Analysis of Induction Cooker Coil

Dr. A.K.M. Al-Shaikli* & Hassan A. Al-Ssadi*

Received on: 26/10/2009
Accepted on: 6/5/2010

Abstract
The use of induction heating in industrial applications is old and wide. In recent years, this technology was entered to domestic use. This research deals with the analysis and applications of induction heating for cooking food, as these techniques have safe and economy, as well as cleanliness and the atmosphere of work comfortable. Knowledge of the electrical resistance of induction heating system is very important to designers working in the high frequency-supplies. This research also analyzes and studies this resistance, by used solid, litz and twisted wire, and offers simplification of equations derivation help anyone interested in this subject. The MATHCAD V.14 program has been employed to obtain the results. The results presented as general curves for three types of wires (solid, litz and twisted wire) so that a designer can use them without lengthy and complex calculations.

Keywords: induction heating; cooker coil; induction cooker; eddy current; skin effect; proximity effect; solid wire; litz wire; twisted wire;

1. Introduction
The induction cookers use induction heating for cooking. The first research about induction cooker was Moreland’s paper [1]. The principle of the work depends on the flow of ac. alternate current in the coil, which generates a time varying magnetic field intersects with cooking vessel metal leads to generate eddy currents and then heating the vessel of cooking. The induction cooker come in a variety of forms, from portable one-hob hot plates to full-sized four-hob ranges. Each hob contains a coil. This means a flameless method of heating. This type of flameless cooking has many advantages over traditional hobs as it provides faster heating, higher efficiency, greater consistency and controllability. Induction cookers are safer to use than other hobs since there are no open flames and the hob itself only gets marginally hot (due to heat conduction from the pan). Also, no
heat is transferred from the hob to the air, keeping the kitchen cooler [2].

These cookers are also easier to clean because the food can not burn if it drops onto the cooking surface as it is not hot. Induction cooker coils are made of many types of wire such as solid, foil, hollow, litz and twisted.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>ac</td>
<td>Conduction area or cross section area (m²)</td>
</tr>
<tr>
<td>d</td>
<td>Distance between load and coil (m)</td>
</tr>
<tr>
<td>f</td>
<td>Frequency of current (Hz)</td>
</tr>
<tr>
<td>J₀ , J₁</td>
<td>Bessel functions</td>
</tr>
<tr>
<td>Jₑ</td>
<td>Current density (A/m²)</td>
</tr>
<tr>
<td>n</td>
<td>Number of turns in coil</td>
</tr>
<tr>
<td>nₛ</td>
<td>Number of strands</td>
</tr>
<tr>
<td>G</td>
<td>Packaging factor</td>
</tr>
<tr>
<td>rₖ</td>
<td>Radius of bundle (m)</td>
</tr>
<tr>
<td>rₑ</td>
<td>Radius of the equivalent solid wire for twisted wire</td>
</tr>
<tr>
<td>rₛ</td>
<td>Radius of strand (m)</td>
</tr>
<tr>
<td>u</td>
<td>Radius of single loop (m)</td>
</tr>
<tr>
<td>u₁</td>
<td>Radius of first Internal loop of multi turns coil (m)</td>
</tr>
<tr>
<td>uᵢ</td>
<td>Radius of final external loop of multi turns coil (m)</td>
</tr>
<tr>
<td>Z</td>
<td>Total impedance (Ω)</td>
</tr>
<tr>
<td>η</td>
<td>Efficiency of coil+load (%)</td>
</tr>
<tr>
<td>ϕₛ</td>
<td>Factor of Skin effect</td>
</tr>
<tr>
<td>ϕₚₑ</td>
<td>Factor of proximity effect</td>
</tr>
</tbody>
</table>

Recently, the studies dropped to study main wires (solid, litz and twisted) which has lower losses, in high frequency operation, compared with other types of wire. The coil is connected to a medium / high frequency inverter. Knowing the inverter load is very important for its design. The inverter resonant load consists of the pan and the induction coil. The coupling between the coil and the pan is modeled as the series connection of a resistor (R_T,L), see fig. (1), and to insulate the load from the coil, ceramic is very suitable to protect cooker coil from heat transfer which generated in the vessel, also see figure (1).

One of the main problems related to the design of the inductors is the calculation of the winding ac resistance. Many efforts, summarized in [2,3], have been made to derive expressions for (R_T).

The objective of this work is to study the variation of (R_T,L) as a function of frequency, radius of strands, number of strands … etc. to be useful for different induction cookers. The main wires used for induction cooker coils are:

1. **Solid wire**: also called solid-core or single-strand wire. At high frequencies, current travels near the surface of the wire because of the skin effect, resulting in increased power loss in the wire. Solid wire commonly used and consider the basic to all other wires or cables.

2. **Litz wire**: is a special type of wire used in high frequency applications. The wire is designed to reduce the skin effect and proximity effect losses in conductors. It consists of many strand wires, individually coated with an insulating film and woven together, see fig. (2).

3. **Twisted wire**: is also a special type of wire, like litz wire. It is clear from figure (3) transaction of strands, taking into account twisting strands only or the strands possess only angular transposition,
twisted wire different from litz wire in this point only.

2. Determination of the Resistance:

Figure (1) shows the equivalent circuit of a planar induction system defined by the values of $R_{T,L}$ which is defined as:

$$R_{T,L} = R_T + R_L$$  \hspace{1cm} (1)

The ac resistance ($R_T$) of solid, litz and twisted wire consist of two components [3], the conduction and induction resistances.

$$R_T = R_c + R_{in}$$  \hspace{1cm} (2)

Where $R_c$ and $R_{in}$: conduction and induction resistance respectively.

In strands wire, the total field over a strand in a particular turn has two components: the field created by the another turns of the winding and the field created by the neighboring strands in the same turn. Therefore $R_{in}$ will be [4]:

$$R_{in} = R_i + R_e$$  \hspace{1cm} (3)

Therefore, the ac resistance of litz wire represent by three components: the resistance due to the skin effect which called conduction resistance $"R_c"$, the resistance due to the proximity effect of external fields which called external induction resistance $"R_e"$ and the resistance due to the proximity effect of individual strands in the bundle on each other which called internal induction resistance $"R_i"$, i.e.:

$$R_T = R_c + R_i + R_e$$  \hspace{1cm} (4)

As the solid wire does not have individual strands. Therefore, the ac resistance of solid wire is:

$$R_T = R_c + R_e$$  \hspace{1cm} (5)

This is applied to ac resistance of twisted wire also; it is because the twisted wire can be considered as equivalent solid wire [3]. The ac resistance of twisted wire will be:

$$R_T = R_c + R_{et}$$  \hspace{1cm} (6)

3. Mathematical Formulation of the Resistances:

3.1. Analysis and Modeling of Litz-Wire Winding Resistance:

a. Conduction Resistance:

The conduction resistance per unit length $R_{c,ul}$, of $(n_s)$ strands litz wire is given by [3]:

$$R_{c,ul} = \frac{k}{2\pi n_s \sigma \delta} * \Phi_c(\alpha r_2)$$  \hspace{1cm} (7)

Where:

$$\Phi_c(\alpha r_2) = \frac{ber(\alpha r_2) - bei(\alpha r_2) - ber'(\alpha r_2) * bei'(\alpha r_2)}{ber^2(\alpha r_2) + bei^2(\alpha r_2)}$$  \hspace{1cm} (8)

$$k = \sqrt{\mu_0 \sigma} = \frac{\sqrt{2}}{\delta}$$  \hspace{1cm} (9)

Where $ber$, $bei$, $ber'$, and $bei'$ are Kelvin functions [5] and $\delta$ the radius of strand, is the skin depth [6].

$$\delta = \sqrt{\frac{2}{\mu_0 \omega \sigma}}$$  \hspace{1cm} (10)

Where
Analysis of Induction Cooker Coil

\( \sigma \): conductivity (mΩ)^{-1}
\( \mu \): permeability of the object
\( \omega \): angular frequency of the current flowing through the object.

\( r_s \): radius of strand.

b. Internal Induction Resistance:

Take a litz wire of \((r_b)\) bundle radius, the resistance due to internal induction per unit length [3]:

\[
R_{L_{\text{int}}} = -\frac{n_s \cdot k \cdot r_s}{2 \pi \cdot r_b^2 \cdot \sigma} \Phi_{\text{in}}(k r_s)
\]

(11)

\[
\Phi_{\text{in}}(k r_s) = \frac{b e r^2(k r_s) + b e t^2(k r_s) - b e r^2(k r_s)}{b e r^2(k r_s) + b e t^2(k r_s)}
\]

(12)

Pointing that \( \Phi_{\text{in}}(k r_s) = \Phi_{\text{c}}(k r_s) \) where \( \Phi_{\text{c}}(k r_s) \) is defined in equation (8). \( h_{l_{\text{in}}} \) and \( h_{a_{\text{in}}} \) are the second order of Kelvin functions [5].

c. External Induction Resistance:

The effect of the magnetic field of each turn (created by the rest of the turns) at other \( l \)-turns, see figure (4), can be calculated by different ways, numerically using a finite element analysis, or analytically.

This field may change by three different reasons: first, the presence of the load; second, the proximity of the neighboring turns; and third, depending on the conduction area of the bundle and the packaging factor of litz wire.

Packaging factor \( G = \frac{n_s \cdot r_s^2}{r_b^2} \) ...

(13)

Let a strand in the \( i \)-turn of the planar inductor as it is shown in Figure (5). The external field applied to each strand will be [7]:

\[
H_{u,i} = H_{u_{0},i} a_r + H_{u_{2},i} a_z
\]

(14)

The external field in each turn is caused by the total current \( I \) in the inductor, and then we can associate these losses with a resistance. Let \( H_{u_{0},i} \) the magnetic field generated over the \( i \)-turn of the winding when a current amplitude of 1 A is circulating in the inductor. And can rewrite the average of the squared field as:

\[
(H_{u_{0},i}^2) = (3r_e^2) \times I^2
\]

(15)

The external induction resistance per unit length [3] is:

\[
R_{u_{\text{ext}}} = -\frac{n_u}{2 \pi \cdot r_s^2 \cdot \sigma} k \Phi_{\text{c}}(k r_s) \Phi_{\text{in}}(k r_u)
\]

(16)

Total Resistance Calculation:

According to (4), (7), (11), and (16), and considering the length of the whole winding, we can calculate the total resistance as follows:

\[
R_c = \frac{k \Phi_{\text{e}(k r_s)}}{2 \pi \cdot r_s^2 \cdot \sigma} \sum_{i=1}^{n} 2 \pi \cdot n_i
\]

(17)

\[
R_i = -\frac{k \cdot \frac{r_s^2}{2 \pi \cdot r_b^2 \cdot \sigma}}{\Phi_{\text{in}}(k r_s)} \sum_{i=1}^{n} 2 \pi \cdot n_i
\]

(18)
3.2 Analysis and Modeling of Twisted-Wire Winding Resistance:

Let \( r_b \) the radius of twisted wire containing \( n_s \) strand, each one of them having a radius \( r_s \).

The radius of the equivalent solid wire \( r_e \) (see figure (6)) is defined by:

\[
r_e = \sqrt{n_s r_s^2} \tag{21}
\]

Where \( G \) is the packing factor of wire given by eq. (13).

For bundle, the conduction resistance per unit length is:

\[
R_{ct,ul} = -\frac{2\pi k r_e^2 \Phi_{in}}{G} \tag{26}
\]

The factor \( \Phi_{e,(r_e)} \) takes into account the skin effect at bundle level and the factor \( \Phi_{e,(r_e)} \) accounts for skin effect at strands level.

b. External Induction Resistance of Twisted Wire:

\[
R_{ce} = \frac{k}{2\pi r_e G} \Phi_{e,(r_e)} \tag{22}
\]

For the complete bundle of twisted wire, the currents were not equally distributed in every strand, as it occurs in true litz wire. Therefore, at equivalent solid wire, the current density at the coordinate \( r \) is given by [4]:

\[
I_0(r) = \frac{Q \cdot I \cdot I_s(q_r)}{2\pi \cdot I_c(q_r) \cdot r_e} \tag{23}
\]

Where \( I \) is the amplitude of the driven current and \( I_s \) and \( I_c \) are Bessel functions. \( Q \) is a parameter related with the skin depth which defined as:

\[
Q = k \cdot \sqrt{\frac{3\pi}{a}} \tag{24}
\]

\[
k = |Q| = \sqrt{\frac{|k\sigma\omega|}{\|}} \tag{25}
\]

Twisted wire can be considered as an equivalent solid wire. Therefore, the internal induction resistance is negligible, and the external induction resistance only can be taken into account as an induction losses. The induction losses represents the power dissipation in a wire under a varying magnetic field, \( H_{o,i} \), which created by the rest of \( i_{turn} \). The induction losses are:

\[
P_{is,ul} = -\frac{2\pi k r_e H_{o,i}^2}{G} \Phi_{in,(q_r)} \tag{27}
\]
The $H_{o_i}$ is calculated analytically and the current density in equivalent solid wire is [3]:

$$J_{o(i,\theta)} = \frac{2kH_{o_i}n_1(qr)}{J_0(qr)} \sin(\theta)$$  \hspace{2cm} (28)

The total induced losses will be the sum of the losses in each differential of area:

$$P_{et} = \frac{G \pi^2 k r_b (H_{o_i}^2)}{4\sqrt{2} r_b} \Phi_{in(qr)}$$

At eq. (29), the average of magnetic field applied due to the total current in the inductor. Let magnetic field generated over the $i$-turn of the winding, when a current amplitude of 1A is circulating in the inductor as in equation (18).

The external induction resistance of whole inductor is:

$$R_{et} = R_{et} + R_{et}$$  \hspace{2cm} (30)

3.3. Analysis and Modeling of load resistance:

Assuming the electric field $E_n$ is linearity with $n$ turns [7], the total voltage amplitude induced at the winding position $z = -d$ is the sum of the voltage induced in each turn, and it is calculated as follows:

$$v = \int E_d dI = -\int_{-\pi}^{\pi} R_0 (r = u_I, z = -d) u_I dI$$  \hspace{2cm} (34)

Therefore, the equivalent impedance is:

$$Z = \frac{v}{I}$$  \hspace{2cm} (35)

The resistance of load will be the real part of impedance:

$$R_L = \text{real}(Z)$$  \hspace{2cm} (36)

4. Computer Results and discussions:

MATHCAD V.14 was applied, which it possess the ability to solve Kelvin function and integral of Bessel function, to perform computer program. This program is prepared to calculate losses and efficiency of induction cooker and to compare among wire types too.

To check the validity of the proposed equations [3], The results are compared with published work [8] which it alone deals with three type of wire in this subject and other published work deals with one type of
wires separately. Very good correlation was obtained as shown in Fig. (7).

Coils of different number of turns have been considered in table (1). The specifications of wires are shown in table (2) these values taking from [9,10]. In this table, we are taking into account the same conduction area "ac = 3 mm$^2$" or across section of copper base for the comparison between three types of wire (solid, litz and twisted wire), and the load properties are shown in table (3). Figs. (8 - 10) show the results of a 25 turns unloaded practical coil. Fig. (8) shows conduction resistance against frequency according to eq. (17) and eq. (31), fig. (9) shows internal induction resistance against frequency according to eq. (18), which only occur in litz wire. This is due to the angular and transpose of strands in litz wire, which making distribution of magnetic field in bundle level is equally.

Fig. (10) shows external induction resistance against frequency according to eq. (19) and eq. (32). At coil level, the external induction resistance $R_e$ increase when number of turn increase as expected.

The total resistance $R_T$ of litz wire ($R_c + R_i + R_e$), solid wire ($R_c + R_e$) and twisted wire ($R_{ct} + R_e$) shown in fig. (11).

The distance $d$ between load (vessel) and the coil is limited by thickness of ceramic and radius of the wire. To avoid low efficiency, the practical value of $d$ does not exceed 10 mm. varying the thickness of ceramic to this value does not affect the total resistance, see figure (12). Thickness of ceramic, in the calculation, is taken to be 5 mm, as a middle value.

The effects of the load are shown in figs. (13 - 15) for magnetic load, fig. (13) shows external induction resistance. At high frequency, the external induction resistance with load is less than the external induction resistance without load, this is clearly shown when compared figure (10) with figure (13).

The efficiency of induction cooker system shown in figure (16), that it when cooker coil loaded by magnetic load. The efficiency is calculated from:

$$\eta (\%) = \frac{R_L \times 100}{R_T}$$

It is known that the different contributions of each kind of load being higher in the case of magnetic load one as expected see figure (17). This attribute to properties of load (Relative magnetic permeability $\mu_{rl}$ and resistivity $\rho_l$). Due to the difference of total resistance $R_T$ of coil, loaded by magnetic or nonmagnetic load, is very small (proximity similar) the total resistance $R_T$ in figure (14) can be considered as total resistance $R_T$ of coil loaded by nonmagnetic load. Also the total resistance with load resistance $R_{T,L}$ is shown in figure (18). In addition, the efficiency of cooker coil loaded by nonmagnetic load shown in figure (19) which clearly shown the difference among wire types.

VI. Conclusions:

First, an extensive review of induction cooker was covered. The characteristics of the induction cookers, their benefits in compared with other types of cookers and types of wire, use in induction cooker coil, were presented too.

The loss mechanisms due to the skin and proximity effects on a single and multi strands were derived. This by used an analytical model to
calculate resistance in different wire planar windings for induction heating cooker. The resistance has been separated into two parts: conduction resistance and induction resistance (with internal and external components for litz wire). In order to compute the induction resistance, a calculation of the magnetic field in the inductor is required. In addition to the comparison of result with other work, discussion the obtained results and the used of resistance curves are presented.

In general, we can consider the twisted wire the best when compared with other wires for whole frequencies, this appear in efficiency cases in figures (16) and (19). Moreover, litz wire has all right efficiency but in limited frequencies, as appear in the same figures (16) and (19).

MATHCAD v.14 proved to be suitable, efficient and accurate in solving and calculating the complex equations of resistance.

References:
Table (1) shows the dimension of turns.

<table>
<thead>
<tr>
<th>25-turns (mm)</th>
<th>Radius of internal turn ( u_i )</th>
<th>Radius of external turn ( u_e )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25</td>
<td>109</td>
</tr>
</tbody>
</table>

Table (2) show the number and radius of strands, bundle, equivalent also include packing factor of each wire.

<table>
<thead>
<tr>
<th>Load</th>
<th>( \rho_L ) (( m\Omega ))</th>
<th>( \mu_{le} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetic material</td>
<td>( 1.25 \times 10^{-7} )</td>
<td>150</td>
</tr>
<tr>
<td>Nonmagnetic material</td>
<td>( 0.33 \times 10^{-7} )</td>
<td>1</td>
</tr>
</tbody>
</table>

Table (3) load properties

<table>
<thead>
<tr>
<th>Wire type</th>
<th>Packing factor</th>
<th>Number of strand</th>
<th>Conduction area ( ac ) (mm²)</th>
<th>Radius of strand ( r_s ) (mm)</th>
<th>Radius of bundle ( r_b ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>solid</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>0.980</td>
<td>0.980</td>
</tr>
<tr>
<td>litz</td>
<td>0.6</td>
<td>20</td>
<td>3</td>
<td>0.220</td>
<td>1.270</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50</td>
<td>3</td>
<td>0.140</td>
<td>1.270</td>
</tr>
<tr>
<td>twisted</td>
<td>0.25</td>
<td>1000</td>
<td>3</td>
<td>0.030</td>
<td>1.897</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>2000</td>
<td>3</td>
<td>0.022</td>
<td>2.200</td>
</tr>
</tbody>
</table>
Figure (1) Elements of induction cooker. The flux and eddy current also shown.

Figure (2) litz wire structure.

Figure (3) twisted wire structure.
Figure (4) Planar inductor of $n$ turns loaded by a medium characterized by its Conductivity $\sigma_i$ and relative permeability $\mu_r$. The coordinate system is also shown.

Figure (5) External magnetic field $H_{or,i}, H_{oz,i}$ in a strand situated in the i-turn.
Figure (6) Equivalent solid wire of twisted wires as a function of its packing factor. The external magnetic field and distribution of the induced currents produced by the equivalent solid wires is also shown.

Figure (7) Comparison between MATHCAD results and results of reference [8].
Figure (8) variation of conduction resistance $R_c$ of 25-turn with frequency, $ac=3\text{mm}^2$.

Figure (9) variation of internal induction resistance $R_i$ of 25-turn with frequency, $ac=3\text{mm}^2$.

Figure (10) variation of external induction resistance $R_e$ of 25-turn with frequency, $ac=3\text{mm}^2$. 

PDF created with pdfFactory Pro trial version www.pdffactory.com
Figure (11) variation of total resistance $R_T$ of 25-turn with frequency without load, $ac=3\text{mm}^2$.

Figure (12) the resistance against distance between load and coil.

Figure (13) variation of external induction resistance $R_e$ of 25-turn with frequency, $ac=3\text{mm}^2$ loaded coil by magnetic load.
Figure (14) variation of conduction resistance $R_c$ of 25-turn with frequency, when the coil loaded by magnetic load $ac=3\text{mm}^2$.

Figure (15) variation of (total+load) resistance $R_{T,L}$ of 25-turn with frequency, when the coil loaded by magnetic load, $ac=3\text{mm}^2$. 
Figure (16) efficiency of induction cooker system when the coil loaded by magnetic load, 25-turn, ac=3mm².

Figure (17) Comparison of external induction resistance $R_e$ between magnetic (points) and non-magnetic (red line) load for coil has 25-turns by used litz wire $n_s=20$, $r_s=0.22$mm, ac=3mm².

Figure (18) variation of (total+load) resistance $R_{T,L}$ of 25-turn with frequency, when the coil loaded by non-magnetic load, ac=3mm².
Figure (19) efficiency of induction cooker system when the coil loaded by non-magnetic load, 25-turn, ac=3mm².