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Boundary Element Analysis of Capped Pile Groups

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Abstract

The boundary element method (BEM) has become one of the most powerful numerical techniques which has already established itself within the scientific community. The most striking feature of this technique is that, in principle, only the boundaries of the region being investigated have to be discretized, which therefore leads to many fewer discrete elements than any scheme requiring internal subdivision of the whole body. This means that the number of unknowns is reduced dramatically, especially for 3D problems, as the unknowns occurred only on the boundary of the problem.

This paper is devoted to make use of the boundary element method (BEM) as a practical problem solving tool to analyze a soil - structure interaction problem. The program (MRBEM) is adopted in this study for the analysis process. It is a general purpose boundary element method program for solving elasticity and potential problems with multiple regions. This program is written by FORTRN-90 language and developed during this study to solve a three dimensional problem represented by a group of piles. The results were compared with those findings in some experimental and theoretical researches and good agreements were obtained.

It was found that when using the BEM in the analysis, the stresses and displacements need only to be calculated where the details of interest occur on the boundary or are localized to a particular part of the domain, and hence an entire domain solution is not required. Moreover, boundary conditions at infinity can be modeled exactly without the need to extend the region a long distance away or to apply artificial boundary conditions as a result to the arbitrary truncation of the outer region.

Keywords: Boundary element, pile group, pile cap.

التحليل بطريقة العناصرالمتاخمة لمجاميع الركائز المربوطة بغطاء

الخلاصة

أصبحت طريقة العناصر المتاخمة (BEM) واحدة من أقوى التقنيات العددية والتي أسست لنفسها أن تكون ضمن المجتمع العلمي. الصفة الأكثر تميزا" لهذه التقنية أن عملية تجزئة المنطقة المراد التحري عنها الى عناصر يقتصر على تخوم هذه المنطقة, الأمر الذي يؤدي الى أن عدد العناصر المجزئة يكون أقل من مخطط التجزئة الداخلي المطلوب للجسم كله. هذا يعني أن عدد المجاهيل سيقل بصورة هائلة, خصوصا" للمسائل ثلاثية الأبعاد, بسبب اقتصار ظهور المجاهيل عند تخوم المسألة.

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هذا البحث مكرس للأستفادة من طريقة العناصر المتاخمة كوسيلة حل عملية لتحليل مسائل التداخل بين التربة والمنشآت. تم اعتماد البرنامج (MRBEM) لأجراء عمليات التحليل في هذه الدراسة. و هو برنامج متعدد الأغراض ويعتمد على طريقة العناصر المتاخمة لتحليل مسائل المرونة والطاقة لمناطق متداخلة متعددة. كتب هذا البرنامج بلغة فورتران-90 وقد تم تطويره خلال مرحلة البحث لحل مسألة ثلاثية الأبعاد متمثلة بمجموعة من الركائز. تم مقارنة النتائج المستحصلة مع بعض الأبحاث العملية والنظرية وأثبتت توافقا" جيدا" مع هذه الأبحاث.

وجد أنه عند استعمال طريقة العناصر المتاخمة فأن الاجهادات والازاحات سيقتصر احتسابها عند مناطق مهمة محددة على التخم أو أجزاء معينة ضمن مجال المنطقة دون الحاجة لأيجاد حل لكامل الأجزاء الداخلية للمسألة. اضافة الى ذلك, فأن تمثيل الشروط التخومية للمسألة في المناطق البعيدة يمكن تمثيلها بصورة دقيقة دون اللجوء الى بسط المنطقة لمسافات بعيدة وتطبيق شروط تخومية غير حقيقية نتيجة" للقطع العشوائي للمنطقة الخارجية.

Introduction

With the modern development of experimental research, theoretical analysis becomes very powerful and soundly based. A successful theoretical analysis is capable of producing accurate and reliable results without the risk or expense required to build a test rig and carry out an actual experimental investigation (El-Zafrany, 1992).

There are many textbooks which describe the mathematical background of BEM (Beer, 2001). Unfortunately, most texts to date have been written by mathematicians or engineers with a strong background in mathematics and, therefore, they tend to dwell on the theoretical treatment of the method rather than concentrating on physical meaning and implementation. Furthermore, many books include simple programs in appendix; few have examples on how the theory is translated into a computer program. Since it is obvious that the methods would not be useful without computers, the lack of emphasis on computer implementation is surprising. In contrast to many mathematicians, who are happy just to prove the existence of a solution and error bounds, the engineer is interested in application of the method in solving real problems.

The Program MRBEM

The program MRBEM (Multi-Region Boundary Element Method) is a general purpose BEM program for solving elasticity and potential problems with multiple regions. This program is suitable for solving soil-structure interaction problems in which different material models are prescribed. For the solution of non- homogeneous domains, the place where cannot obtain a fundamental solution, the concept of multiple regions is adopted where the domain is subdivided into subregions, much in the same way as with the FEM. Since at the interfaces between the regions, both tractions and displacements are not known, the number of unknowns is increased and additional equations are required to be able to solve the problem. These equations can be obtained from the conditions of equilibrium and compatibility at the region interfaces.

There are two approaches which can be taken in the implementation of the method. In the first, the assembly procedure is modified so that larger systems of equations are obtained including the additional unknowns at the interfaces. The second method is similar to the approach taken by the finite element method (FEM). Here, a stiffness matrix K is constructed for each region, the coefficients of which are the fluxes or tractions due to unit temperatures/ displacements. The matrices K for all regions are then assembled in the same way as with the FEM. The second method is more efficient and more amenable (Beer, 2001).

Description of the program MRBEM

The first part of the program reads input data. There are three types of data: job specification, geometry, and boundary data. They are read in by calling three separate subroutines, Jobin MR, Geomin, and BC input. The flow chart of the program MRBEM is shown in Figure (1).

- Subroutine Jobin MR: presents the job information of multi-region which consist of the Cartesian dimension of the problem (2-D or 3-D), the type of region (finite or infinite), whether it is potential or elasticity problem, the type of elements used (linear or quadratic), the properties, that is conductivity for potential problems and modulus of elasticity and Poisson's ratio for elasticity problems, and number of elements / nodes.
- Subroutine Geomin: represents the geometrical information which consists of the coordinates of the nodes and the element incidences.
- Subroutine BCinput: represents the boundary conditions where input.
- Subroutine Mirror: this subroutine has been written to generate elements across symmetry planes. It returns the incidence, destination and coordinate vector of the mirrored element, as well as multiplication factors for the assembly. In this subroutine, it is assumed that if points are on the symmetry plane, then they have a zero coordinate, and one must ensure that this is actually the case.
- Subroutine Assembly: a subprogram for assembling the coefficient matrices using a vector of incidences or destinations, as well as information about the type of boundary and symmetry condition is easily written. The information about the boundary condition is supplied for each node or each degree of freedom of an element, and the code is 0 for Neumann and 1 for Dirichlet condition.
- Subroutine Solve: after assembly and the computation of the diagonal coefficients, a system of simultaneous equations is obtained. These equations are solved by numerical approximation using the Gauss elimination method which is employed in this subroutine.

For a boundary value problem, either the displacement u or the traction t is specified and the other is the unknown to be determined by solving the integral equations. The specified boundary condition of displacement u is also known as the *Dirichlet* boundary condition, whereas the specification of flow t or traction t is often referred to as a *Numann* boundary condition. The assumption of the program is that all nodes have a *Numann* boundary condition with zero prescribed value by default. All nodes with Dirichlet boundary conditions and all nodes having a Numann boundary condition with non-zero prescribed values have to be input.

Axial load transfer for piles in sand

A pile test program was carried out in 1984 at the Baghdad University Complex, Iraq, close to the bank of the River Tigris. An instrumented square test pile consisting of 285 ^{mm} diameter, 12.0 m long, precast concrete pile was driven 11.0 m into sand deposit. The purpose of the test was to obtain information for use in the design of pile foundations for the expansion of the university campus (Altaee et al., 1992).

The soil profile next to the test pile location consisted of two main soil layers. One upper, 3.0 m layer of clayey silt sand deposited on a lower, thick layer of uniform sand with some silt. The standard penetration test (SPT) index (N-index) was obtained from a borehole located 7.0 m from the test pile. Figure (2) illustrates the results of standard penetration test (SPT) and cone penetrometer test (CPT) which shows that the soil to be of compact (medium) condition depending on the N-index values which vary through 25 blows/0.3 m and an average CPT friction ratio of 2.4%.

The pile compression loading test consists of three stages. At the first two stages, the load on the pile was applied in increments of 100 kN to a maximum load of 1000 kN and unloading. In the third stage, the pile was loaded until it reached failure at a maximum load 1600 kN.

The BEM is used to simulate this problem by trying to solve the pile-soil interaction using the program MRBEM. For the reason of incapability to give an exact identification to those three stages of loading, especially the unloading condition, it is decided to deal only with the first stage of loading. The axial load is applied to the pile head and the settlement of the pile is calculated as the average of settlement values for the four nodes at the pile head.

Three regions are considered to represent this problem. The upper and lower soil layers are represented as independent regions so as to include the difference in their properties. The third region is represented by the square concrete pile which lies inside the soil regions. The basic idea is to consider a number of regions which are connected to each other as much like pieces of puzzle.

Figure (3) shows the pile-soil interaction system which represents the problem to be solved by the BEM. Each region is defined by linear boundary elements describing its boundary, and the mesh generation, as it appears from this figure, is restricted on the boundary without the need to discretize the entire body. At the interface between regions, one can notice that there are two boundary elements; each belongs to its original region, in other words, the boundary element which represents the properties of region. These interface elements are identical on both regions, except that the sequence of node numbers is reversed. The boundary of the pile-soil system, represented by the three regions, is discretized into 62 and 102 elements respectively, so as to study the effect of increasing the number of elements on the accuracy of the results (Al-Soud, 2008).

As it was mentioned previously, there are two kinds of boundary conditions describing the unknowns per each node at the boundary, a Neumann boundary condition (t) and Dirichlet boundary condition (u). In this problem, the traction (Neumann boundary condition) is a known value at the top of the pile which is equal to the applied force, while the displacement (Dirichlet boundary condition) at the pile tip is equal to zero.

Figures (4) and (5) illustrate the results of load-displacement diagram and the load distribution in the pile respectively. These figures reveal that the results of MRBEM program got a maximum percentage difference about 20% as compared to the experimental work done by Altaee et. al. (1992), and they come closer with a difference percentage of 16% by using a finer mesh. It can be noticed that at smaller loads (within the elastic range), the prediction of settlement by the BEM is very close to the measured settlement.

The good agreement of the previous results with the experimental works gives an encouragement towards the use of the program MRBEM to deal with a wider range of soil-structure interaction problems.

The above test is repeated on a single and group of circular piles, 1, 2, 4 piles respectively with and without ground contacting caps, which were adopted by Butterfield and Banerjee (1971). A boundary element mesh is generated first to simulate a capped or uncapped single pile and its interaction with the surrounding soil. Then, this single pile mesh could be enlarged to simulate a couple of piles or a group of 4 piles by taking the advantage of symmetry about one or two axes, which is one of the MRBEM program's features.

In order to be identical to the previous work of Altaee et al. (1992), the circular pile used in the present study is converted to a square pile by taking an equivalent diameter, as shown in Figure (6), (Al-Soud, 2008).

The boundary element mesh for a single capped pile with the surrounding soil is shown in Figure (7). The results were normalized by taking the factor K_p (stiffness factor) represented by the equation:

$$K_p = \frac{P}{GWD} \qquad (1)$$

where P = axial load,
G = shear modulus of the soil medium,
W = displacement of head of pile, and
D = shaft diameter.

The effects of length to diameter ratio, pile compressibility ratio λ (which is the ratio of E for the pile material to G for the medium), the distribution of the load between the cap and the individual piles in the group have been studied and typical results are presented in Figures (8) to (13).

These figures reflect the good approximation of the results gained by the program MRBEM as compared with those obtained from Butterfield and Banerjee (1971). The convergence with the original work is increased by increasing the number of elements. The results are also compatible with Paiva and Trondi (1999).

Conclusions:

It is more reliable to say that this paper is concerned with bringing out the role of the BEM in the soil-structure interaction problems rather than the analysis of the soil-structure interaction problems by this technique. The following conclusions were found n this work:

1. Only the boundary discretization is needed which leads to simple data preparation and less storage requirements. These facilities are quite suitable for 3D geotechnical problems.
2. Stresses and displacements need only to be found where required.
3. Boundary conditions at infinity can be modeled exactly without the need to the arbitrary truncating of the outer region.
4. Generally, the settlement of the pile group increases with increasing the number of piles and the amount of the applied load. The settlement decreases with increasing pile diameter, pile length and the spacing between piles.

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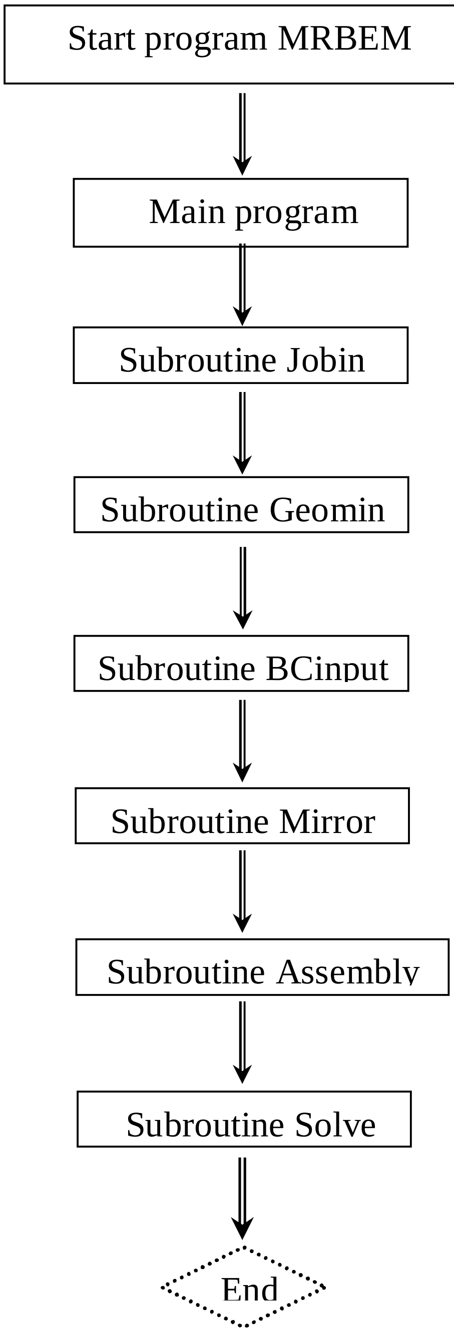
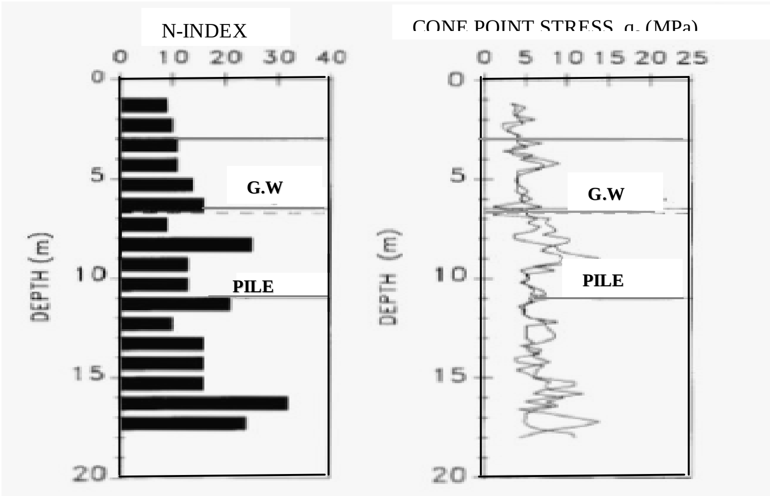


Figure (1): Flow chart of boundary element program MRBEM.



(a) (b)

Figure (2): Soil field test at Baghdad University complex.
(a) Standard penetration test. (b) Cone penetration test
(after Altaee et al., 1992).

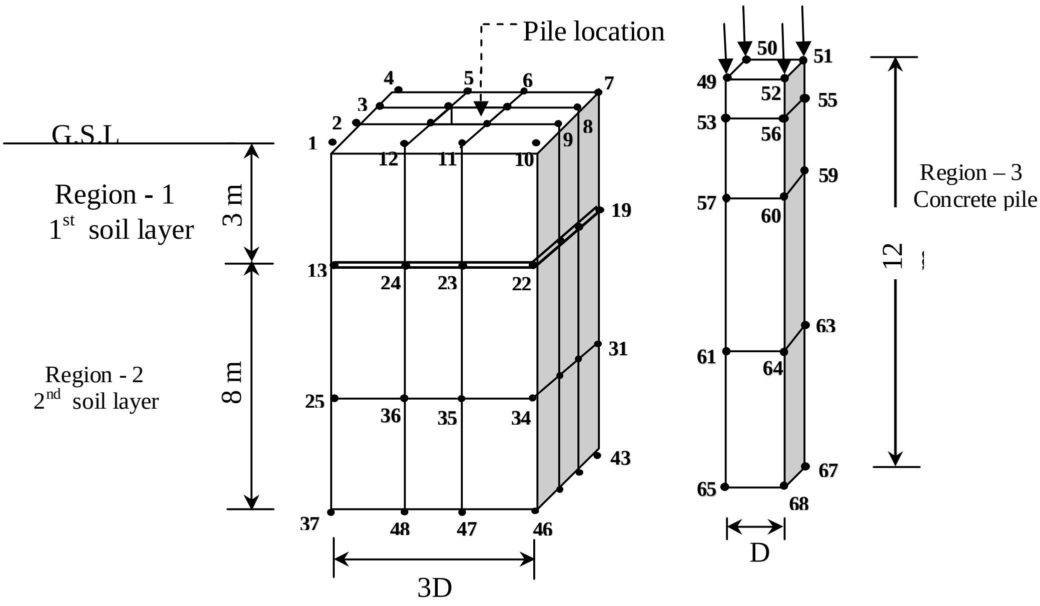


Figure (3): Pile – soil mesh system.

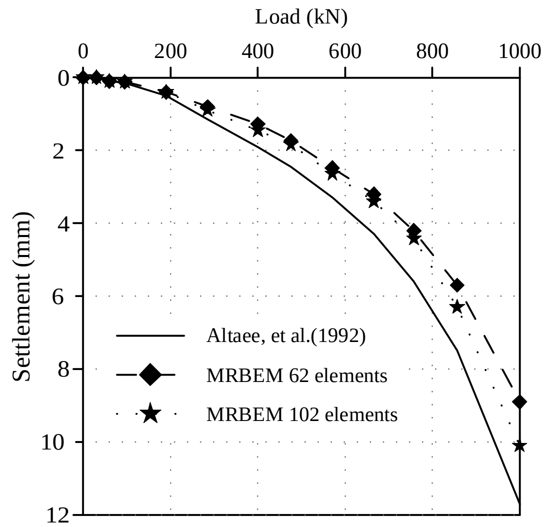
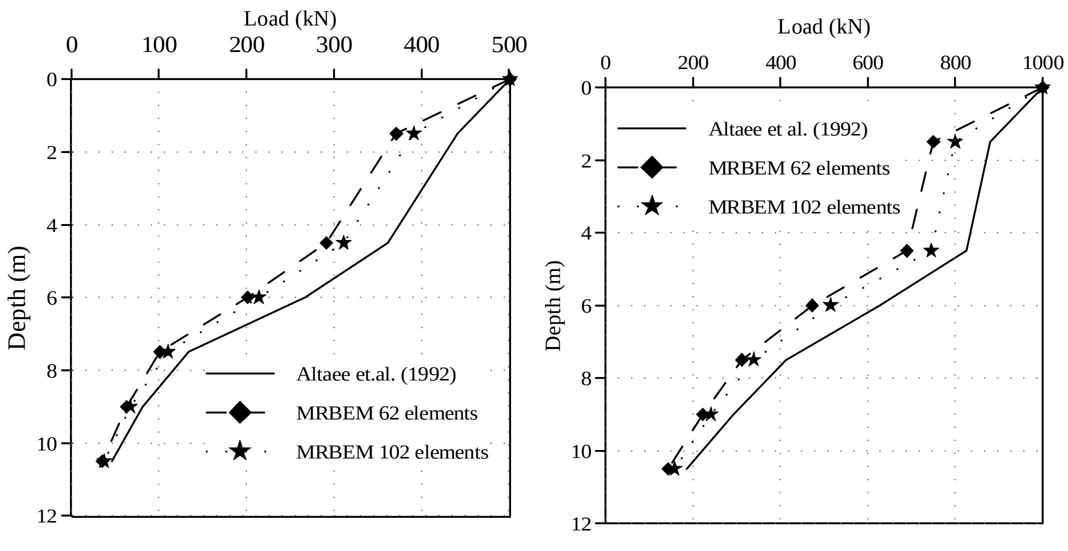


Figure (4): Pile head load-movement diagram.



(a)　　　　(b)

Figure (5): Load distribution along the pile (a) Max. load 500 kN (b) Max. load 1000 kN.

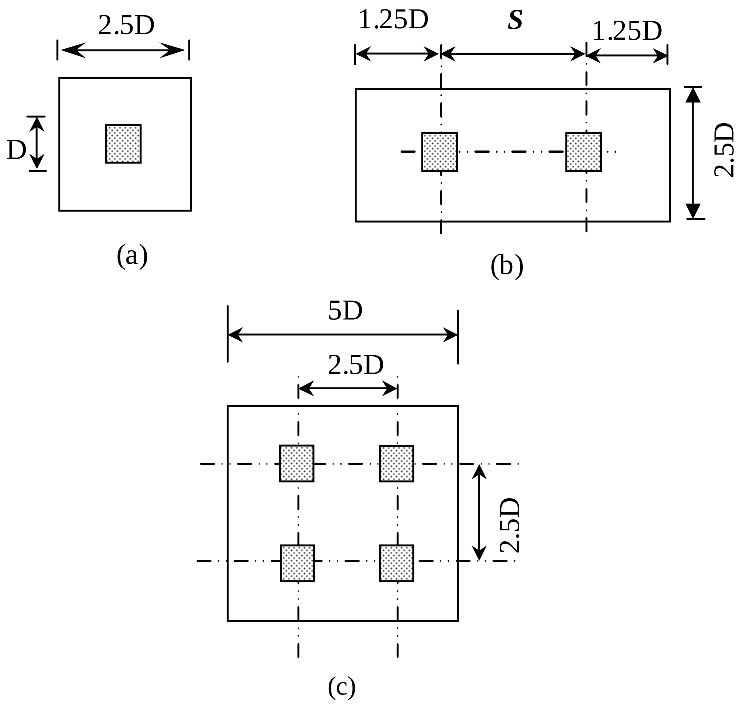
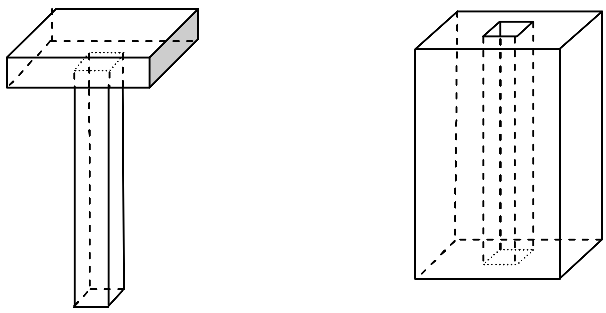
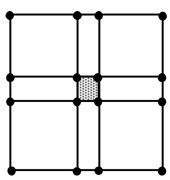


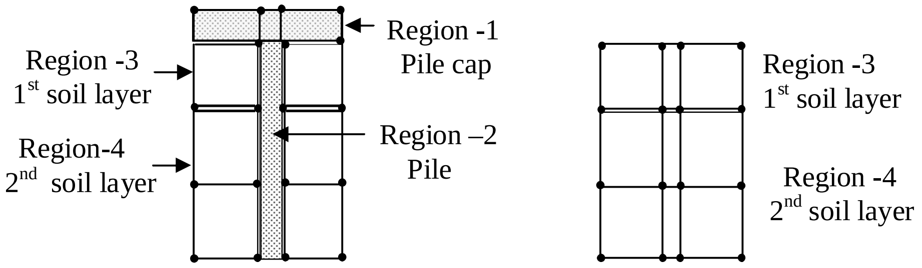
Figure (6): Capped pile group system (a) Single pile (b) Two- pile group (c) Four-pile group.



a) General description

b) The cap mesh





c) Pile cap – soil discretization

d) Soil boundary discretization

Figure (7): Boundary element mesh for a capped single pile.

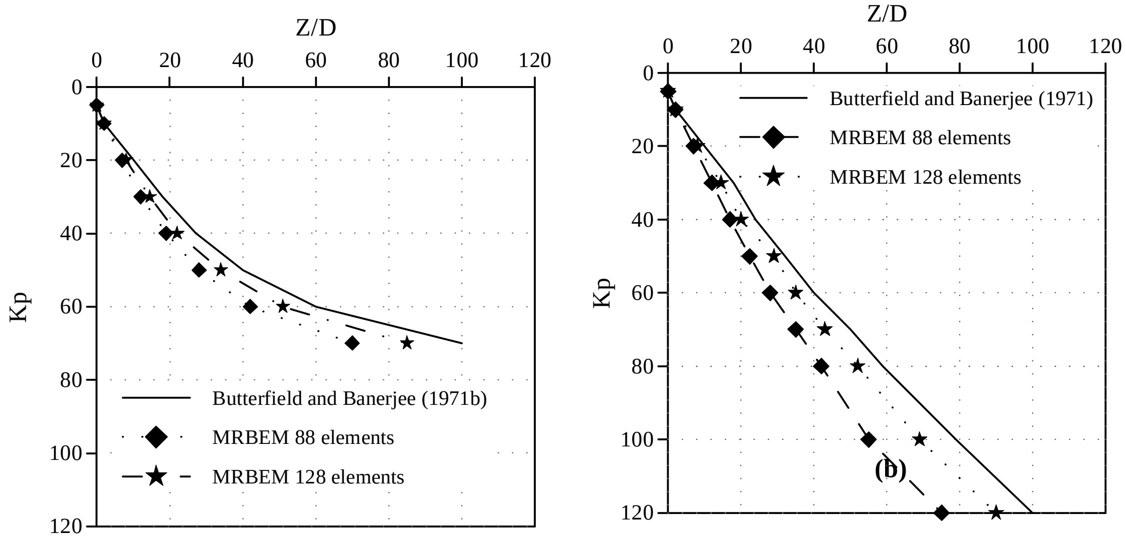


Figure (8): Load-displacement characteristics of a single uncapped pile. (a) λ = 6000 (b) λ = ∞

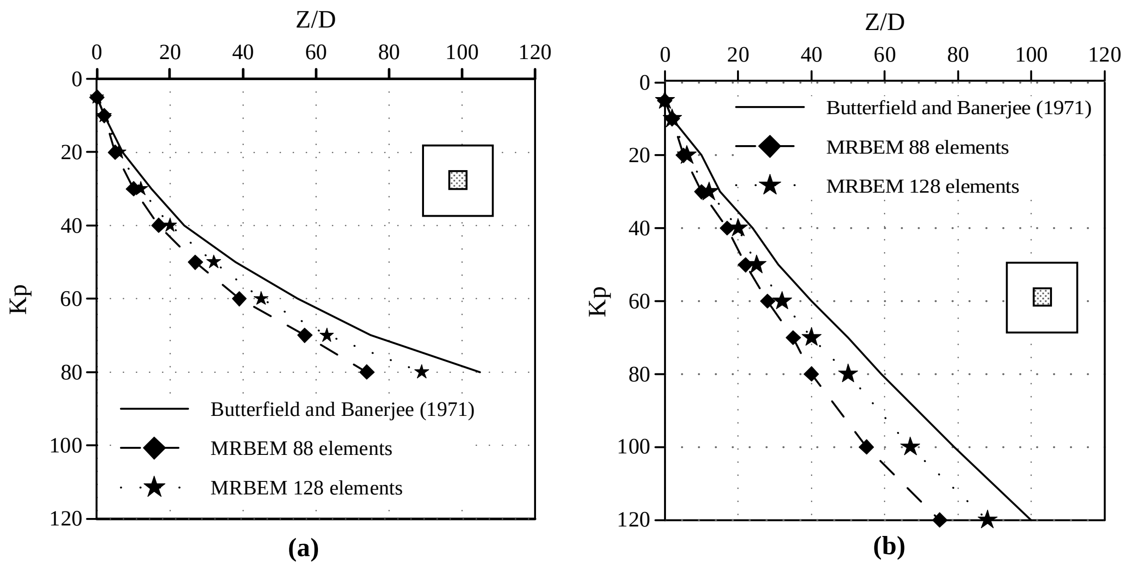


Figure (9): Load-displacement characteristics of a single capped pile. (a) λ = 6000 (b) λ = ∞

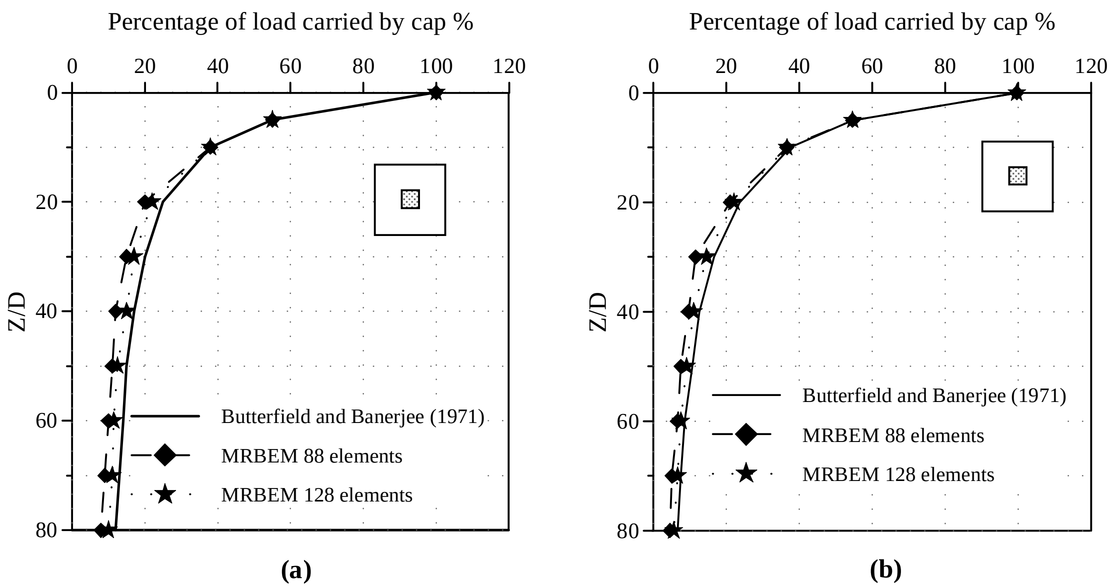
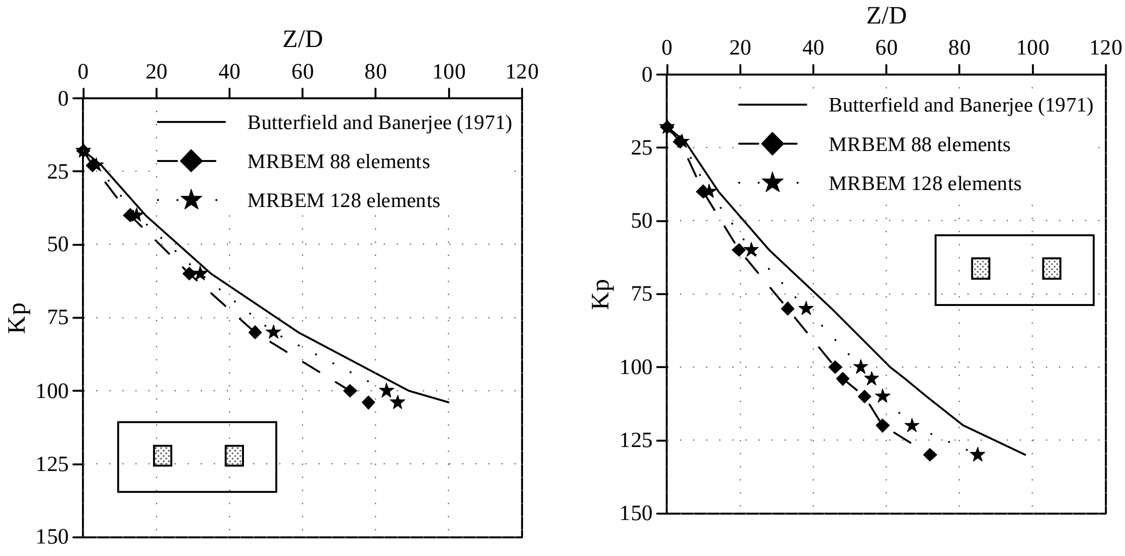
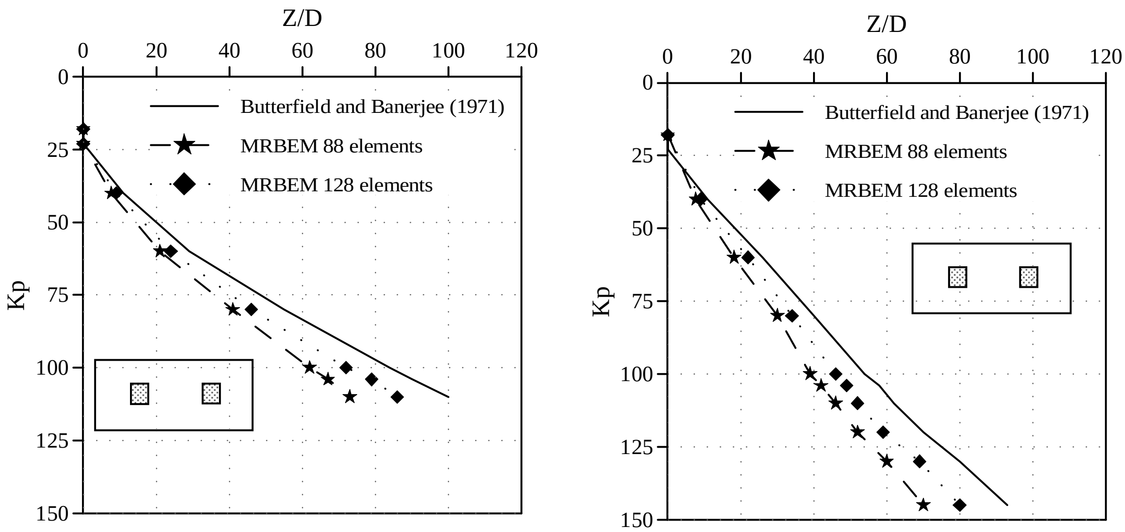


Figure (10): Load distribution between the square cap and a single pile, (a) λ=6000 (b) λ= ∞.



(a) (b)

Figure (11): Load-displacement characteristics of a capped group of two piles with S/D =2.5, (a) λ =6000 (b) λ= ∞.



(a) (b)

Figure (12): Load-displacement characteristics of a capped group of two piles with S/D = 5. (a) λ =6000 (b) λ= ∞.

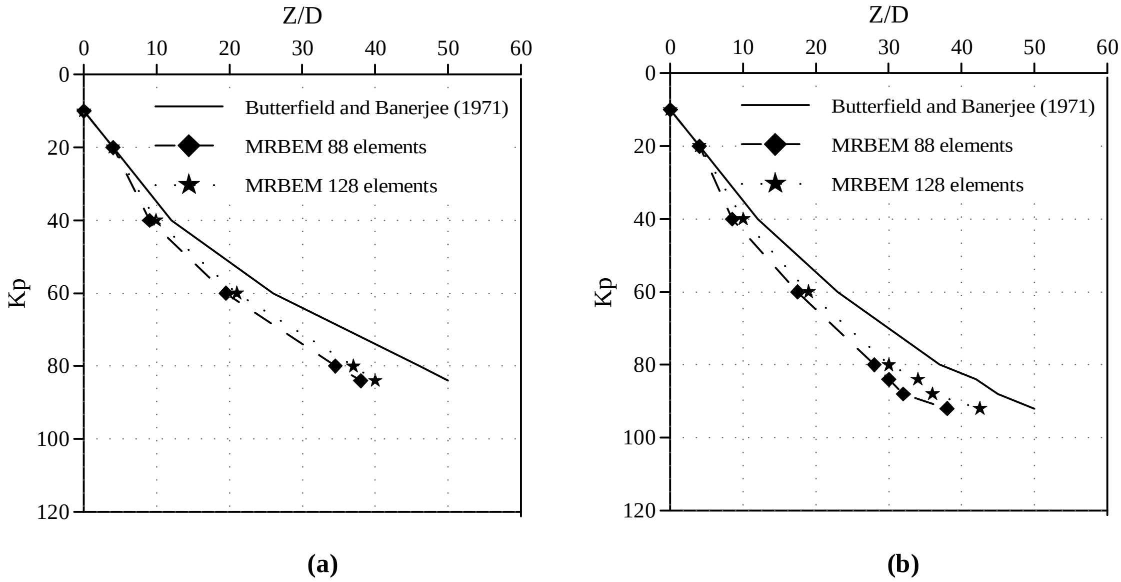


Figure (13): Load-displacement characteristics of uncapped group of four piles with S/D = 5. (a) λ =6000 (b) $\lambda = \infty$.

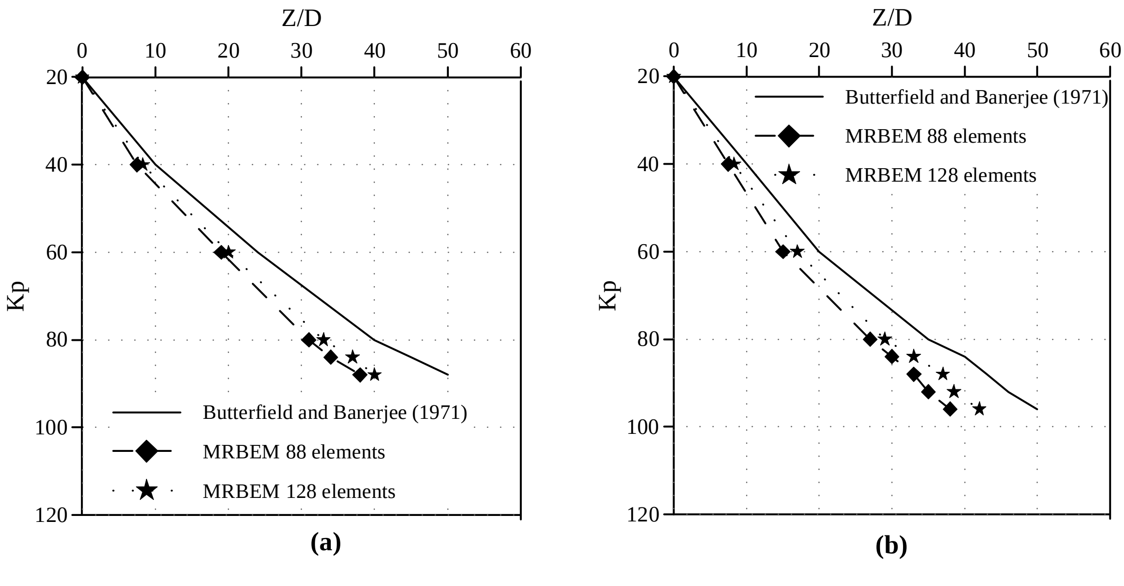


Figure (14): Load displacement characteristics of a capped group of four piles with S/D = 5. (a) λ =6000 (b) $\lambda = \infty$.