State Estimation of Two-Phase Permanent Magnet Synchronous Motor

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Abstract

The goal of this paper is to estimate the states of two-phase permanent magnet synchronous motor (PMSM). The system is highly nonlinear and one therefore cannot directly use any linear system tools for estimation. However, if one can linearize the system around a nominal (possibly time-varying) operating point then linear system tools could be used for control and estimation. Firstly, the error covariance matrices of measurement and process would be derived when the system inputs and outputs are subjected to uncertain variations. Then, the corrupted-noise nonlinear model of the system will be discretized and extended to be suitable for applying standard discrete Kalman filter (KF) for state estimation purpose. The entire state estimated system has been modeled using MATLAB/SIMULINK blocks. The state estimation algorithm and the motor discretized model are coded inside special S-functions of m-file type.

Keywords: Two-Phase Permanent Magnet Synchronous Motor, Kalman Filter, Extended Kalman Filter, Modelling, Matlab/Simulink/s-function.

تخمين متغيرات المحرك التزامني ثنائي الطور ذو المغانيط الدائمية

الخلاصة

ان غاية هذا البحث هو تخمين متغيرات المحرك التزامني ثنائي الطور ذو المغانيط الدائمية. يتميز الانموذج الرياضي لمنظومة المحرك التزامني بدرجة عالية من اللاخطية. لذلك لا يمكن استخدام وسائل المنظومات الخطية بصورة مباشرة لغرض تخمين منظومة المحرك التزامني. مع ذلك، اذا تمكنا من جعل المنظومة (الانموذج الرياضي للمحرك) خطية عند نقطة الاشتغال الطبيعية (والتي قد تكون متغيرة مع الزمن) فانه من الممكن استخدام طرق المنظومة الخطية لتخمين متغيرات منظومة المحرك التزامني اللاخطية.
تم استخدام مرشح كالمان القياسي المتقطع (standard discrete Kalman filter) لغرض عملية التخمين. لذلك تم تقطيع (discretization) وتوسيع (Extension) الانموذج اللاخطي ليكون ملائم لهذا التطبيق. تم نمذجة منظومة المحرك والمنظومة التخمينية باستخدام كتل (Matlab/Simulink blocks)، ثم ادراج برنامج خوارزمية التخمين بملف نوع (m-file) ويتم استدعائه باسنخدام دالة (s-function)

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Introduction

In controlling AC machine drives, speed transducers such as tacho-generators, resolvers, or digital encoders are used to obtain speed information. Using these speed sensors has some disadvantages [1]

- They are usually expensive,
- The speed sensor and the corresponding wires will take up space,
- In defective and aggressive environme-nts, the speed sensor might be the weakest part of the system.

Especially the last item degrades the systems reliability and reduces the advantage of an induction motor drive system. This has led to a great many speed sensorless vector control methods.

On the other hand, avoiding sensor means use of additional algorithms and added computational complexity that requires high-speed processors for real time applications. As digital signal processors have become cheaper, and their performance greater, it has become possible to use them for controlling electrical drives as a cost effective solution [2].

Estimation of unmeasurable state variables is commonly called observation. A device (or a computer program) that estimates or observes the states is called a state-observer. An observer can be classified according to the type of representation used for the plant to be observed [1].

If the plant is deterministic, then the observer is a deterministic observer; otherwise it is a stochastic observer. The most commonly used observers are Luenberger and Kalman types [2]. The Luenberger observer (LO) is of the deterministic type, and the Kalman Filter (KF) is of the stochastic type. The basic Kalman filter is only applicable to linear stochastic systems, and for non-linear systems the extended Kalman filter (EKF) can be used, which can provide estimates of the states of a system or of both the states and parameters [1,2].

The EKF is a recursive filter (based on the knowledge of statistics of both the state and noise created by measurement and system modelling), which can be applied to non-linear time varying stochastic systems. EKF is insensitive to parameter changes and used for stochastic systems where measurement and modeling noise is taken into account.

Model of PMSM and Development of Error Covariance Matrices

The continuous-time electromechanical model of two-phase permanent magnet (PM) synchronous motor is fourth order, nonlinear and can be described by [3]

$$\begin{aligned}
\dot{i}_a &= -(R_a/L)\, i_a + (\lambda/L)\,\omega_r \sin\theta_r + (1/L)\, u_a \\
\dot{i}_b &= -(R_a/L)\, i_b - (\lambda/L)\,\omega_r \cos\theta_r + (1/L)\, u_b \\
\dot{\omega}_r &= -(3\lambda/2J)\, i_a \sin\theta_r + (3\lambda/2J)\, i_b \cos\theta_r \\
&\quad - (F/J)\,\omega_r - (1/J)\, T_L \\
\dot{\theta}_r &= \omega_r
\end{aligned} \qquad (1)$$

where i_a and i_b are the currents through the two windings, R_a and L are the resistance and inductance of the windings, θ_r and ω_r are the angular position and velocity of the rotor, λ is the flux constant of the motor, u_a and u_b are the voltages applied across the two windings, J is the moment of inertia of the rotor and its load, F is the viscous friction of the rotor, and T_L is the load torque [3,4].

However, the system is highly nonlinear and one therefore cannot directly use any linear system tools for estimation. However, if one can linearize the system around a nominal (possibly time-varying) operating point then linear system tools could be used for control and estimation. We start by defining a state vector as $\mathbf{x}=[i_a \quad i_b \quad \omega_r \quad \theta_r]^T$ and the output vector as $\mathbf{y}=[i_a \quad i_b]^T$. With this definition, Eq.(1) can be written compactly as $\dot{\mathbf{x}} = [\dot{x}_1 \quad \dot{x}_2 \quad \dot{x}_3 \quad \dot{x}_4]^T$

$$\dot{\mathbf{x}} = \boldsymbol{A}\mathbf{x} + \boldsymbol{B}\mathbf{u} + \boldsymbol{M}\, T_L \qquad (2)$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} \qquad (3)$$

where

$$\boldsymbol{A}=\begin{bmatrix} -\frac{R_a}{L} & 0 & \frac{l}{L}\sin x_4 & 0 \\ 0 & -\frac{R_a}{L} & -\frac{l}{L}\cos x_4 & 0 \\ -\frac{3l}{2J}\sin x_4 & \frac{3l}{2J}\cos x_4 & -\frac{F}{J} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{B}=\begin{bmatrix} (1/L) & 0 \\ 0 & (1/L) \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \mathbf{M}=\begin{bmatrix} 0 \\ 0 \\ -\frac{1}{J} \\ 0 \end{bmatrix} \text{ and } \mathbf{C}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T$$

The motor Equation (2) is to be discretized for the digital implementation as:

$$x_{k+1} = A_k x_k + B_k u_k + M_k\, T_L \quad (4)$$

$$\boldsymbol{y_k} = \boldsymbol{C_k x_k} \quad (5)$$

$\boldsymbol{A_k}$ and $\boldsymbol{B_k}$ are the discretizd system and input matrices, respectively. They are [1,4,5]

$$\boldsymbol{A_k} = \boldsymbol{e}^{\boldsymbol{AT}} = \boldsymbol{I} + \boldsymbol{AT} + \frac{(\boldsymbol{AT})^2}{2!} + \ldots.$$

$$\cong \boldsymbol{I} + \boldsymbol{AT} \quad (6)$$

$$\boldsymbol{B_k} = \int_0^T \boldsymbol{e}^{\boldsymbol{A}z}\, \boldsymbol{B}dz = \left[\boldsymbol{e}^{\boldsymbol{AT}} - \boldsymbol{I}\right] \boldsymbol{A}^{-1}\boldsymbol{B}$$

$$= \boldsymbol{BT} + \frac{\boldsymbol{ABT}^2}{2!} + \ldots \cong \boldsymbol{BT} \quad (7)$$

$$\boldsymbol{M_k} \cong \boldsymbol{M}\mathrm{T} \quad (8)$$

$$\boldsymbol{C_k} = \boldsymbol{C} \quad (9)$$

where T is the sampling time and **I** is an identity (4×4) matrix. The above approximation is justified due to the small size of sampling time and the presence of increasingly large factorials, which further diminishes the magnitude of the higher-order terms.

$$\boldsymbol{A_k} = \begin{bmatrix} a_{11} & 0 & a_{13}\sin x_4(k) & 0 \\ 0 & a_{11} & -a_{13}\cos x_4(k) & 0 \\ a_{31}\sin x_4(k) & -a_{31}\cos x_4(k) & a_{33} & 0 \\ 0 & 0 & a_{43} & 1 \end{bmatrix}$$

$$\boldsymbol{B_k} = \begin{bmatrix} b_{11} & 0 \\ 0 & b_{11} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ and } \boldsymbol{M_k} = \begin{bmatrix} 0 \\ 0 \\ -\frac{T}{J} \\ 0 \end{bmatrix}$$

where

$$a_{11} = 1 \quad \frac{R_a T}{L},\ a_{13} = \frac{lT}{L},\ a_{31} = -\frac{3lT}{2J}$$

$$a_{33} = 1 - \frac{FT}{J},\ a_{43} = T \text{ and } b_{11} = \frac{T}{L}$$

If the noises Δu_a, Δu_b have corrupted the inputs u_a and u_b, respectively, and the noise $\Delta\alpha$ has been admitted to account for uncertainties in the load torque, then a noise vector will arise in Eq.(4)

$$\boldsymbol{x_{k+1}} = \boldsymbol{A_k x_k} + \boldsymbol{B_k} \begin{bmatrix} u_a + \Delta u_a \\ u_b + \Delta u_b \end{bmatrix}_k + \boldsymbol{M_k}\,(T_L + \Delta T_L)$$

or

$$\boldsymbol{x_{k+1}} = \boldsymbol{A_k\, x_k} + \boldsymbol{B_k\, u_k} + \boldsymbol{M_k}\, T_L + \boldsymbol{w_k} \quad (10)$$

where

$$\boldsymbol{w_k} = \begin{bmatrix} (T/L)\Delta u_a \\ (T/L)\Delta u_b \\ -(T/J)\Delta T_L \\ 0 \end{bmatrix} \quad (11)$$

Similarly, if the measurements i_a and i_b are distorted by noises Δi_a and Δi_b respectively, then Eq.(5) becomes

$$\boldsymbol{y_k} = \boldsymbol{C_k} \begin{bmatrix} i_a + D\, i_a \\ i_b + D\, i_b \\ w_r \\ q_r \end{bmatrix} = \boldsymbol{C_k x_k} + \boldsymbol{C_k} \begin{bmatrix} \Delta i_a \\ \Delta i_b \\ 0 \\ 0 \end{bmatrix}$$

$$= \boldsymbol{C_k x_k} + \begin{bmatrix} \Delta i_a \\ \Delta i_b \end{bmatrix} = \boldsymbol{C_k x_k} + \boldsymbol{v_k} \quad (12)$$

where

$$\boldsymbol{v_k} = \begin{bmatrix} \Delta i_a \\ \Delta i_b \end{bmatrix} \quad (13)$$

where the vectors $\boldsymbol{w_k}$ and $\boldsymbol{v_k}$ are called the process and measurement noises respectively.

The Kalman filter theory and algorithm

The aim in all estimation problems is to have an estimator that gives an accurate estimate of the true state even though one cannot directly measure it. Two obvious requirements should be attained [6]:

- First, the average value of our state estimate is to be equal to the average value of the true state. That is, the estimate has not to be biased one way or another. Mathematically, one would say that the expected value of the estimate should be equal to the expected value of the state.

- Second, the state estimate varies from the true state as little as possible. That is, not only one wants the average of the state estimate to be equal to the average of the true state, but also want an estimator that results in the smallest possible variation of the state estimate. Mathematically, an estimator with the smallest possible error variance is sought.

It so happens that the Kalman filter is the estimator that satisfies these two criteria. But the Kalman filter solution does not apply unless certain assumptions about the noise that affects the system under study must be satisfied:

1. It is firstly to assume that the average value of both w_k and v_k are zero.
2. One has to further assume that no correlation exists between w_k and v_k. That is, at any time k, w_k and v_k are independent random variables. Then the noise covariance matrices S_w and S_v are defined as:

Process noise covariance:

$$S_w = E(w_k w_k^T) \qquad (14)$$

Measurement noise covariance:

$$S_v = E(v_k v_k^T) \qquad (15)$$

where w^T and v^T indicate the transpose of w and v random noise vectors, and E(.) means the expected value.

Substituting Eq.(11) into Eq.(14), and Eq.(13) into Eq.(15), one can get the following process and measurement noise covariance matrices:

$$S_w = E\left(\begin{bmatrix} \left(\frac{T}{L}\right)^2(\Delta u_a)^2 & \left(\frac{T}{L}\right)^2\Delta u_a\Delta u_b & -\left(\frac{T^2}{JL}\right)\Delta u_a\Delta T_L & 0 \\ \left(\frac{T}{L}\right)^2\Delta u_a\Delta u_b & \left(\frac{T}{L}\right)^2(\Delta u_b)^2 & -\left(\frac{T^2}{JL}\right)\Delta u_b\Delta T_L & 0 \\ -\left(\frac{T^2}{JL}\right)\Delta u_a\Delta T_L & -\left(\frac{T^2}{JL}\right)\Delta u_b\Delta T_L & \left(\frac{T}{J}\right)^2(\Delta T_L)^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right) \qquad (16)$$

$$S_v = E\left(\begin{bmatrix} (\Delta i_a)^2 & \Delta i_a\Delta i_b \\ \Delta i_a\Delta i_b & (\Delta i_b)^2 \end{bmatrix}\right) \qquad (17)$$

If the noises Δu_a (Δu_b), ΔT_L, Δi_a (Δi_b) are white, zero mean, uncorrelated, and have known variances s_i^2, s_T^2 and s_M^2, respectively, then the covariance matrices S_w and S_v will become

$$S_w = \begin{bmatrix} \left(\frac{T}{L}\right)^2 s_i^2 & 0 & 0 & 0 \\ 0 & \left(\frac{T}{L}\right)^2 s_i^2 & 0 & 0 \\ 0 & 0 & \left(\frac{T}{J}\right)^2 s_T^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad (18)$$

$$S_v = \begin{bmatrix} s_M^2 & 0 \\ 0 & s_M^2 \end{bmatrix} \qquad (19)$$

One may summarize the recursive state estimation of the discrete Kalman filter as shown in Fig.(1). In the figure, the superscripts "-1", "T", "+" and "-" indicate matrix inversion, matrix transposition, posteriori and priori of variable respectively. The **K** matrix is called the Kalman gain and the **P** matrix is called the estimation error covariance. The flowchart includes the initialization of state $\hat{x}_0$ in the absence of any observed data at *k=0*, and the initial value of the a posteriori covariance matrix P_0 [7].

The timing diagram of the various quantities involved in the discrete optimal filter equations is shown in Fig.(2). The figure shows that after we process the measurement at time (*k-1*), we have an estimate of x_{k+1} (denoted $\hat{x}_{k-1}^+$) and the covariance of that estimate (denoted P_{k-1}^+). When time k arrives, before we process the measurement at time k we compute an estimate of x_k (denoted $\hat{x}_k^-$) and the covariance of that estimate (denoted P_k^-). Then the measurement is processed at time k to refine our estimate of x_k. The resulting estimate of x_k is denoted $\hat{x}_k^+$ and its covariance is denoted P_k^+.

By substituting error covariance update equation into propagation equation, and the state estimate propagation equation into update equation, the algorithm of Fig.(1) will be summarized as [3,4,6,7-11]

$$K_k = P_k C_k^T \left[C_k P_k C_k^T + R_k \right]^{-1} \quad (20)$$

$$\hat{x}_k = A_{k-1} \hat{x}_{k-1} + K_k (y_k - C_k \hat{x}_k) \quad (21)$$

$$P_k = A_{k-1}((I - K_k C_k) P_k) A_{k-1} + Q_{k-1} \quad (22)$$

Extended Kalman Filter (EKF)

The state-space model of Eqs.(10) and (12) can be rewritten in the following form:

$$x_{k+1} = f(x,u,k) + w_k \quad (23)$$

$$y_k = C_d x_k + v_k \quad (24)$$

where

$$f(x,u,k) = \begin{bmatrix} a_{11} x_1(k) + a_{13} x_3(k) \sin x_4(k) + b_{11} u_a \\ a_{11} x_2(k) - a_{13} x_3(k) \cos x_4(k) + b_{11} u_b \\ a_{31} x_1(k) \sin x_4(k) - a_{31} x_2(k) \cos x_4(k) + a_{33} x_3(k) \\ a_{43} x_3(k) + x_4(k) \end{bmatrix}$$

It is clear that *f(x,u,k)* is nonlinear. However, to use nonlinear model with the standard KF, the model must be linearized about the current operating point, giving a linear perturbation model represented by Jacobian matrix $F(x,u,k)$,

$$F(x,u,k) = \left. \frac{\partial f(x,u,k)}{\partial x} \right|_{\hat{x}(k)+u(k)} \quad (25)$$

$$= \begin{bmatrix} a_{11} & 0 & a_{13}\sin x_4(k) & a_{13}x_3(k)\cos x_4(k) \\ 0 & a_{11} & -a_{13}\cos x_4(k) & a_{13}x_3(k)\sin x_4(k) \\ a_{31}\sin x_4(k) & -a_{31}\cos x_4(k) & a_{33} & a_{31}(x_1(k)\cos x_4(k)+x_2(k)\sin x_4(k)) \\ 0 & 0 & a_{43} & 1 \end{bmatrix}$$

By now, the Jacobian matrix is replaced by A_k into Eq.s (21) and (22).

Modeling of Motor State Estimation System Using MATLAB/SIMULINK

SIMULINK is an extension to MATLAB and allows graphical block diagram modeling and simulation of dynamic systems. It is easier to develop state estimator using this package, as many components of the system are already included in the SIMULINK block diagram library [12].

The discretized model of the motor and the state estimation algorithm has been entered into a S-function-type of *m*-file. An *m*-file is a MATLAB program that allows algorithms or equations to be entered in a programming language. An S-function block, from the SIMULINK nonlinear library, links this m-file into a graphical block for use within the overall state estimation system.

Two quadrature sinusoidal waveforms drawn from the SIMULINK library have fed both the blocks of motor dynamic system and the state estimator, as shown in Fig.(3). The load change has been permitted and the repeating sequence block, from the SIMULINK source library, is employed. The S-function block of motor model generates the actual states. The state estimator block receives, in addition to inputs u_a, u_b and T_L, the actual currents i_a and i_b. The estimator produces the estimated states of the motor.

Simulated Results

The parameters of the motor are listed in Table (1). The SIMULINK model of Fig.(3) has been run and the estimated and actual states representing stator currents and rotor velocity and position are shown in Fig.(4). The system was simulated at sampling time (T=2.5 ms). One can easily notice that the EKF estimator could successively estimate the motor states and the estimator showed an excellent noise rejection capability. However, one can observe that the estimator does hardly estimate the speed and the angular position at the motor starting, but there is a perfect overlap at steady state.

Figure (5) shows the outputs of the estimtor when the sampling time is increased to (T=2.95 ms). The performance of estimator shows a great degradation in its responses. This unstable behavior of the estimator is attributed to matrix singularity problems in the Kalman gain matrix. As this would assign the gain K large values, which will reflect directly to updated state estimates.

In Figure (6), a mechanical load having the waveform of Fig.(7) has been applied. It is clear from the figure that the estimated speed state stillwell tracks the actual state at times of load changes. A nice overlapping between states has been observed. One may conclude that the EKF works properly under load conditions.

In figure (8), the standard deviation of measurement noise has been changed and the trace of error covariance matrix, Trace(P), is calculated in each time. Being the covariance matrix P is a measure of how we are certain in the measurements, one can expect that the trace of matrix will show large values for large values of standard deviation of measurement noise. This conclusion has been reported in Fig.(8).

Conclusion

Based on the observations of the simulated results one might highlights the following points:

Inspection of the figure (8) shows that if the measurement noise is large, so $\boldsymbol{P}$ will be large too and we don't have much certainty about the measurement y when computing the next $\hat{x}$. On the other hand, if the measurement noise is small, so $\boldsymbol{P}$ will be small and we will have a lot of certainty to the measurement when computing the next $\hat{x}$.

One can easily conclude that the EKF estimator could successively estimate the motor states and the estimator showed an excellent noise rejection capability.

The application of Kalman Filter is resticted by the limitation of sampling period. Serious stability problems will arise as the sampling time is increased to a specified value. As the Kalman gain K suffers singularity at the increased sampling time.

The EKF estimator shows good tracking performance in spite of load exertion during estimation process.

References

[1] Bilal Akin, "State Estimation Techniques for Speed Sensorless Field Oriented Control of Induction Machine", Master's thesis, The Middle East technical university, 2003.

[2] Bimal K. Bose, "Modern Power Electronics and AC Drive," University of Tennessee, Knoxville, Prentice Hall, 2006.

[3] Dan Simon, "Using Nonlinear Kalman Filtering to Estimate Signals," 2003, (d.j.simon @ csuohio.edu).

[4] Dan Simon, "Optimal State Estimation: Kalman, H_∞, and Nonlinear Approaches," John Wiley & Sons Inc., 2006.

[5] Gene F. Franklin etal, "Digital Control of Dynamic Systems," Addison-Wesly Longman, Inc., 1998.

[6] Dan Simon, "Kalman Filtering," Embedded Systems Programming, JUNE 2001.

[7] A. Gelb etal, "Applied Optimal Estimation," Cambridge, Mass, 1974.

[8] R. G. Brown, "Introduction to Random Signal Analysis and Kalman Filter," Johon Wiley and Sons, Inc., 1983.

[9] Dan Simon, "Kalman Filtering and Neural Networks," Johon Wiley and Sons, Inc., 2001.

[10] Greg Welch and Gary Bishop, "An Introduction to the Kalman Filter," ACM, Inc., 2001.

[11] Mohinder S. Grewal, Angus P. Andrews, "Kalman Filtering: Theory and Practice Using Matlab" John Wiley and Sons Inc., 2001.

[12] "Short Tutorial on Matlab," Tomas Co., 2004.

Table 1:Parameters of two-phase motor

Winding resistance	R_a	2 Ω
Winding inductance	L	3 mH
Motor flux constant	l	0.1
Standard deviation of control input noises	s_i	0.001 A
Standard deviation of load torque noise	s_T	0.05 rad/sec^2
Standard deviation of measurement noise	s_m	0.1 A
Moment of Inertia	J	0.002
Frequency	f	1 Hz

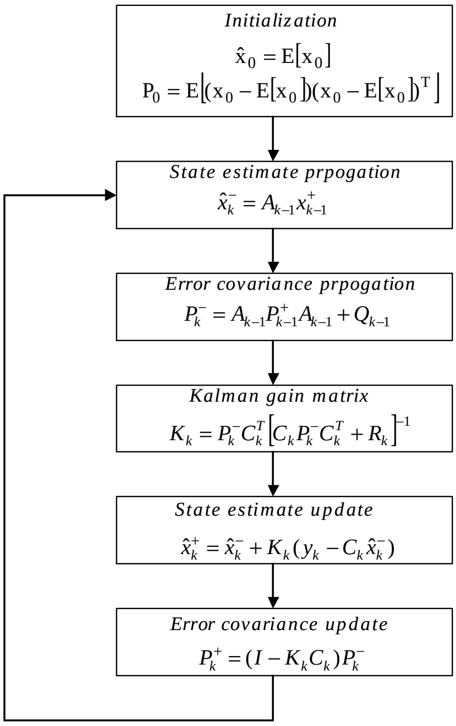

Figure (1) Recursive Algorithm of Discrete Kalman Filter

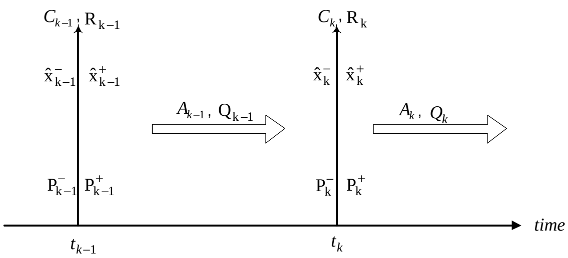

Figure (2) Timeline showing a priori and a posteriori state estimates and estimation- error covariance.

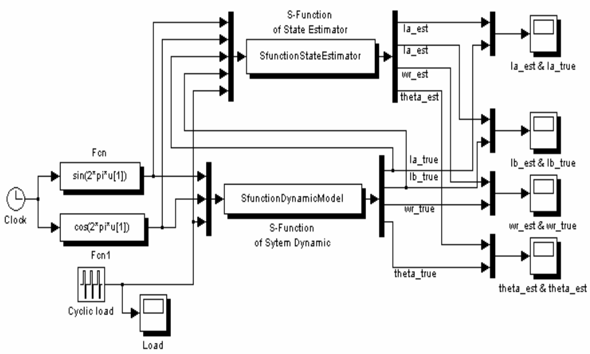

Figure (3) SIMULINK Modeling of Motor State Estimation System

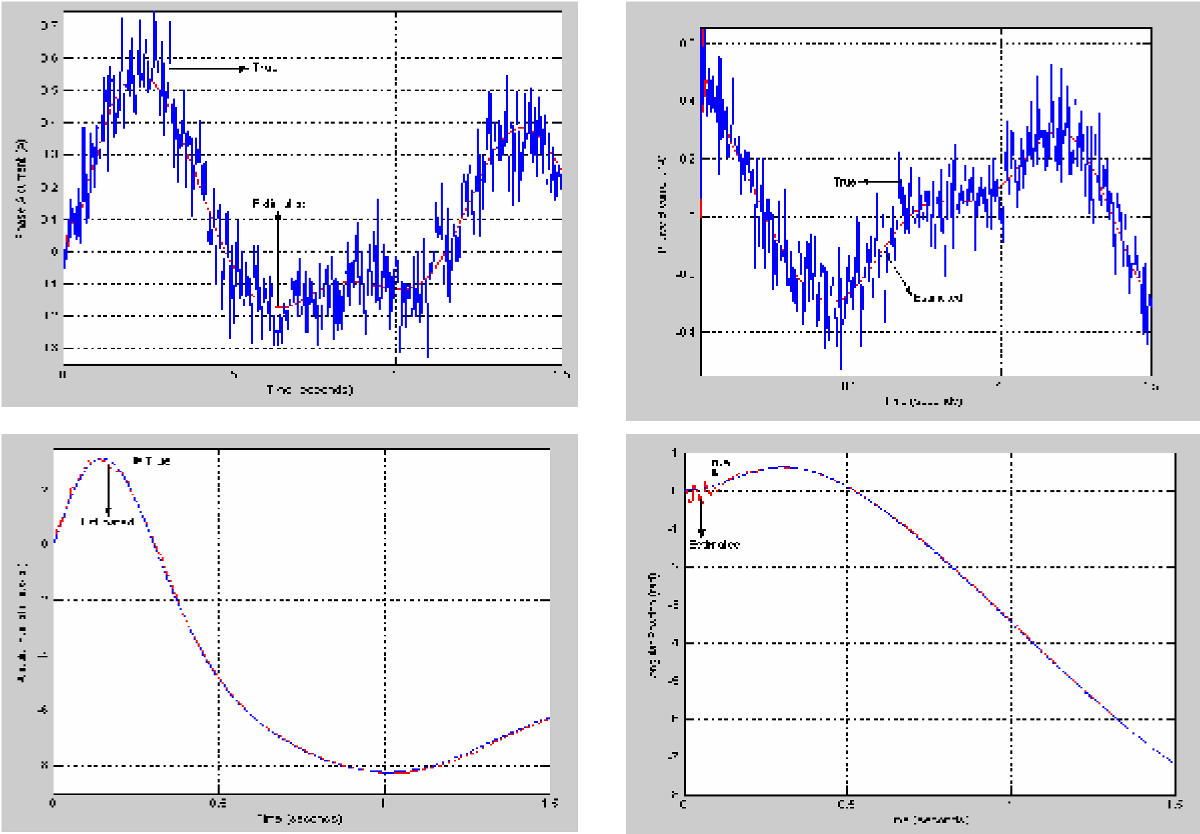

Figure (4) Estimated and actual states of the motor.

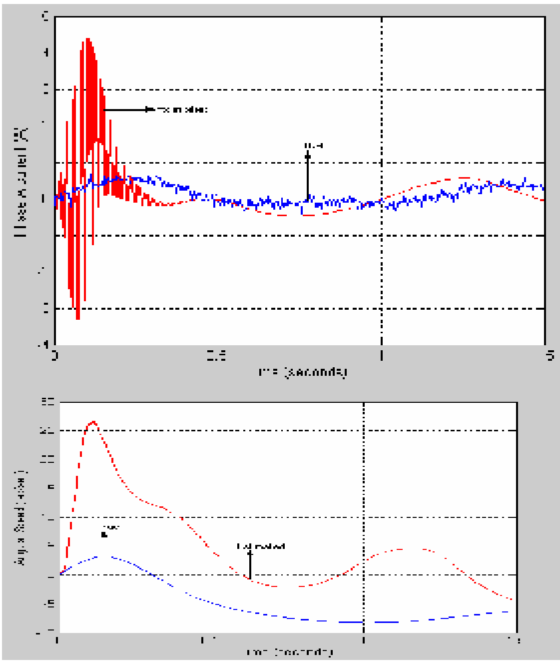

Figure (5) Estimated and actual states (current and speed) of the motor at sampling time
(T=2.93 ms)

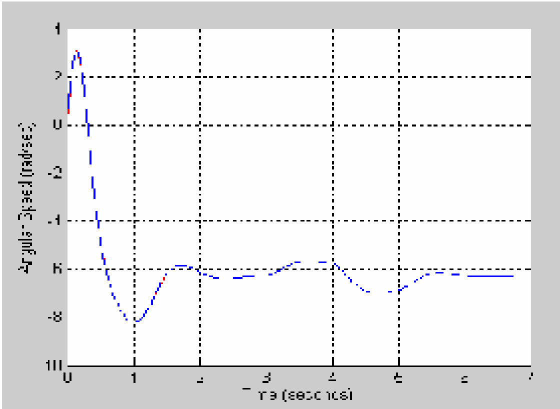

Figure (6) Estimated and actual states of the motor.

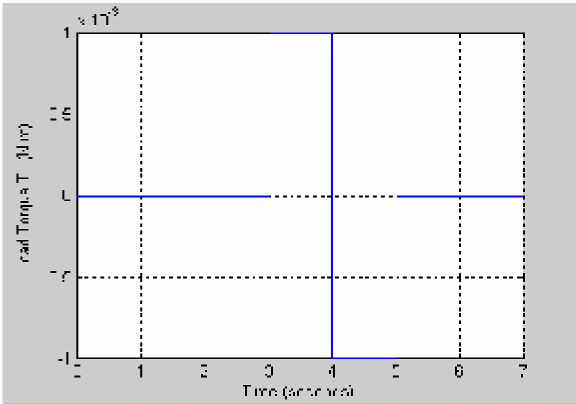

Figure (7) Change of Motor Torque Load

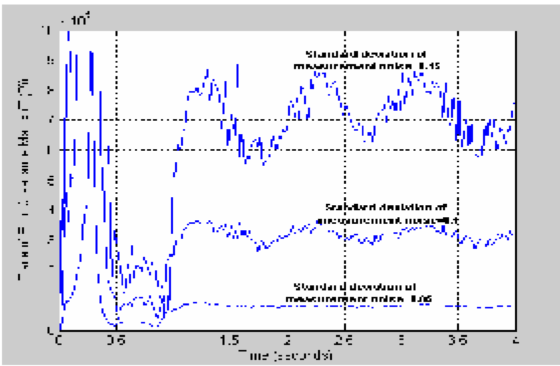

Figure (8) Trace of the error state covariance matrix with different standard deviations of measurement noises