Consolidation Characteristics of Unsaturated Soil

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Abstract

The most common three-phase problem in porous media is the flow of air and water. This is for example found in the unsaturated zone, where water infiltrates through partly saturated pores to the groundwater. Liquid flow in the unsaturated zone is controlled by a combination of gravitational, capillary, and viscous forces.

The mechanical behaviour of partially saturated soils can be very different from that of fully saturated soils. It has long been established that for such soils, changes in suction do not have the same effect as changes in the applied stresses, and consequently the effective stress principle is not applicable. Conventional constitutive models, which are based on this principle, are therefore of limited use when analyzing geotechnical problems that involve the presence of partially saturated soil zones.

In this paper, Al-Mdina trial embankment was the problem analysed. The finite element programs SIGMA/W and SEEP/W were used, and eight nodded isoparametric quadrilateral elements were used for modelling both the soil skeleton and pore water pressure. Parametric study was carried out and different parameters were changed to find their effects on the behaviour of partially saturated soil. The parameters include the modulus of elasticity and permeability of the soil.

It was concluded that the effect of modulus of elasticity on the behaviour of unsaturated soil is apparent at early stages of consolidation and diminishes when the time proceeds. When the clay layer consists of soft clay (E_{soil} <10000 kN/m²), the effect of unsaturated soil is apparent, while the effect of the modulus of elasticity diminishes when the soil is stiff.

Keywords: Consolidation, Unsaturated soil, Finite elements, Embankment.
Introduction:
Partially saturated soil is the most common material encountered in the field of geotechnical engineering. Yet, mechanics of partially saturated soil lags far behind that of saturated soil. A partially saturated soil is a complex multi-phase system consisting of air, water and solid material whose response is a function of the stress state, moisture condition and other internal variables present within the soil (Berney et al., 2003).

The classical definition of soil for the purposes of numerical analysis is one in which the volume of material is comprised of two phases, incompressible solid grains and a pore volume consisting of solely air or water known as a saturated soil. However, the more realistic state of soil is one consisting of four phases: solid, liquid, air and an air-water interface that all interact with one another within a matrix of soil grains known as a partially saturated soil. The former soil definition is used extensively for the design of soil structures, however the latter is necessary in the prediction of actual response of soil structures and their effects with changes in environment (Berney et al., 2003). Conventional soil mechanics theory treats soil as either fully saturated (pores filled with water) or dry (pores filled with air). However, a large number of geotechnical problems involve the presence of partially saturated soil zones where the voids between the soil particles are filled with a mixture of air and water. These zones are usually ignored in practice and the soil is assumed to be either fully saturated or completely dry. It has long been established however that behaviour of partially saturated soils can be very different from that of fully saturated or completely dry soils, (Fredlund, 2006).

Previous Works on Unsaturated Soils
Lloret and Alonso (1980) were one of the first to attempt to generate a model for unsaturated soil that included consolidation behaviour. They provided a general formulation for three-dimensional behaviour that allowed for air and water as separate phases.
However, the data for their analyses were determined from one-dimensional tests, and their examples are limited to such tests. They simulated saturated consolidation, infiltration leading to swelling in an unsaturated soil, and collapse due to loading then wetting of an unsaturated soil, all one-dimensionally.

Their results show the importance of variable permeabilities in governing deformation in unsaturated soil. They appear also to support the use of a three-phase approach, with the air phase explicitly modelled as a separate phase. However, since they did not repeat their analyses with a two-phase model, it is not possible to ascertain the full significance of the three-phase approach.

The 1990s and beyond have become a period where there has been an emphasis on the implementation of unsaturated soil mechanics into routine geotechnical engineering practice. A series of international conferences have been dedicated to the exchange of information on the engineering of unsaturated soils and it has become apparent that the time had come for increased usage of unsaturated soil mechanics in engineering practice. Implementation can be defined as “a unique and important step that brings theories and analytical solutions into engineering practice” (Fredlund, 2000).

Zainal, (2002), studied the consolidation process for partially saturated soil. The method of finite elements is used to find a numerical solution to describe the behaviour of soil during consolidation, also the effects of temperature differences are taken into consideration.

The saturated-unsaturated steady state flow through an earth dam is performed on two separated examples. The first one represents an earth dam with a horizontal drain. The second one represents an earth dam with no horizontal drain.

It was found that equipotential lines extend from the saturated zone through the unsaturated zone. Changes in hydraulic head between equipotential lines demonstrate that water flows in both the saturated and unsaturated zones.

The amount of water flowing in the unsaturated zone depends upon the rate at which the coefficient of permeability changes with respect to matric suction.

**Unsaturated Soil as a Four-Phase Mixture**

An unsaturated soil is commonly referred to as a three-phase mixture (i.e., solids, air, and water) but there is strong justification for including a fourth independent phase called the contractile skin or the air–water interface. The contractile skin acts like a thin membrane interwoven throughout the voids of the soil, acting as a partition between the air and water phases. It is the interaction of the contractile skin with the soil structure that causes an unsaturated soil to change in volume and shear strength. The unsaturated soil properties change in response to the position of the contractile skin (i.e., water degree of saturation). It is important to view an unsaturated soil as a four-phase mixture...
for purposes of stress analysis, within the context of multiphase continuum mechanics. Consequently, an unsaturated soil has two phases that flow under the influence of a stress gradient (i.e., air and water) and two phases that come to equilibrium under the influence of a stress gradient (i.e., soil particles forming a structural arrangement and the contractile skin forming a partition between the fluid phases) (Fredlund and Rahardjo, 1993).

**Additional Material Properties for Unsaturated Coupled Analysis**

Fully - coupled approach to the behaviour of unsaturated soils. This approach makes use of Biot (1941) and Dakshanamurthy et al. (1984) theory to generate coupled consolidation equations for unsaturated behaviour. Consolidation analysis can be done by coupling displacements and pore water pressures. In a consolidation analysis, it is interested in how displacement and pore-water pressure change simultaneously.

When coupled, the analysis contributes to forming a common global characteristic (stiffness) matrix. Three equations are created for each node in the finite element mesh. Two are equilibrium (displacement) equations and the third is a continuity (flow) equation. Solving all three equations simultaneously gives both displacement and pore-water pressure change simultaneously.

For a coupled analysis involving unsaturated soils, two additional material properties \( H \) and \( R \) need to be defined. \( H \) is a modulus relating to the change of volumetric strain in the soil structure to a change in suction. \( R \) is another modulus relating the change in volumetric water content to suction; therefore, it is given by the inverse of the slope of the soil water characteristic curve.

In this section, a procedure to obtain the \( H \) modulus parameter from the slope of a void ratio \( (e) \) versus matric suction \( (u_a - u_w) \) curve is described.

For a soil element, a change in its volume can be decomposed into two parts:

\[
dV = dV_s + dV_v \quad \text{...(1)}
\]

where:

\[
dV_s = \text{the change in volume of the soil particles}
\]

\[
dV_v = \text{the change in the volume of voids}.
\]

If the volume change in the soil particles, \( dV_s \), is small and thus neglected, the volumetric strain can be approximated as follows:

\[
dV = dV_v \quad \text{...(2)}
\]

From the definition of void ratio, \( e \), a change in void ratio, \( de \), is given by:

\[
de = d \left[ \frac{V_v}{V_i} \right] = \frac{dV_v}{V_i} = \frac{dV}{(1-n)V} = \frac{d\kappa_v}{(1-n)} \quad \text{...(3)}
\]

where:

\[
n = \text{the porosity of the soil}.
\]

The slope of a void ratio versus matrix suction curve can be written as:
In an unsaturated soil element, when only a change in matric suction occurs, the incremental volumetric strain, $d\varepsilon_v$, can be written as:

$$d\varepsilon_v = d\varepsilon_x + d\varepsilon_y + d\varepsilon_z = \frac{3d(u_h - u_w)}{H} \quad \text{.........(5)}$$

or

$$d\varepsilon_v = \frac{3}{H} \quad \text{.........(6)}$$

After substituting Equation (6) into Equation (4), it can be seen that the slope of a void ratio versus matric suction curve is \(\frac{3}{(1-n)H}\) (User's Guide Manual of SIGMA/W, 2002).

Problem Description

Al-Mdaina trial embankment was constructed in the southern part of Iraq alongside of the road between Talha and Al-Mdaina (near Al-Qurna in Al-Basrah province) in 1979. The height of the embankment is (5 meters), the width at the top is (12 meters), and the width at the bottom is (37 meters) with side slopes of (2.5:1).

Problem Profile

The site investigation of the foundation zone of Al-Mdaina trial embankment revealed that: the first (6 m) is a brown clayey silt with organic material. From (6-12 m) under the ground surface the soil is a gray silty clay, and (12-18 m) from the ground surface the soil is brown to gray silty clay.

Below (18 m) there is a layer of dense sand, Figure (1) shows the cross-section of Al-Mdaina trial embankment.

Modelling and Material Properties

The soil of Al-Mdaina embankment and the soil beneath it have properties as shown in Table (1).

In this work, the model foundation soil is considered to be homogeneous, the properties of one layer (the first layer) are considered for all the depths of the soil.

The finite element mesh is illustrated in Figure (2). Due to symmetry, (210) elements are used for modelling half of the soil and embankment geometry. Eight noded isoparametric quadrilateral elements are used for modelling both the soil skeleton and pore water pressure.

The right and left hand edges of the mesh were restricted to move horizontally and the bottom of the mesh was restricted in both horizontal and vertical directions. The top edge is free in both directions. In addition, the side boundaries are assumed to be impermeable, i.e. no flow is allowed through these sides.

The soil is assumed to follow linear-elastic stress-strain relationship.

Analysis and Results

For analysis, different cases are considered. At first, different water table levels are tried while the same modulus of elasticity of the problem (2870 kN/m$^2$) is used. The water table levels were selected to be at the ground level (fully saturated), and (2 m, 4 m, 6 m and 8 m) below the ground level.
The problem was solved under two-way drainage condition by defining the drainage conditions in the program SEEP/W.

Then, the modulus of elasticity is changed from (2870 kN/m$^2$) to (10000 kN/m$^2$) and (20000 kN/m$^2$), and for each modulus of elasticity, the problem was re-solved with the same water table levels.

The soil suction requires special apparatus for measurement and the soil water characteristic curve (SWCC) is not easy to be determined, therefore, the H-modulus values were either assumed or taken from the program SIGMA/W library for typical soils as shown in Figure (3).

Pore Water Pressure
Figures (4), (5), (6), and (7) show the relationship between the pore water pressure and modulus of elasticity at point A (in Figure 2) at different elevations of water table.

Figures (8), (9), (10), and (11) show the relationship between the pore water pressure and modulus of elasticity at point B (in Figure 2) at different elevations for water table.

In these figures, $u_p$ refers to pore water pressure for the case of partially saturated soil while $u_f$ refers to pore water pressure in the case of fully saturated soil.

From Figure (4) it can be noticed that the difference between the pore water pressure in the case of partially saturated soil and fully saturated soil decreases with time. The percentage of pore water pressure ($u_p$) to ($u_f$) is about (0.68) at a time of (10 days), this percent is reduced to about (0.18) at the time of (2000 days). This is also the case in Figure (5).

It can be concluded that the effect of unsaturated soil on consolidation characteristics appears at early stages of consolidation. In addition, when the clay layer consists of soft clay ($E_{soil} <10000$ kN/m$^2$), the effect of unsaturated soil is apparent, while effect of the modulus of elasticity diminishes when the soil is stiff.

It was also noticed that, with increase in the modulus of elasticity, the difference between percentage of pore water pressure of partially saturated soil and pore water pressure of fully saturated soil is decreased clearly. For instance, in Figure (4), at time (10 days) the percentage of ($u_p/u_f$), when the modulus of elasticity value is (2870 kN/m$^2$), is equal to (0.68), while this percentage becomes (0.6) when the modulus of elasticity is (10000 kN/m$^2$) and reaches about (0.58) when the modulus of elasticity becomes (20000 kN/m$^2$). Moreover, it is noticed that the effect of modulus of elasticity on pore water pressure decreases with time. It was concluded that, the difference between the values of ($u_p/u_f$) at the time (10 days) reaches (0.1) for the case of modulus of elasticity of (2870 kN/m$^2$) and (20000 kN/m$^2$). This difference decreases up to (0.02) at time (2000 days). This is also clear in Figure (5).

For clayey soil, with a relatively high degree of saturation, from about (90 %) (the case of W.T. at 2 m from the ground surface), the air in the soil is occluded and can often be assumed to have little effect on the pore water pressure. In such a case, the unsaturated soil will tend to behave as if it were saturated and the effective stress can be
assumed to be equal to \((\sigma-u_w)\). The exception is a fine grained soil near to but on the dry side of optimum where the air may not be occluded. In this case, the effective stress will not even be approximately equal to \((\sigma-u_w)\) (Smith and Smith, 1998).

Figure (6) illustrates different behaviours of the soil; the percentage \((u_p/u_f)\) increases with time until reaching \((190\, \text{days})\), after that, the percentage returns to decrease again until it becomes compatible with the previous cases at time of \((2000\, \text{days})\). This means that the effect of modulus of elasticity on the behaviour of unsaturated soil is apparent in early stages of consolidation and diminishes when the time proceeds. It is also noticed that, the percentage of \((u_p/u_f)\) increases at the same time with the increasing of the value of modulus of elasticity. For example, at the time \((190\, \text{days})\), the difference reaches about \((1.1)\) when the modulus of elasticity was changed form \((2870\, \text{kN/m}^2)\) to \((20000\, \text{kN/m}^2)\), but however, this difference decreases after \((190\, \text{days})\).

In Figure (7), the soil behaves as in Figure (6), with exception that, the increase of the percentage \((u_p/u_f)\) was continued until the time of \((70\, \text{days})\) and after that returned to decrease. In all cases, the effect of partial saturation of the soil diminishes at the end of consolidation, i.e. at time of about 2000 days. Due to what was mentioned above, it can be concluded that the behaviour of the soil, when the depth of water is \((6\, \text{m})\) and below, is different from that when the depth of water is less. When negative value of pore water pressure is generated, the behaviour can be explained by that the value of negative pressure after \((4\, \text{m})\) was overriding the phenomenon of Mendel-Cryer which leads to increase in the value of initial pore water pressure and this will eliminate the negative value of water pressure up to a depth of \((4\, \text{m})\). Figure (8) represents the pore water pressure at a point on the embankment toe. At the time \((10\, \text{days})\), it is noticed clearly that the percentage \((u_p/u_f)\) increases with increasing the value of modulus of elasticity. At the time \((70\, \text{days})\) and afterward, it was noticed that the unsaturation has no effect on changing the pore water pressure.

Figure (9) reveals that the percentage \((u_p/u_f)\) is increasing toward the positive values at the time \((10\, \text{days})\) with increase in modulus of elasticity. This increase is reflected negatively toward the time \((70\, \text{days})\) with a few differences in this percentage of increase in modulus of elasticity. After \((70\, \text{days})\) the percentage \((u_p/u_f)\) reaches zero, and therefore, the effect of increasing of modulus of elasticity will decrease.

In Figure (10), the percentage \((u_p/u_f)\) increases at time \((10\, \text{days})\) with increasing of modulus of elasticity and. At time \((70\, \text{days})\), the percentage \((u_p/u_f)\) becomes greater in comparison with the value at time \((10\, \text{days})\) at the same modulus of elasticity, and this attitude continues until reaching a value of \((16000\, \text{kN/m}^2)\), where after this value, a conversion will occur and the percentage \((u_p/u_f)\) becomes greater at time \((10\, \text{days})\) in comparison with \((70\, \text{days})\). At time \((190\, \text{days})\) and after, the curves were compacted on each other at a value approaches zero, nearly with modulus of elasticity of \((2870\, \text{kN/m}^2)\).
and with a slight increase in the percentage toward the negative with the increase of the modulus of elasticity.

Figure (11) reveals that at time (10 days), the value of the percentage \( \frac{u_p}{u_f} \) increases with increasing values of modulus of elasticity, and the value is changed from (30) at modulus of elasticity of \((2870 \text{ kN/m}^2)\) to (100) at modulus of elasticity value of \((20000 \text{ kN/m}^2)\). However, this increase becomes lower at time (70 days) as it will range between (20) at modulus of elasticity of \((2870 \text{ kN/m}^2)\) and (45) at modulus of elasticity of \((20000 \text{ kN/m}^2)\). At time (190 days) and afterward, the curves will be compacted again on each other with reducing of the difference of the percentage \( \frac{u_p}{u_f} \) with increasing of modulus of elasticity, and the value of \( \frac{u_p}{u_f} \) is between \((-2.5)\) at the modulus of elasticity of \((2870 \text{ kN/m}^2)\) and \((-4)\) at modulus of elasticity of \((20000 \text{ kN/m}^2)\).

This behaviour can be explained by the fact that voids of unsaturated soil are filled either with water, a water and air mixture or simple air. With air and water filled voids, small lenses of water from menisci around the particle contacts. With clays, there will also be absorbed water, so strongly attached to the soil particles that it can be regarded as being part of the soil skeleton.

Although the volume of meniscus water in an unsaturated soil may be very small, it can have a dramatic effect on the mechanical behaviour of the soil, an such effect that cannot be estimated.

**Vertical displacement**

Figures (12), (13), (14) and (15), present the relationship between the vertical displacement and modulus of elasticity at point (A) (shown in Figure 2) at different elevations of water table.

Figures (16), (17), (18), (19), show the relationship between the vertical displacement and modulus of elasticity at point (B) (shown in Figure 2) at different elevations of water table.

In these figures, \( \delta_p \) and \( \delta_f \) refer to vertical displacement in the case of partially and fully saturated soil, respectively.

Figures (12) to (15) generally illustrate a similar behaviour, when the modulus of elasticity is \((2870 \text{ kN/m}^2)\), the percentage of vertical displacement of partially saturated soil to vertical displacement of fully saturated soil \( \left( \frac{\delta_p}{\delta_f} \right) \) decreases with time. This is also the case when the modulus of elasticity varies between \((10000 – 20000 \text{ kN/m}^2)\). It is noticed that at a certain time, the percentage \( \left( \frac{\delta_p}{\delta_f} \right) \) was decreased with increasing the value of modulus of elasticity. It is also noticed that with increasing the depth of water, the percentage \( \left( \frac{\delta_p}{\delta_f} \right) \) at a certain time and at a certain value of modulus of elasticity is decreased. For example, in Figure (12), the value of \( \left( \frac{\delta_p}{\delta_f} \right) \) at time (10 days) and when the modulus of elasticity is \((10000 \text{ kN/m}^2)\) is \(0.967\) while the value was \(0.86\) in Figure (15) where the water table is at depth \((8 \text{ m})\).

Figures (16) to (19) reveal the same previous behaviour of the soil but with a less percentage of \( \left( \frac{\delta_p}{\delta_f} \right) \) for a certain depth. For example, in Figure (18) at time (800 days) and when the modulus of elasticity is \((20000 \text{ kN/m}^2)\), the value of \( \left( \frac{\delta_p}{\delta_f} \right) \) is \(0.63\) while it becomes \(0.83\) in Figure (14) for the same time and modulus of elasticity value.
This means that the effect of unsaturation becomes greater at the middle of the clay layer and near the center line of the embankment where the load concentrates more than at its toe.

Generally the settlement of fully saturated soil is greater than partially saturated soil because in the state of partially saturated soil friction between the soil particles is greater and the excess pore water pressure is the smallest.

Conclusions:
From the finite element analysis carried out on a trial embankment, the following conclusions can be drawn:

1. The effect of modulus of elasticity on the behaviour of unsaturated soil is apparent at early stages of consolidation and diminishes when the time proceeds. At (10 days) the difference of \( \frac{u_p}{u_f} \) (\( u_p \) refers to pore water pressure for the case of partially saturated soil while \( u_f \) refers to pore water pressure in the case of fully saturated soil) between modulus of elasticity of (2870 kN/m\(^2\)) and (20000 kN/m\(^2\)) is about (0.081), while at time (2000 days) the difference becomes approximately zero.

2. When the clay layer consists of soft clay (\( E_{\text{soil}} <10000 \text{ kN/m}^2 \)), the effect of unsaturated soil is apparent, while effect of the modulus of elasticity diminishes when the soil is stiff. At (10 days) the difference of \( \frac{\delta_{vp}}{\delta_{vf}} \) (\( \delta_{vp} \) and \( \delta_{vf} \) refer to vertical displacement in the case of partially and fully saturated soil, respectively) between modulus of elasticity of (2870 kN/m\(^2\)) and (10000 kN/m\(^2\)) is about (0.05), while after modulus of elasticity of (10000), the difference becomes approximately zero.

3. The phenomenon of Mendel-Cryer which leads to increase in the value of initial pore water pressure will eliminate part of negative pore water pressure at Aeration zone. The excess pore water pressure of fully saturated soil is greater than partially saturated soil and dissipation starts at fast rate and becomes slow with time.

References:
John Wiley and Sons, New York, U.S.A.

Table (1) Material properties for Al-Mdaina trial embankment (after Al-Hamrany, 1980 as cited by Kadhim, 2008).

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<th>Position</th>
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<td></td>
<td>Poisson’s ratio ((v))</td>
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<td>—</td>
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<td></td>
<td>Total unit weight ((\gamma_t))</td>
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<td>kN/m³</td>
</tr>
<tr>
<td>Foundation Soil</td>
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<td>kN/m²</td>
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<tr>
<td></td>
<td>Poisson’s ratio ((v))</td>
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<td>Total unit weight ((\gamma_t)) (calculated)</td>
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<td></td>
<td>Cohesion (c)</td>
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<td>kN/m²</td>
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</table>

Figure (1) Cross-section of Al-Mdaina trial embankment, (from Kadhim, 2008).
Figure (2) Finite element mesh of Al-Mdaina trial embankment.

Figure (3) Typical soil water characteristic curve (Ba-Te et al., 2005).
Figure (4) The relation between \( \frac{u_p}{u_l} \) and soil modulus of elasticity at point (A) with water table at (2 m) from the ground level.

Figure (5) The relation between \( \frac{u_p}{u_l} \) and soil modulus of elasticity at point (A) with water table at (4 m) from the ground level.
Figure (6) The relation between (up/uf) and soil modulus of elasticity at point (A) with water table at (6 m) from the ground level.

Figure (7) The relation between (up/uf) and soil modulus of elasticity at point (A) with water table at (8 m) from the ground level.
Figure (8) The relation between \((up/uf)\) and soil modulus of elasticity at point (B) with water table at (2 m) from the ground level.

Figure (9) The relation between \((up/uf)\) and soil modulus of elasticity at point (B) with water table at (4 m) from the ground level.
Figure (10) The relation between (up/uf) and soil modulus of elasticity at point (B) with water table at (6 m) from the ground level.

Figure (11) The relation between (up/uf) and soil modulus of elasticity at point (B) with water table at (8 m) from the ground level.
Figure (12) The relation between $\frac{\delta v_p}{\delta v_f}$ and soil modulus of elasticity at point (A) with water table at (2 m) from the ground level.

Figure (13) The relation between $\frac{\delta v_p}{\delta v_f}$ and soil modulus of elasticity at point (A) with water table at (4 m) from the ground level.
Figure (14) The relation between $\frac{\delta V_p}{\delta V_f}$ and soil modulus of elasticity at point (A) with water table at (6 m) from the ground level.

Figure (15) The relation between $\frac{\delta V_p}{\delta V_f}$ and soil modulus of elasticity at point (A) with water table at (8 m) from the ground level.
Figure (16) The relation between \( \frac{\delta v_p}{\delta v_f} \) and soil modulus of elasticity at point (B) with water table at (2 m) from the ground level.

Figure (17) The relation between \( \frac{\delta v_p}{\delta v_f} \) and soil modulus of elasticity at point (B) with water table at (4 m) from the ground level.
Figure (18) The relation between \( \frac{\delta v_p}{\delta v_f} \) and soil modulus of elasticity at point (B) with water table at (6 m) from the ground level.

Figure (19) The relation between \( \frac{\delta v_p}{\delta v_f} \) and soil modulus of elasticity at point (B) with water table at (8 m) from the ground level.