2 having maximum pressure angle \(45^\circ\) of un-symmetric cam profile.
4. There is no deflection on the points of duel region profile because the radius of curvature, force, and acceleration were zero.

REFERENCES