Interference Drag Between Cylindrical Particles in Stokes Flow

Dr. Abbas H. Sulaymon* & Dr. Sawsan A. M. Mohammed*

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Abstract

In this study, the interference effects when two cylinders are placed in series or parallel in a low Reynolds number flow are predicted. For two cylinders with their line of centers perpendicular to the flow, the drag force is lower than an isolated cylinder at small gaps but is greater at large gaps; a maximum is found at a gap of approximately of 7 cylinder diameters. For cylinders with their line of centers parallel to the flow, the drag on the trailing body is less than the leading body, which in turn is less than the drag on an isolated cylinder.

Keywords: drag force, two cylinders, Stokes flow

قوة السحب التداخلية بين جسيمتين اسطوانيتين في جريان ستوكس

الخلاصة

تم ايجاد التاثيرات التداخلية بين اسطوانتين متماثلتين مرتبة بالاتجاه الطولي مع الجريان مرة وبشكل متعامد مع الجريان مرة اخرى في مدى صغير من رقم رينولدز. دلت النتائج للحالة الاولى على ان قوة السحب على الاسطوانة الخلفية اقل من تلك على الاسطوانة المتقدمة والتي هي اقل من قيمة قوة السحب على الاسطوانة المعزولة, اما بالنسبة للحالة الثانية فان قوة السحب تكون اقل من الاسطوانة المعزولة عندما تكون المسافة الفاصلة بين الاسطوانتين قليلة وتزداد بازدياد المسافة الفاصلة حتى تصل الى قيمة عظمى عند مسافة حوالي 7 مرات بقدر قطر الاسطوانة.

Introduction

There have been great advances in the measurement, prediction and understanding of the flows around bluff bodies over the past decade. This has been stimulated by the widespread liquid solid flows.

Liquid – solid flows occur in many industrial processes. The behavior of the flow and the motions of the particles in these fluids are dependent on the outcomes of the many interactions that occur continually between particles and bounding walls. In the liquid-solid flows where the interstitial fluid is important, the effective coefficient of restitution is important in describing collisions. In this case, it is required to take into account the viscous dissipation and kinetic energy needed to displace the fluid between the surfaces in addition to inelasticity of the contacts [1].

Some studies, theoretical, experimental and applied have been successfully performed for predicting motions of non-spherical particles, especially particle deposition in two-phase flows. Most of the existing studies are for tiny and non reacting particles [2].

Low Reynolds number flow around objects in proximity has been the subject of research for many years. A mathematical model was constructed by

Chemical Department, College of Engineering, University of Baghdad/ Baghdad

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2412-0758/University of Technology-Iraq, Baghdad, Iraq

Stimson and Jeffery [3]. The solution was based on determining the Stokes stream function of the motion of the fluid:

$$F_Z = \pi \mu \int d^3 \frac{\partial}{\partial n}\left[\frac{E^2 \psi}{d^2}\right] ds$$

(1)

where n is the normal drawn outward from the solid and d is the distance from the axis.

Happel and Brenner [4] simplified this further to

$$F_Z = 6\pi \mu a u \lambda \qquad (2)$$

where a is the radius of either sphere and λ can be defined as the a ratio of the drag of one of the spheres to the drag of a sphere in an unbounded fluid.

Experiments were also performed by Happel and Brenner [4] to investigate the change of drag ratio with varying distance between two spheres both parallel and perpendicular to the line of centers.

Kendoush et al [5] measured the drag force between two co-aligned particles for Reynolds number varying from 10 to 10^3. In the present research, the change of drag of two cylinders in fluid flow with respect to the change of distance between them is investigated at low Reynolds numbers.

Experimental Work

The experimental apparatus is shown in figure 1.

The experiments were performed in a quick visible flow (QVF) cylindrical column of length 2.0 m and a diameter of 0.3 m. A glycerin – water solutions of approximately 80 wt% ($\rho_f = 1208.5\ kg/m^3$, $\mu_f = 60.1\ mPa.s$), is circulated upward through the column. With this viscous fluid, the drag force acting on the particle can be detected accurately by using a micro-balance, which has the resolution of 1 mg.

A polyvinyl chloride (PVC) cylindrical particles ($\rho_p = 1366\, kg/m^3$) of 5.41 mm in diameter and 100 mm length were used. Reynolds number was set equal to 1.

The test particle is attached to an electronic balance (Sartorius BL 210S) through a thin rod (about 1.6 mm diameter) such that the drag acting on the test particle can be measured.

Drag force measurements were conducted on the test particle attached to the rod and on the rod only. The particle drag force can be obtained by subtracting the drag force of the rod from that of the test particle with the rod.

The total drag on the test particle and its supporting rod was obtained by deducting the gravitational and buoyancy contributions from the total force measured by the balance.

The gravitational and buoyancy forces are pre-determined in the stationary fluid. The drag coefficient of the particle can be calculated from:

$$F_D = \frac{1}{2} C_D \rho_f S_{eff}\, u^2$$

(3)

where S_{eff} is the particle area normal to the direction of the drag force[4].

Results and Discussion

Flow perpendicular to the line of centers

The drag on each cylinder relative to the isolated cylinder case is plotted for different gaps in Fig.2. The relative drag exceeds unity when the gap is greater than approximately 3 cylinder diameters, reaches a maximum at a gap of 7 cylinder diameters then asymptotes slowly towards unity as the gap is increased. This is similar to a particle falling in a fluid close to a wall where the wall has a retarding effect, causing an increase in drag.

The theoretical value of the drag coefficient for a circular cylinder is given in [6] as:

$$C_D = 8p/[\mathrm{Re}\ln(7.4/\mathrm{Re})] = 12.56 \qquad (4)$$

for Re=1

This result is fundamentally different from the case of two spheres with their line of centers perpendicular to the flow as shown in Fig. 3. In that case at very close separation distances, the drag force increased due to increase in velocity as the flow is forced between the neighboring particles. Beyond separation distances of 5 to 7 particle diameters, the effects become negligible on the measured particle.

Flow parallel to the line of centers

In this case, the wake of the leading cylinder exerts a strong influence on the trailing cylinder, as shown in Fig. 4. Both cylinders experience a lower drag as the gap decreases due to the symmetry however the drag on the trailing cylinder is significantly lower than that on the leading cylinder.

An empirical relation is obtained to describe the effect of the inter-particle distance l/d on the dimensionless drag of the trailing and leading particles. The empirical equations take the exponential form:

For the leading cylinder

$$\frac{C_D}{C_{D0}} = 1 - 0.26\exp\left(-0.26\frac{l}{d}\right) \qquad (5)$$

for ($l/d \geq 1$)

For the trailing cylinder

$$\frac{C_D}{C_{D0}} = 1 - 0.61\exp\left(-0.057\frac{l}{d}\right) \qquad (6)$$

for ($l/d \geq 1$)

Equations 5 and 6 are examined at two limits of separation distances between two cylinders. As l goes to infinity, the second term in the equations vanishes and the drag ratio becomes unity as expected. At contact (l/d =1), the drag ratio equals 0.8 for the leading cylinder and 0.42 for the trailing cylinder, which give the minimum value of the drag ratio.

In the case of two spheres when the line of centers are parallel to the flow, the drag on each individual body is similar and less than the drag of an isolated body for all gaps as shown in Fig. 5.

Conclusions

1. For two cylinders with their line of centers perpendicular to the flow, the drag force is lower than an isolated cylinder at small gaps but is greater at all other gaps; a maximum is found at a gap of approximately 7 cylinder diameters.
2. When the lines of centers are parallel to the flow, the drag on the leading cylinder is significantly greater than that on the trailing cylinder.

Nomenclature

a Sphere radius (m)

C_D Drag coefficient (—)

C_{DO} Drag coefficient of an isolated cylinder (—)

d Cylinder diameter (m)

d_p Particle diameter (m)

$E^2 \equiv d \frac{\partial}{\partial d}\left(\frac{1}{d}\frac{\partial}{\partial d}\right)+\frac{\partial^2}{\partial z^2}$ (An operator)

F_D Drag force (N)

g Gravitational acceleration (m /s^2)

l The distance between the centers of cylinders (m)

R_e Reynolds number ($r_f u d_p / m_f$) (–)

S_{eff} Particle area normal to the direction of the drag force (m^2)

u Particle velocity (m /s)

z Cylindrical co-ordinate

Greek letters

d Distance from the axis (m)

m_f Dynamic viscosity of fluid (kg/m.s)

r_f Density of fluid (kg/m^3)

r_p Density of particle (kg/m^3)

y Stokes stream function

References

[1]-Michaelides, E. E., 2003, "*Hydrodynamic Force and Heat/Mass Transfer from Particles, Bubbles and Drops- The Freeman Scholar Lecture*", ASME J. Fluid Eng., 125, pp 1-30.

[2]-Loewenberg, M., 1993," *Stokes resistance, added mass, and Basset force for arbitrarily oriented, finite-length cylinders*", Phys. Fluids A, 5 (3), pp 765-767.

[3]-Stimson, M. and Jeffery, G. B., 1926, "*The Motion of Two Spheres in a Viscous Fluid*", Proc. Roy. Soc., A111, pp 110-116.

[4]-Happel, J. and Brenner, H., 1973, "*Low Reynolds Number Hydrodynamics*", Noordhroff Int. Publishing, Leyden.

[5]-Kendoush, A.A., Sulaymon, A.H. and Mohammed,S.A.M.,2007,"*Experimentl evaluation of the virtual mass of two solid spheres accelerating in fluids*", Exp. Thermal Fluid Sci., 31, pp 813-823.

[6]-Yin, C., Rosendahl,L.,Kaer, S.K.,and Sorenson H., 2003,"*Modelling the Motion of Cylindrical Particles in a Non-Uniform Flow*", Chem. Engn. Sci., 58, pp 3489-3498.

[7]-Rowe, P. N. and Henwood, G. A., 1961, "*Drag Forces in a Hydraulic Model of a Fluidized Bed-Part I*", Trans. Instn. Chem. Engrs., 39, pp 43-54.

[8]-Liang, S.-C., Hong, T. and Fan, L-S., 1996, "*Effects of Particle* Arrangements on the Drag Force of a Particle in the Intermediate Flow *Regime*", Int. J., Multiphase Flow, 22, pp 285-306.

[9]-Liang, S.-C. and Fan, L.-S., 1994, "*Particle Wake Effect on the Drag Force of an Interactive Particle*", Int. J. Multiphase Flow, 20, pp 117-129.

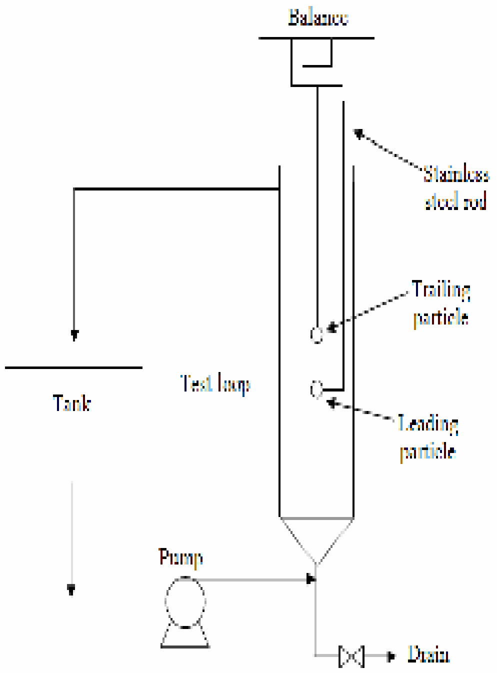

Fig. 1 Schematic diagram of experimental apparatus

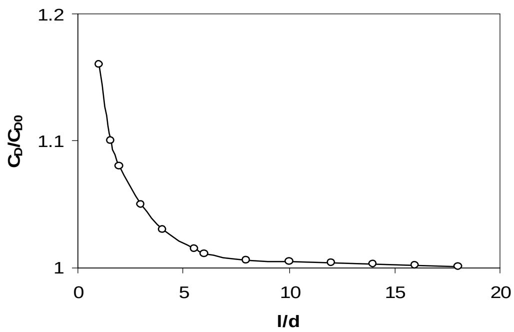

Fig.3.Drag ratio vs. spacing for two spheres in a flow perpendicular to the line of centers [7].

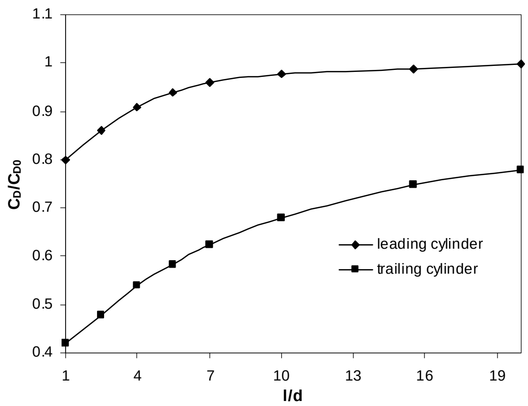

Fig.4. Drag ratio vs. distance for two cylinders in a flow parallel to the line of centers.

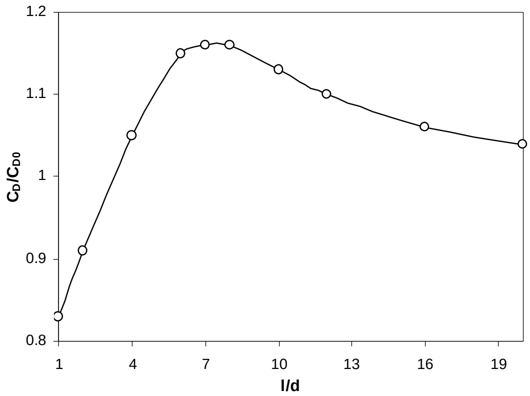

Fig.2.Drag ratio vs. spacing for two cylinders in a flow perpendicular to the line of centers.

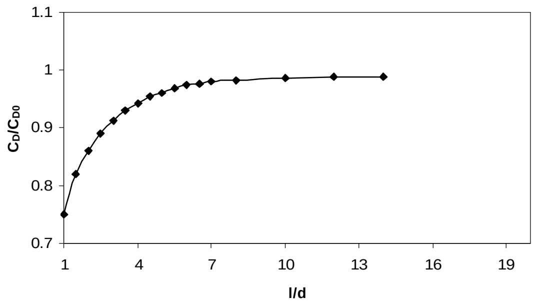

Fig.5. Drag ratio vs. spacing for two spheres in a flow parallel to the line of centers [4], [8], and [9].