Evaluation of Elastic Deflections and Bending Moments for Orthotropically Reinforced Concrete Rectangular slabs Supported on Three Edges Only

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Abstract

This paper deals with the evaluation of the elastic deflections and bending moments for orthotropically reinforced concrete rectangular slabs supported on three variously restrained edges with the fourth edge free and subjected to uniformly distributed load. Six cases are considered for such slabs to cover all possible restraining conditions at the three supported edges. Based on the finite difference approach, equations are derived to calculate the maximum values of the positive and negative bending moments as well as the maximum deflection in any of these six slab cases caused by the applied uniform load.

Keywords: finite difference, free edge, orthotropic, reinforced concrete, slab

حساب عزوم الأنحناء والأود المرن للبلاطات الخرسانية المستطيلة المسندة على ثلاث حافات وذات تسليح مختلف في الأتجاهين المتعامدين

الخلاصة

تم في هذا البحث أشتقاق معادلات لحساب الأود المرن وعزوم الأنحناء البلاطـــات الخرســـانية المستطيلة الشكل مسلحة تسليحا مختلفآ بالأتجاهين المتعامدين والمسندة على ثلاث حافات مختلفة التقييـــد مع حافة رابعة حرة ومعرضة لحمل منتشر. تم دراسة ستة حالات مختلفة لهذه البلاطات لتغطي جميع أنواع التقييد عند الحافات المسندة الثلاثة.أستخدمت طريقة الفروقات المحددة لأشتقاق المعادلات الازمـــة لحساب القيم العظمى لعزوم الأنحناء الموجبة والسالبة والأود الأعظم لجميـــع الحـــالات الـــسته لهـــذه البلاطات تحت تأثير الحمل المنتظم المسلط.

Notations

D_x , D_y	Flexural rigidity in x and y direction, respectively.
l_x	Span of rectangular slab in the x direction.
l_y	Span of rectangular slab in the y direction.
m	Ratio of the span in y direction to the span in x direction = l_y / l_x .
M_{ox}, M_{oy}	Yield bending moment per unit-width of slab, in x and y direction respectively.
V_x , V_y	Shear force per unit-width of slab, in x and y direction respectively.
q	Uniformly distributed load per unit area.
w	Vertical deflection of slab.
u	Poisson 's ratio.

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Introduction

It is still customary for structural engineers to use elastic methods in calculating induced bending moments in reinforced concrete (RC) slabs due to applied loading. This is simply done by referring to different codes of practice where tables are provided for direct calculations of elastic moments in reinforced concrete slabs but these are, however, restricted to either uniformly loaded one-way slabs or uniformly loaded two-way rectangular slabs supported on all sides. The elastic solution presented by other authors such as *Reynolds and Sleedman*, [(1)] and *Jofriet*[(2)], for calculating bending moments in rectangular slabs having other types of boundary conditions and subjected to either uniformly distributed load or triangular load, are also continuously referred to.

In the routine engineering practice, however, the designer is often faced with cases of panels having complex shapes, loading configuration and boundary conditions including free edges. In such cases the yield line theory due to *Johansen*[(3)] can be best consulted which makes it possible to determine the real load carrying capacity of a slab as well as identifying the actual mode of failure. This inelastic method of Johansen has already been recognised by different codes of practice as a powerful method of slab analysis and design; C.P 110[(4)] and the new edition B.S 8100[(5)] for instance recommend using the yield line theory in slab design. The application of the yield line theory in the analysis of different shapes of slabs under different types of loading has been carried out by several investigators. Among them are, for instance, *AL-Hassani and AL-Shadidi* [(6)] who adopted the virtual work method of the yield line theory approach to derive equations for predicting the collapse loads of orthotropically reinforced rectangular concrete slabs that have three edges supported with different moment restraints and a fourth edge free. Two familiar types of loading were considered; namely; uniformly distributed load and triangular loading, each covering the full slab area.

Based on the finite difference approach, equations are derived in this paper for the estimation of deflections and bending moments in orthotropically reinforced concrete rectangular slabs having one free edge and three supported edges and subjected to uniformly distributed load. Different boundary conditions are considered to exist at the three supported edges such that all the six possible cases of support conditions described in Figure (1) are covered.

The adopted method of analysis of this paper has much in common with the one presented by *Timoshenko and Woinowsky-Krieger* [(7)] for the analysis of isotropically reinforced concrete rectangular plates which have either all edges simply supported or all edges fixed and subjected to uniform load. Therefore his terminology has been followed wherever possible.

Finite differences

The method of finite differences is considered one of the most important numerical methods of approach. This technique eventually requires the solution of a system of linear algebraic equations. Such calculations are commonly performed by means of a digital computer employing matrix methods.

The solution of the deflection and bending problem in rectangular plates by using finite difference approach is achieved by dividing the plate into a square mesh. By this way the problem is reduced to the simultaneous solution of a set of algebraic equations, written for every nodal point within the plate.

The differential equation of the deflection surface for orthotropic plates was given by *Timoshenko and Woinowsky-Krieger*[(7)] as :

$$D_x \frac{\partial^4 w}{\partial x^4} + 2\sqrt{D_x D_y}\,\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = q \qquad \ldots(1)$$

where D_x , D_y = flexural rigidity in x and y direction, respectively.

Equation (1) can be used to investigate the bending of orthotropic rectangular plates having nonhomogeneous material such as reinforced concrete [(7)].

This equation in finite difference becomes;

See Eq.(1) in Appendix A

Timoshenko and Woinowsky-Krieger[(7)] have shown that the values of the bending moments for orthotropic rectangular plates are as follows :

$$M_{ox} = -\left(D_x \frac{\partial^2 w}{\partial x^2} + u \sqrt{D_x D_y} \frac{\partial^2 w}{\partial y^2} \right) \quad \ldots(2)$$

$$M_{oy} = -\left(D_y \frac{\partial^2 w}{\partial y^2} + u \sqrt{D_x D_y} \frac{\partial^2 w}{\partial x^2} \right) \quad \ldots(3)$$

where u = Poisson 's ratio

These two equations in finite difference become:

See Eqs.(2 and3) in Appendix A

While the differential equations for shear of orthotropic rectangular plates have been given by Timoshenko and Woinowsky-Krieger[(7)] as follows:

$$V_x = -\left[D_x \frac{\partial^3 w}{\partial x^3} + (2-u) \sqrt{D_x D_y} \frac{\partial^3 w}{\partial x \, \partial y^2} \right] \quad \ldots(4)$$

$$V_y = -\left[D_y \frac{\partial^3 w}{\partial y^3} + (2-u) \sqrt{D_x D_y} \frac{\partial^3 w}{\partial x^2 \, \partial y} \right] \quad \ldots(5)$$

By applying the principles of the finite difference method, equations (4) and (5) become

See Eqs.(4 and5) in Appendix A

The procedure of using these equations to determine deflections and bending moments in an orthotropic reinforced concrete rectangular slab can be best demonstrated through the following example.

Example Consider the case of a uniformly loaded rectangular plate with two adjacent edges simply supported, the third edge free, and the fourth edge built in (slab case 5). Use the finite difference method with i=a/4 to obtain the deflection (w) at various points. Take D_x=D, D_y=0.5D, and u =0.2.

Solution The domain is divided into twelve squares of sides a/4 (Figure 2)

At the nodes of the simply supported and the built-in edges, the deflection w=0. The finite difference formula corresponding to Eq. (1) is applied to all interior nodes and to nodes 16 and 24 on the free edge, where the deflections are to be found. This application gives eight equations involving thirty-six nodes, indicated in the mesh on the figure. The boundary conditions for each edge, replaced by the finite difference forms, are listed below.

A. At the two simply supported edges ab and ad

$w=0 \rightarrow w_5 = w_{12} = w_{20} = w_{27} = w_{28} = w_{29} = w_{30} = w_{31} = 0$

$\left(\frac{\partial^2 w}{\partial y^2}\right)=0 \rightarrow w_{13} = -\ w_{11},\ w_{21} = -\ w_{19}$

$\left(\frac{\partial^2 w}{\partial x^2}\right)=0 \rightarrow w_{21} = -\ w_{33},\ w_{22} = -\ w_{34}$, $w_{23} = -\ w_{35}$,$w_{24} = -\ w_{36}$

continuity of zero curvature $\left(\frac{\partial^2 w}{\partial y^2} = 0\right)$ along ab

$\rightarrow w_{30} = -\ w_{32} = 0$

B. At the clamped edge bc

$w=0 \rightarrow w_6 = w_7 = w_8 = w_9 = 0$

$\left(\frac{\partial w}{\partial x}\right)=0 \rightarrow w_{13} = w_1$, $w_{14} = w_2$, $w_{15} = w_3$, $w_{16} = w_4$

continuity of zero rotation $\left(\frac{\partial w}{\partial y} = 0\right)$ along bc $\rightarrow$

$w_8 = -\ w_{10} = 0$

C. At the free edge cd

$V_{y=0}$ at nodes 16 and 24

Applying Eq. (5-5) to the two nodes 16 and 24 give respectively;

$0.5\ (w_{14} - w_{18}) + 1.273\ (w_{23} - w_{25}) +3.546\ (w_{17} - w_{15}) = 0$

$0.5\ (w_{22} - w_{26}) + 1.273\ (w_{15} - w_{17}) + 3.546\ (w_{25} - w_{23}) = 0$

$M_y = 0$ at nodes 16 and 24

Applying Eq. (5-3) to the two nodes 16 and 24 give respectively;

$0.5\ (w_{15} + w_{17}) + 0.1414\ w_{24} - 1.2828\ w_{16} = 0$

$0.5\ (w_{23} + w_{25}) + 0.1414\ w_{16} - 1.2828\ w_{24} = 0$

By applying Eq. (1) to nodes 13, 14, 15, 16, 21, 22, 23 and 24 and inserting the deflections given in cases A and B, eight simultaneous equations are obtained containing twelve unknown values of w at nodal points 13 through 18 and 21 through 26. Case C provides four equations in terms of the same unknown values of w. The resulting twelve independent expressions are represented in the following matrix form.

A solution of these equations gives:-

$w_{13} = 0.29213\ qi^4/D$

$w_{14} = 0.4156\ qi^4/D$

$w_{15} = 0.45786\ qi^4/D$

$w_{16} = 0.49918\ qi^4/D$

$w_{17} = 0.64987\ qi^4/D$

$w_{18} = 0.96685\ qi^4/D$

$w_{21} = 0.34509\ qi^4/D$

$w_{22} = 0.49944\ qi^4/D$

$w_{23} = 0.55580\ qi^4/D$

$w_{24} = 0.61228\ qi^4/D$

$w_{25} = 0.87415\ qi^4/D$

$w_{26} = 2.268359\ qi^4/D$

Eqs. (2) and (3) are used to evaluate the bending moments at all nodes of the plate and the maximum values of the positive and negative bending moments in the plate are found to be;

$$M_{ox}^{+} = 0.69631\ qi^2 \quad = 0.0435\ q\,a^2$$

$$M_{oy}^{+} = 0.15166\, qi^2 \quad = 0.00959\, q\,a^2$$

$$and \quad M_{ox}^{-} = 0.99836\, qi^2$$

$$= 0.0624\,qa^2$$

By using the same procedure, the coefficients of the maximum positive and maximum negative bending moments in the six slab cases are obtained (see next article).

Introducing bending moment coefficients

Figure (3) shows a two-way reinforced concrete rectangular slab supported on three edges with the fourth edge free and having spans (l_x) and (l_y), where (l_x) is parallel to the free edge and (l_y) perpendicular on it.

When the slab is subjected to a uniformly distributed load of intensity q, per unit area the maximum negative and maximum positive bending moments of the middle strips running in the two orthogonal directions of the slab are;

$$\begin{array}{cc|}
0 & 0 \\
0 & 0 \\
0 & 0 \\
1.414 & 0 \\
0 & 0 \\
0 & 0 \\
0.5 & 0 \\
-4.828 & 0.5 \\
-1.273 & 0 \\
3.546 & -0.5 \\
0 & 0 \\
0.5 & 0
\end{array}
\begin{Bmatrix} w_{13} \\ w_{14} \\ w_{15} \\ w_{16} \\ w_{17} \\ w_{18} \\ w_{21} \\ w_{22} \\ w_{23} \\ w_{24} \\ w_{25} \\ w_{26} \end{Bmatrix}
= qi^4/D
\begin{Bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

15.157	-4.828	0.5	0	0	0	-6.828	1.414	0	0
-4.828	15.657	-4.828	0.5	0	0	1.414	-6.828	1.414	0
0.5	-4.828	15.657	-4.828	0.5	0	0	1.414	-6.828	1.414
0	0.5	-4.828	15.657	-4.828	0.5	0	0	1.414	-6.828
-6.828	1.414	0	0	0	0	13.157	-4.828	0.5	0
1.414	-6.828	1.414	0	0	0	-4.828	13.657	-4.828	0.5
0	1.414	-6.828	1.414	0	0	0.5	-4.828	13.657	-4.828
0	0	1.414	-6.828	1.414	0	0	0.5	-4.828	13.657
0	0.5	-3.546	0	3.546	-0.5	0	0	1.273	0
0	0	1.273	0	-1.273	0	0	0.5	-3.546	0
0	0	0.5	-1.283	0.5	0	0	0	0	0.1414
0	0	0	0.1414	0	0	0	0	0.5	-1.283

$$\left.\begin{aligned} M_{ox}^{-} &= C_x^{-}\ q\ l_x^2 \\ M_{oy}^{-} &= C_y^{-}\ q\ l_x^2 \\ M_{ox}^{+} &= C_x^{+}\ q\ l_x^2 \\ M_{oy}^{+} &= C_y^{+}\ q\ l_x^2 \end{aligned}\right\} \qquad \ldots(6)$$

and the values of the coefficients C_x^-, C_y^-, C_x^+ and C_y^+ (for the six different cases of reinforced concrete rectangular slabs) are presented in Tables (1 to 6). For a particular slab case, these bending moment coefficients are given for each aspect ratio *m* of the slab (where *m* =*ly*/*lx*).

Conclusions

Methods for the analysis and design of orthotropically reinforced concrete rectangular slabs supported on three variously restrained edges with the fourth edge free and subjected to uniformly distributed load are rare. In this paper, coefficients are presented in tables for direct calculations of deflection and bending moments in such slabs. These tables are therefore quite useful and serve as powerful tools for simplifying the analysis and design of RC rectangular slabs having one free edge.

Note:

This research is part of a Ph.D thesis(8) presented recently to the Building and Construction Engineering Department of the University of Technology, Baghdad.

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Table (1) Numerical factors a , C_x^+, C_y^+, C_x^- and C_y^- for slab case (1) subjected to uniformly distributed load. $u = 0.2$ (concrete).

m	$w_{max} = a \frac{ql_x^4}{D}$ a	$M_{x\,max}^+ = c_x^+ q l_x^2$ C_x^+	$M_{y\,max}^+ = c_y^+ q l_x^2$ C_y^+	$M_{x\,max}^- = c_x^- q l_x^2$ C_x^-	$M_{y\,max}^- = c_y^- q l_x^2$ C_y^-
2.0	0.00416	0.04836	0.0163	0.0825	0.04728
1.9	0.00415	0.04835	0.01629	0.08248	0.04728
1.8	0.00414	0.04833	0.01627	0.0824	0.04727
1.7	0.00413	0.04832	0.01626	0.0823	0.04726
1.6	0.00412	0.04827	0.01624	0.08211	0.04723
1.5	0.00411	0.04822	0.01621	0.0820	0.04721
1.4	0.00410	0.04821	0.0160	0.0819	0.04720
1.3	0.00409	0.04815	0.0159	0.08181	0.04715
1.2	0.00408	0.04791	0.0156	0.08162	0.04708
1.1	0.00407	0.04741	0.0154	0.0815	0.04683
1.0	0.00405	0.0470	0.0153	0.08141	0.04670
0.9	0.00402	0.04601	0.0149	0.08126	0.04660
0.8	0.00400	0.04562	0.0148	0.08111	0.04649
0.7	0.00397	0.0452	0.01467	0.08101	0.04638
0.6	0.00395	0.0448	0.0145	0.0807	0.04629
0.5	0.00391	0.0443	0.01432	0.0805	0.04617

Table (2) Numerical factors a , C_x^+, C_y^+ and C_x^- for slab case (2) subjected to uniformly distributed load. $u = 0.2$ (concrete).

m	$w_{max.} = a \frac{ql_x^4}{D}$ a	$M_{x\,max.}^+ = c_x^+ q l_x^2$ C_x^+	$M_{y\,max.}^+ = c_y^+ q l_x^2$ C_y^+	$M_{x\,max.}^- = c_x^- q l_x^2$ C_x^-
2.0	0.00425	0.04849	0.02080	0.08330
1.9	0.00424	0.04846	0.02060	0.08321
1.8	0.00422	0.04845	0.02050	0.08300
1.7	0.00420	0.04841	0.02030	0.08280
1.6	0.00418	0.04835	0.02010	0.08250
1.5	0.00417	0.04833	0.01980	0.08323
1.4	0.00416	0.04830	0.01950	0.08322
1.3	0.00415	0.04824	0.01920	0.08320
1.2	0.00414	0.04820	0.01900	0.08316
1.1	0.00414	0.04818	0.01880	0.08313
1.0	0.00413	0.04792	0.01860	0.08310
0.9	0.00412	0.04784	0.01848	0.08307
0.8	0.00411	0.04777	0.01826	0.08304
0.7	0.00409	0.04768	0.01804	0.08302
0.6	0.00408	0.04761	0.01782	0.08299
0.5	0.00406	0.04753	0.01760	0.08297

Table (3) Numerical factors a , C_x^+ ,C_y^+ ,C_x^- and C_y^- for slab case (3) subjected to uniformly distributed load. $u = 0.2$ (concrete).

m	$w_{max} = a \frac{ql_x^4}{D}$ a	$M_{x\max}^+ = c_x^+ q l_x^2$ C_x^+	$M_{y\max}^+ = c_y^+ q l_x^2$ C_y^+	$M_{x\max}^- = c_x^- q l_x^2$ C_x^-	$M_{y\max}^- = c_y^- q l_x^2$ C_y^-
2.0	0.00704	0.07045	0.02080	0.12040	0.06408
1.9	0.00703	0.07036	0.02072	0.12038	0.06408
1.8	0.00703	0.07030	0.02071	0.12037	0.06404
1.7	0.00702	0.07021	0.02065	0.12034	0.06398
1.6	0.00700	0.06998	0.02059	0.12023	0.06392
1.5	0.00696	0.06960	0.02036	0.11993	0.06386
1.4	0.00691	0.06902	0.02008	0.11940	0.06374
1.3	0.00684	0.06817	0.01978	0.11854	0.06359
1.2	0.00672	0.06691	0.01947	0.11714	0.06337
1.1	0.00655	0.06506	0.01907	0.11492	0.06305
1.0	0.00631	0.06239	0.01843	0.11150	0.06257
0.9	0.00618	0.06080	0.01804	0.10981	0.06231
0.8	0.00604	0.05930	0.01751	0.10813	0.06206
0.7	0.00591	0.05706	0.01727	0.10645	0.06180
0.6	0.00578	0.05641	0.01689	0.10531	0.06154
0.5	0.00564	0.05517	0.01651	0.10309	0.06130

Table (4) Numerical factors a , C_x^+ ,C_y^+ and C_y^- for slab case (4) subjected to uniformly distributed load. $u = 0.2$ (concrete).

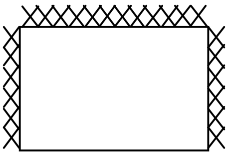
m	$w_{max.} = a \frac{ql_x^4}{D}$ a	$M_{x\max.}^+ = c_x^+ q l_x^2$ C_x^+	$M_{y\max.}^+ = c_y^+ q l_x^2$ C_y^+	$M_{y\max.}^- = c_y^- q l_x^2$ C_y^-
2.0	0.01456	0.12776	0.03243	0.09792
1.9	0.01445	0.12682	0.03230	0.09783
1.8	0.01433	0.12572	0.03225	0.09772
1.7	0.01417	0.12428	0.03200	0.09757
1.6	0.01396	0.12238	0.03170	0.09736
1.5	0.01369	0.11994	0.03150	0.09704
1.4	0.01334	0.11682	0.03100	0.09661
1.3	0.01290	0.11287	0.02990	0.09598
1.2	0.01235	0.10789	0.02830	0.09507
1.1	0.01165	0.10164	0.02610	0.09374
1.0	0.01079	0.09389	0.02372	0.09176
0.9	0.01021	0.08868	0.02216	0.09060
0.8	0.00963	0.08347	0.02070	0.08950
0.7	0.00905	0.07823	0.01905	0.08840
0.6	0.00847	0.07320	0.01750	0.08730
0.5	0.00789	0.06781	0.01596	0.08610

Table (5) Numerical factors a , C_x^+ , C_y^+ and C_x^- for slab case (5) subjected to uniformly distributed load. $u = 0.2$ (concrete).

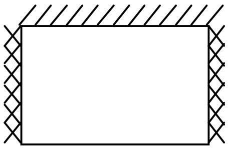
m	$w_{max.} = a \frac{ql_x^4}{D}$ a	$M_{x\,max.}^+ = c_x^+ \, q l_x^2$ C_x^+	$M_{y\,max.}^+ = c_y^+ \, q l_x^2$ C_y^+	$M_{x\,max.}^- = c_x^- \, q l_x^2$ C_x^-
2.0	0.00705	0.07050	0.02412	0.12050
1.9	0.00704	0.07049	0.02402	0.12052
1.8	0.00704	0.07047	0.02400	0.12051
1.7	0.00703	0.07044	0.02393	0.12047
1.6	0.00703	0.07036	0.02391	0.12043
1.5	0.00702	0.07021	0.02390	0.12040
1.4	0.00701	0.06995	0.02380	0.12037
1.3	0.00696	0.06955	0.02365	0.12009
1.2	0.00691	0.06890	0.02334	0.11955
1.1	0.00682	0.06791	0.02292	0.11859
1.0	0.00668	0.06641	0.02241	0.11698
0.9	0.00661	0.06584	0.02220	0.11628
0.8	0.00653	0.06490	0.02188	0.11558
0.7	0.00646	0.06413	0.02160	0.11487
0.6	0.00640	0.06335	0.02130	0.11418
0.5	0.00634	0.06260	0.02100	0.11347

Table (6) Numerical factors a , C_x^+ and C_y^+ for slab case (6) subjected to uniformly distributed load. $u = 0.2$ (concrete).

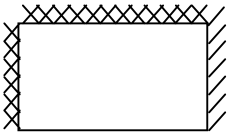
m	$w_{max.} = a \frac{ql_x^4}{D}$ a	$M_{x\,max.}^+ = c_x^+ \, q l_x^2$ C_x^+	$M_{y\,max.}^+ = c_y^+ \, q l_x^2$ C_y^+
2.0	0.0147	0.1286	0.03622
1.9	0.0146	0.12826	0.03610
1.8	0.0145	0.12756	0.03564
1.7	0.0144	0.12695	0.03500
1.6	0.0143	0.12541	0.03460
1.5	0.0141	0.12382	0.03420
1.4	0.01389	0.12177	0.03380
1.3	0.0136	0.11915	0.03330
1.2	0.0132	0.1158	0.03321
1.1	0.0128	0.11152	0.03311
1.0	0.0121	0.10607	0.03310
0.9	0.0112	0.1024	0.03286
0.8	0.0108	0.09894	0.03263
0.7	0.0104	0.0954	0.03241
0.6	0.0098	0.09183	0.03222
0.5	0.0094	0.8831	0.03198



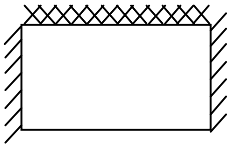
<u>Case (1)</u> Three Edges Fixed-One Edge Free



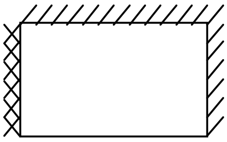
<u>Case (2)</u> Two Parallel Edges Fixed - One Edge Simply Supported-One Edge Free



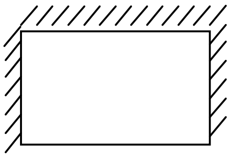
<u>Case (3)</u> Two Adjacent Edges Fixed- One Edge Simply Supported-One Edge Free



<u>Case (4)</u> Two Parallel Edges Simply Supported- One Edge Fixed-One Edge Free



<u>Case (5)</u> Two Adjacent Edges Simply Supported -One Edge Fixed -One Edge Free



<u>Case (6)</u> Three Edges Simply Supported-One Edge Free

Free edge

simply supported or discontinuous edge

fixed (clamped) or continuous edge

Figure (1) The six possible cases of a RC rectangular slab supported on three edges only with the fourth edge free.

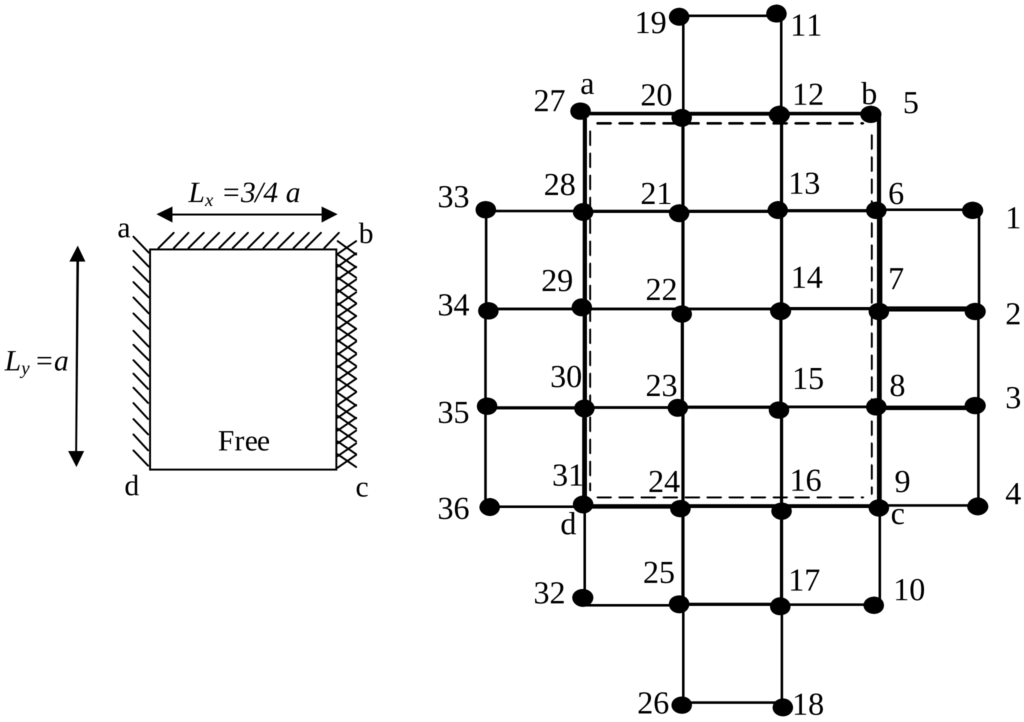
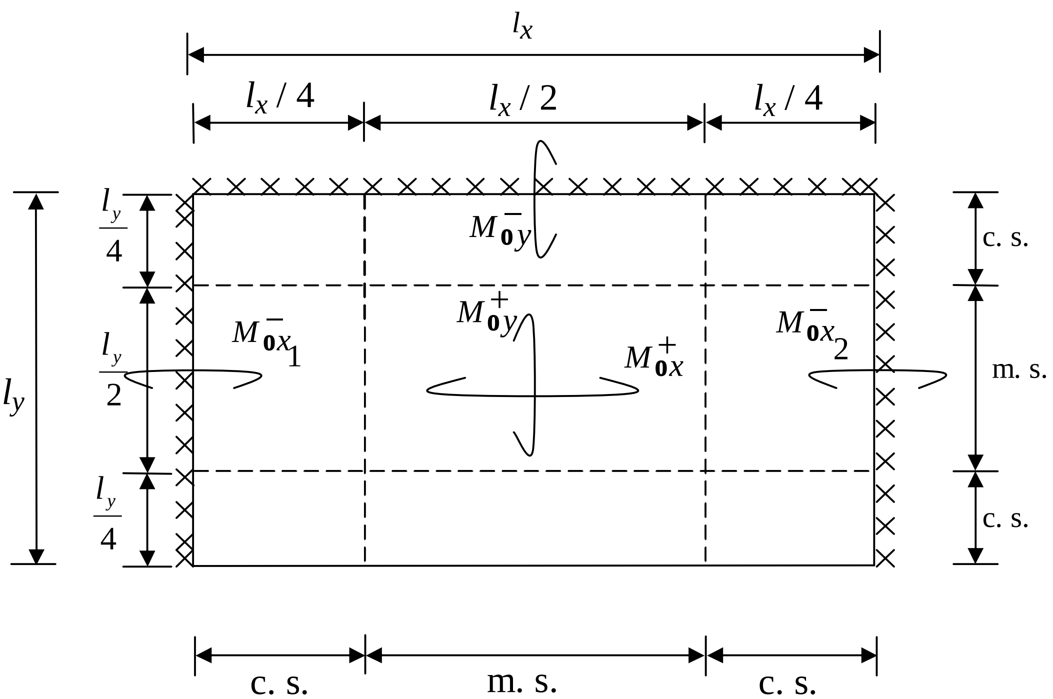


Figure (2) Rectangular plate divided into twelve square mesh.



m. s. = middle strip
c. s. = column strip

Figure (3) Distribution of bending moments in a two-way rectangular slab supported on three edges with the fourth edge free.

Appendix A : (set of equation based on the finite difference approach)

$$q \approx \frac{1}{i^4}\begin{array}{ccccc} & & D_y & & \\ & 2\sqrt{D_x D_y} & -4(\sqrt{D_x D_y}+D_y) & 2\sqrt{D_x D_y} & \\ D_x & -4(\sqrt{D_x D_y}+D_x) & 6(D_x+D_y)+8\sqrt{D_x D_y} & -4(\sqrt{D_x D_y}+D_x) & D_x \\ & 2\sqrt{D_x D_y} & -4(\sqrt{D_x D_y}+D_y) & 2\sqrt{D_x D_y} & \\ & & D_y & & \end{array} \quad \ldots(1)$$

where i is the spacing between nodes in the square mesh.

$$M_{ox} = -\frac{1}{i^2}\begin{array}{ccc} & u\sqrt{D_x D_y} & \\ D_x & -2D_x - 2u\sqrt{D_x D_y} & D_x \\ & u\sqrt{D_x D_y} & \end{array} \quad \ldots(2)$$

$$M_{oy} = -\frac{1}{i^2}\begin{array}{ccc} & D_y & \\ u\sqrt{D_x D_y} & -2D_y - 2u\sqrt{D_x D_y} & u\sqrt{D_x D_y} \\ & D_y & \end{array} \quad \ldots(3)$$

$$V_x \approx -\frac{1}{2i^3}\begin{array}{ccccc} & & 0 & & \\ & \boxed{-(2-u)\sqrt{D_xD_y}} & 0 & \boxed{(2-u)\sqrt{D_xD_y}} & \\ -D_x & \boxed{2D_x+2(2-u)\sqrt{D_xD_y}} & 0 & \boxed{-2D_x-2(2-u)\sqrt{D_xD_y}} & D_x \\ & \boxed{-(2-u)\sqrt{D_xD_y}} & 0 & \boxed{(2-u)\sqrt{D_xD_y}} & \\ & & 0 & & \end{array} \quad \ldots(4)$$

$$V_y \approx -\frac{1}{2i^3}\begin{array}{ccccc} & & D_y & & \\ & \boxed{(2-u)\sqrt{D_xD_y}} & \boxed{-2D_y-2(2-u)\sqrt{D_xD_y}} & \boxed{(2-u)\sqrt{D_xD_y}} & \\ 0 & 0 & 0 & 0 & 0 \\ & \boxed{-(2-u)\sqrt{D_xD_y}} & \boxed{2D_y+2(2-u)\sqrt{D_xD_y}} & \boxed{-(2-u)\sqrt{D_xD_y}} & \\ & & -D_y & & \end{array} \quad \ldots(5)$$