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Behavior of High-Rise Steel Building With The Inclusion of Warping

Dr. Hisham M. AL-Hassani*, Dr. Haitham H. Muteb
& Dr. Najla'a H. Shareef**

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Abstract

The mathematical procedure in this study covers the calculation of sectorial properties of the equivalent cross-sectional storey for high-rise building frames. The formulation is efficiently used to obtain the free vibration analysis of high-rise buildings which are constructed from several columns, beams, shear walls and bracing etc. the analysis is based on transformation the complex system to a simple tall column to represent a cantilevered tall building structure. This is partitioned to nodes one of which indicates a storey with equivalent cross-sectional properties for all storeys' elements after calculation of these properties with respect to the shear center of high-rise building. A thin walled bar finite element with seven degrees of freedom at each node is assumed. A new formulation of the stiffness and consistent mass matrices of the thin- walled element is presented in this study. The effect of cross sectional warping and its properties on the flexural, torsional and axial properties was investigated, using discrete element approach in idealizing the structure in high rise building. For the purpose of the present study, it is assumed that the cross-sectional types under condition are only of thin-walled sections. Algorithm method was developed which covers the calculation of sectorial properties of the cross section for floor plan in high-rise building, to study the share of columns for lateral shear force resistance, and investigate the behavior of different types of high-rise building with inclusion of warping restraint. The effect of natural frequency with height of tall buildings, and the mode shape for different cross-sectional plans of high-rise building was studied. To check the efficiency and accuracy, the mathematical procedure is demonstrated for static and dynamic examples by comparing the results with those obtained by using software ANSYS program. A difference of 15% is shown. An eigen value problem is analyzed and numerical examples are discussed.

Keywords: High-rise building, finite element method, thin-walled sections, torsion, warping restraint, stiffness and mass matrices, natural frequencies.

* Building and Construction Engineering Department, University of Technology/ Baghdad
** Engineering College, University of Babylon/Babylon

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2412-0758/University of Technology-Iraq, Baghdad, Iraq

سلوك البنايةِ الفولاذيةِ العاليةِ مَع إدراج تَشويه

الخلاصة

في هذه الدراسة تم الحصول على إجراء رياضي لاحتساب الخواص القطاعية (sectorial properties) لخواص المقطع المكافئ لهياكل طوابق الأبنية العالية. الصياغة استعملت بشكل كفوء للحصول على الاهتزاز الحر (free vibration) وتحويل الأبنية العالية المكونة من العديد من الأعمدة والأعتاب و جدران قص ونظم إسناد إلى عمود طويل مؤلف من مجموعة من العقد وكل عقدة تمثل طابق له مواصفات مكافئة لجميع خواص أعضاء الطابق بعدما تم حساب هذه الخواص بالنسبة إلى مركز القص للبناية. تم فرض عنصر الجدار الرقيق (thin-wall element) للعنصر المحدد مع اعتبار درجة حرية سابعة في كل عقدة. ولأغراض دراسة تشوه اللي الطولي (Warping) وتأثيره على الخواص الانثنائية ، اللي و المحورية تم اعتماد نظام التمثيل المنفصل المميز (Direct element approach) للأبنية العالية . ولأغراض الدراسة الحالية، تم فرض المقاطع تحت شرط رقيقة الجدران. تم تطوير خوارزمية للحصول على خواص المقاطع للطابق الواحد في الأبنية العالية و كذلك دراسة حصة العمود الواحد من قوى القص الجانبية وتحليل سلوكية أنواع مختلفة من الأبنية العالية مع التقييد الضمني لتشوه اللي الطولي وأيضا دراسة أشكال أنماط الاهتزاز للأبنية العالية ذات المقاطع المختلفة. و لتدقيق كفاءة الإجراء الرياضي وإظهار صحته في التحليل الساكن والديناميكي قورنت النتائج مع برنامج (ANSYS) حيث أظهرت اختلافات لا تتجاوز 15%. تم عمل تحليلا للقيم المميزة وتم مناقشة أمثلة عددية.

1. Introduction

High-rise buildings have become one of the impressive reflections of today's civilization. The outlook of cities all over the world has been changing with these tall and slender structures.

High-rise structural problems, like most other practical engineering problems, involve complex material property, loading and boundary conditions. The engineers introduce assumptions and idealizations deemed necessary to make the problem mathematically manageable, but still capable of providing sufficiently accurate solutions and satisfactory results from the point of view of safety and economy.

Analysis of the effect of warping restraint on the behavior of high-rise building has been a major topic of structural engineering research for many years. Most of the early work on this problem was concentrated on the response of isolated members in which warping was assumed to be either completely free or fully prevented at supports [2, 3]. When the warping effect has been taken into account in space frame analysis, the joints have again been assumed to be either free to warp or completely prevented from warping [4, 9]. The global stiffness matrix of the core system on the basis of an assumed solution for the governing differential equations of three-dimensional non-planar core element with seven degrees of freedom at each node to idealize the non-planar wall unit (used in static analysis of core systems), in which, the seventh degree of freedom accounts for cross-sectional warping [10,11]. The same finite element idealization introduced by Taranath as a method for verifying free vibration response of cores obtained using the finite strip idealization [3]. A generalized coordinate method for the analysis of core system, the overall stiffness equation is obtained in generalized coordinates. A transformation was performed at the member, and computations involving large matrices were reduced [14]. The natural frequencies and corresponding mode shapes are evaluated for thin-walled elements of constant and open cross-section using the finite element method. Following the classical thin-walled bar theory, warping and Saint-Venant torsional rigidities were accounted [12]. A concept to increase the lateral stiffness of wall-frame tall building structures by stiffening a storey of the frame system either at the top or at an intermediate optimized level[1]. The warping deformation is considered as a seventh degree of freedom in space frame elements for static and dynamic analysis. The main conclusions drawn from his study were: a) warping deformation has a significant influence on the coupled lateral-torsional and the uncoupled torsional response. b) Shear deformations affects both lateral and torsional core response, especially, square core system. c) The discrete element idealization procedure with negligible warping can never be a reliable approach in estimating shear cores and tubular structures response, especially when they are subjected to loading that causes torsional deformations [1]. An efficient and practical random vibration approach for seismic response analysis of a super tall building with a large number of degrees of freedom, the seismic responses of tall buildings subjected to random earthquake excitations were evaluated by using the Pseudo-excitation method[8].

The aim of this study is to investigate the behavior of different types of high-rise buildings with inclusion of warping restrained. The mathematical procedure in this study covers the calculation of sectorial properties of the equivalent cross-sectional storey for high-rise building frames.

A new formulation presented to obtain the free vibration analysis and transformation of high-rise buildings which are constructed from several columns, beams, shear walls and bracing etc. to a simple tall column to represent a cantilevered tall building structure. This is partitioned to nodes one of which indicates a storey with equivalent cross-sectional properties for all storeys' elements after calculation of these properties with respect to the shear center of high-rise building.

2. Analysis of High-Rise Building

In general, the normal stresses vary from point to point along the member, hence, they are accompanied by a non uniform shearing stress distribution which, in turn, alters the twist of the section, as a result, the twisting moment developed on each section is no longer proportional to the rate of twist and final shearing stresses cannot be obtained by supper-imposing those produced by unrestrained torsion and bending. To complicate matters, such "warping" stresses are also developed by subjecting thin walled member to eccentric axial load; consider, for example, the high-rise building in Figure (1), which is composed of four stringers of equal area connected to thin skin. For simplicity, it is assumed that the stringers develop on normal stresses and the skins develop only shearing stresses [7].

According to the Bernoulli-Euler theory of bending, the eccentric load (P) produces the axial force (Nx) and the bending moments (Mz) and (My) shown in Figure (3-1). No additional stress results are given by the elementary theory because it is based on the assumption that planes remain plane. The super-position of cases (b), (c) and (d) in Figure (1), does not lead to a force (P) in bottom stringer; the additional force system shown Figure (1, e) is also necessary.

In thin walled open sections, stresses produced by restrained warping diminish very slowly from their points of application and may constitute the primary stress system developed in the structure. High-rise buildings are modeled by using thin-walled open section and analyzed as a space structure. Due to he fact that effect of warping is important in the analysis of such structures, seven degrees of freedom are considered at a joint, as shown in Figure(2). Six of these are the familiar translations and rotations about three orthogonal axes and the seventh one is the warping displacement [11].

3. System Analysis

An C-storey tall building shown in Figure (3, a) is considered. The origin of the principal coordinate system (x,y,z) is arbitrary chosen of the first floor, the positive direction of z-axis is upward.

The structure is comprised of vertical members (columns and shear walls) and bracing. The column and shear wall at which the vertical axis crosses the floor are named an element.

An element is denoted by (i) and the shear walls are divided into a number of small elements, each containing one layer of rebar as shown in Figure (3,b) and the length of each element is equal to the spacing between the bars. Although in practice there are usually two rebar placed near the edge of walls of core, in this study these two bars are theoretically represented by a single bar which is placed in the middle of each element.

This assumption is made by considering the fact that the thickness of wall of core structure is quite small compared with its other dimensions

3.1 Computations of sectional properties

The sectional and sectorial properties of floor plan in high-rise building are calculated by initially dividing the plan into a number of columns and shear wall. At the same time the shear walls are divided into a number of small elements , which contains a rebar at its center as shown in Figure.(3-3b). The procedure is based on Vlasov's theorems [13] and similar to that used by Miranda [6]. The steps of procedure are given as follows:

1. Divided the shear wall cross section into a number of small elements in which each element is defined by two node numbers. k represents a node number.
2. Choose an arbitrary coordinate system (Xr and Yr) (principal axis) and obtain the coordinates of nodes according to this system.
3. Calculate the area for each small equivalent element of the shear wall cross section :

$$A_i^{sh} = A_{ci} + A_{sti} \qquad (1)$$

where

A_{ci} is the concrete element cross section area ;

A_{sti} is the steel element area .

4. Calculate the total area for storey cross section (plan):

$$A_t = \sum_{i=1}^{n-1} A_i^{sh} + \sum_{j=1}^{m} A_j^c \qquad (2)$$

where

A_i^{sh} is the cross section area of shear wall number i ;

A_j^{c} is the cross section area of column j.

The concrete area must be transformed to equivalent area of steel as below;

$$A_{ci} = (1-0.002)\frac{t_i s_i}{n}$$

$$A_{ci} = \frac{0.998 t_i}{n}\sqrt{(x_{r,(k+1)} - x_{r,k})^2 + (y \quad \textbf{(3)}$$

Also the area of steel which is computed by:

$$A_{sti} = 0.002 t_i s_i \qquad (4)$$

where:
t_i= thickness of each small element.
s_i= length of each small element.
x_r and y_r=principal coordinate of nodes to arbitrary coordinate system Xr and Yr .
n=$\frac{Es}{Ec}$ is the modular ratio in which (Es) and (Ec) are the modulus of elasticity of concrete and steel; respectively.

5. Transform the arbitrary coordinate system Xr, Yr into centroidal coordinate system X, Y

$$S_x^{sh} = \sum_{i=1}^{n-1} A_i^{sh} * \frac{(y_{r,(k+1)} + y_{r,k})}{2} \qquad (5)$$

$$S_x^{c} = \sum_{j=1}^{m} A_j^{c} * y_r \qquad (6)$$

$$S_x = S_x^{sh} + S_x^{c} \qquad (7)$$

$$S_y^{sh} = \sum_{i=1}^{n-1} A_i^{sh} * \frac{(x_{r,(k+1)} + x_{r,k})}{2} \qquad (8)$$

$$S_y^{c} = \sum_{j=1}^{m} A_j^{c} * x_r \qquad (9)$$

$$S_y = S_y^{sh} + S_y^{c} \qquad (10)$$

$$x' = \frac{S_y}{A_t}; y' = \frac{S_x}{A_t} \qquad (11)$$

where:
S_x and S_y = sectional moment about arbitrary coordinate system X_r, Y_r.
x' and y'=coordinate of centroidal are shown in Fig. (3b)
n=number of steel columns.
m=number of small elements in shear wall.

6. Calculate the moment of inertia of the section with respect to centroidal coordinate system.

$$I_{x'_i} = I_{x_i} + A_i^{sh}(dy_i)^2 \qquad (12)$$

$$I_{y'_i} = I_{y_i} + A_i^{sh}(dx_i)^2 \qquad (13)$$

$$I_{x'y'_i} = I_{xy_i} + A_i^{sh} dx_i dy_i \qquad (14)$$

Doing that for both steel columns and shear walls and the overall moment of inertia are: (15)

$$I_{x'} = \sum_{i=1}^{n-1} I_{x'_j} + \sum_{j=1}^{m} I_{x'_j} \qquad (16)$$

$$I_{y'} = \sum_{i=1}^{n-1} I_{y'_j} + \sum_{j=1}^{m} I_{y'_j} \qquad (17)$$

$$I_{x'y'} = \sum_{i=1}^{n-1} I_{x'y'_j} + \sum_{j=1}^{m} I_{x'y'_j}$$

The same as for the steel column section;

7. Locate the pole at the center of gravity *C* of the section. for simplicity select C1 as the initial radius of the sectorial coordinate.

The total sectorial coordinates $w_{c1,(k+1)}^{c}$ for concrete elements are:

$$w_{c1,(k+1)}^{c} = w_{c1,k}^{c} + w_{c1,k+1}^{c} \qquad (18)$$

In which:

$$w_{c1,k+1}^{c} = (|dx_i * \Delta y_i| + |dy_i * \Delta x_i|) \qquad (19)$$

Also, the total sectorial coordinates $w^{s}_{c1,(k+1)}$ for steel elements are:

$$w^{s}_{c1,(k+1)} = w^{s}_{c1,k} + w^{s}_{c1,k+1} \qquad (20)$$

In which:

$$w^{s}_{c1,k+1} = \left(\left| dx_i * \Delta y_{si} \right| + \left| dy_i * \Delta x_{si} \right| \right) \qquad (21)$$

where

$w^{c}_{c1,k}, w^{s}_{c1,k}$ =sectorial coordinates of point (k) from arbitrarily chosen pole C and an initial radius of C1 for equivalent concrete and steel elements, respectively. (Note $w^{c}_{c1,k}, w^{s}_{c1,k}$ are equal to zero at point 1). Are calculating from:

$$w^{c}_{c1,k} = \left(\left| \frac{y_{r,k+1} + y_{r,k}}{2} * \Delta x_i \right| + \left| \frac{x_{r,k+1} -}{2} \right. \right. \qquad (22)$$

$$\Delta x_i = x_{k+1} - x_k ; \Delta y_i = y_{k+1} - y_k \qquad (23)$$

The area of reinforcement bars is converted into square areas so that sectorial coordinate of each bar can be calculated easily. The sides of this equivalent square are taken as:

$$\Delta x_{si} = \Delta y_{si} = \sqrt{A_{sti}} \qquad (24)$$

To find the sectorial area of steel columns by dealing with discrete elements idealized, where each element is bounded by two nodes taken at adjacent floor levels. That is to say, each column behaves alone with respect to centroid.

$$w_j = \left(\sqrt{(dx_j)^2 + (dy_j)^2} \right) * d_j \qquad (25)$$

where:

dj=effective depth of column's cross-section.

8. Determine the location of the shear centre, or torsion centre.

it is the point in the plane of the cross-section about which twisting take place .The shear centre location is required for calculating the warping tensional constant. It also requires determining the stabilizing or disabling effect of gravity loading applied below or above the shear center. For this, first calculate the sectorial statical moments of the section.

$$Sw^{c}_{i} = w^{c}_{ci,i} * A_{ci} \qquad (26)$$

$$Sw^{s}_{i} = w^{s}_{ci,i} * A_{sti} \qquad (27)$$

$$Swx_i = \left(Sw^{c}_{i} + Sw^{s}_{i} \right) * dy_i \qquad (28)$$

$$Swy_i = \left(Sw^{c}_{i} + Sw^{s}_{i} \right) * dx_i \qquad (29)$$

where:

$$w^{c}_{c1,i} = 0.5\left(w^{c}_{c1,k} + w^{c}_{c1,(k+1)} \right) \qquad (30)$$

$$w^{s}_{c1,i} = 0.5\left(w^{s}_{c1,k} + w^{s}_{c1,(k+1)} \right) \qquad (31)$$

and

$$w_i = w^{c}_{c1,i} + w^{s}_{c1,i} \qquad (32)$$

For column element, the sectorial statical moment is obtained by:

$$Swx_j = w_j * dy_j * A^{c}_{j} \qquad (33)$$

$$Swy_j = w_j * dx_j * A^{c}_{j} \qquad (34)$$

Then, calculate overall plan section:

$$Swx = \sum_{i=1}^{n-1} Swx_i + \sum_{j=1}^{m} Swx_j \qquad (35)$$

$$Swy = \sum_{i=1}^{n-1} Swy_i + \sum_{j=1}^{m} Swy_j \qquad (36)$$

The location of the principle pole is then obtained from:

$$ax = -\left(\frac{I_{y'} Swx - I_{x'y'} Swy}{I_{x'} I_{y'} - I_{x'y'}^{\;2}} \right) \qquad (37)$$

$$ay = \left(\frac{I_{x'} Swy - I_{x'y'} Swx}{I_{x'} I_{y'} - I_{x'y'}{}^2} \right) \quad (38)$$

The principal pole is the shear center of the section and its coordinates (ax) and (ay) are shown in Figure. (3b).

9. **Calculate the principal sectorial coordinates and principal sectorial moment of inertia.**

This is achieved by finding non-principal sectorial coordinates of the section using A1 as the initial radius. Note that $w^c_{A1,k}$ and $w^s_{A1,k}$ are equal to zero at point (1).

$$w^c_{A1,(k+1)} = w^c_{A1,k} - w^c_{c_1,(k+1)} - \left(|ax| * \Delta y_i + |ay| * \Delta x_i\right) \quad (39)$$

$$w^s_{A1,(k+1)} = w^s_{A1,k} - w^s_{c_1,(k+1)} - \left(|ax| * \Delta y_{si} + |ay| * \Delta x_{si}\right) \quad (40)$$

where:

$$w^c_{A1,k} = |dx'_i - ax|(\Delta y_i) + |dy'_i - ay \quad 41$$

$$w^s_{A1,k} = |dx'_i - ax|(\Delta y_{si}) + |dy'_i - a \quad 42)$$

and

$$\left. \begin{aligned} w^c_{A1,i} &= 0.5\left(w^c_{A1,k} + w^c_{A1,(k+1)}\right) \\ w^s_{A1,i} &= 0.5\left(w^s_{A1,k} + w^s_{A1,(k+1)}\right) \end{aligned} \right\} \quad (43)$$

The same as for the steel column;

$$w'_j = \left(\sqrt{(dax_j)^2 + (day_j)^2} \right) * d \quad 44)$$

w' =sectorial area for each column and small element in shear wall with respect to shear centre.

$$Sax_j = w'_j * day_j \quad (45)$$
$$Say_j = w'_j * dax_j$$

For determining the sectorial statical moments S^c_w & S^s_w for equivalent concrete and steel bar in shear wall:

$$S^c_w = \sum_{i=1}^{n} w^c_{A1,i} * A_{ci} ; S^s_w = \sum_{i=1}^{n} w^s_{A1} \quad 46)$$

The total sectorial statical moments for each small element in shear wall are equal to:

$$S^{sh}_w = S^c_w + S^s_w \quad (47)$$

The amount of coordinate transformation required to render ω_{A1} into the principal ω_{AO} is calculated from:

$$w_{AO} = \frac{S^{sh}_w}{A^{sh}_t} \quad (48)$$

The principal sectorial coordinate for each point k and center point of each element is determined from

$$w^c_{AO,k} = w^c_{A1,k} - w_{AO} ; w^s_{AO,k} = w^s_{A1,k} - w_{AO} \quad (49)$$

$$w^c_{AO,(k+1)} = w^c_{A1,(k+1)} - w_{AO} ; w^s_{AO,(k+1)} = w^s_{A1,(k+1)} - w_{AO} \quad (50)$$

$$w^c_i = \frac{w^c_{AO,k} + w^c_{AO,(k+1)}}{2} ; w^s_i = \frac{w^s_{AO,k} + w^s_{AO,}}{2} \quad (51)$$

(52)

$$w'^{sh}_i = w^c_i + w^s_i \quad (53)$$

$$S^{sh}_{ax} = \sum_{i=1}^{n-1} w'^{sh}_i * d_{ay} \quad (54)$$

$$S^{sh}_{ay} = \sum_{i=1}^{n-1} w'^{sh}_i * d_{ax}$$

where:

S^{sh}_{ax} and S^{sh}_{ay} = statical moment of shear wall about shear center.

Now w can get the shear correction factor in both directions x and y for shear wall:

(39)

(40)

$$C_x^2 = \frac{\left(\sum_{i=1}^{n-1} \mathrm{I}_{ax,i}\right)^2 * b^2}{\left(S_{ax}^{sh}\right)^2 * \left(\mathrm{A}_t^{sh}\right)^2} \quad (55)$$

$$C_y^2 = \frac{\left(\sum_{i=1}^{n-1} \mathrm{I}_{ay,i}\right)^2 * b^2}{\left(S_{ay}^{sh}\right)^2 * \left(\mathrm{A}_t^{sh}\right)^2} \quad (56)$$

where:

b= width of shear wall.

10. **Calculate the warping properties**

Finally warping properties by calculating the principal sectorial moment of inertia is obtained from:

$$\mathrm{I}_w^{sh} = \sum_{i=1}^{n} \left(w_i^c\right)^2 * \mathrm{A}_{ci} + \left(w_i^s\right)^2 * \mathrm{A} \quad 57$$

$$\mathrm{I}_w^{col} = \sum_{j=1}^{m} \left(w_j'\right)^2 * \mathrm{A}_j^c \quad 58)$$

The total warping constant for floor plan cross-section is equal to:

$$\mathrm{I}_w = \mathrm{I}_w^{sh} + \mathrm{I}_w^{col} \quad (59)$$

For calculating the product moment in inertia:

$$\mathrm{I}_{wx} = \sum_{i=1}^{n-1} d_{ay,i}\left(w_i^c * \mathrm{A}_{ci} + w_i^s * \mathrm{A}_{sti}\right) + \sum_{j=1}^{m} w_j' * \mathrm{A}_j * d_{ay,j} \quad 60$$

$$\mathrm{I}_{wy} = \sum_{i=1}^{n-1} d_{ax,i}\left(w_i^c * \mathrm{A}_{ci} + w_i^s * \mathrm{A}_{sti}\right) + \sum_{j=1}^{m} w_j' * \mathrm{A}_j * d_{ax,j} \quad 61$$

$$S_{w'}^T = \sum_{i=1}^{n-1} \left(w_i^c * \mathrm{A}_{ci} + w_i^s * \mathrm{A}_{sti}\right) + \sum_{j=1}^{m} w_j' * \mathrm{A}_j \quad 62$$

where:

$S_{w'}^T$ = product moment of area.

3.2 Thin-Walled Finite Element Members

When a thin-walled member having one or more cross section constrained against warping is subjected to a general system of external loads, a complex distribution of longitudinal stresses is developed that cannot be evaluated using the elementary theories. The assumption that plane sections remain plane during deformation is no longer valid, and applications of Saint Venant's may lead to serious errors [7].

The most popular approach used by the greater part of researchers is employing of principle-generalized coordinates; the mass matrix will have a simple form. When such coordinates are assumed, some quantities are referred to the shear centre and other to the centriod[12].

Linear theory of thin-walled bars has been well known since the early works of Vlasov, his set of the four ordinary differential equations of the fourth order with constant coefficients describes the behavior of thin-walled member to distributed torque m(z)[13]:

Axial deformation along the longitudinal axis (z) of the member:

$$\mathrm{A}w_c^{ii} + S_y u_c^{iii} - S_x v_c^{iii} - S_w \Theta^{iii} = \quad 63$$

Bending about (y) and (x) axes, respectively:

$$-S_y w_c^{iii} + \mathrm{I}_y u_c^{iv} - \mathrm{I}_{xy} v_c^{iv} - \mathrm{I}_{wx}\Theta^{i} \quad 64$$

$$-S_x w_c^{iii} + \mathrm{I}_{xy} u_c^{iv} - \mathrm{I}_x v_c^{iv} - \mathrm{I}_{wx}\Theta^{iv} \quad 65$$

Torsional behavior of the member:

$$-S_w w_c^{iii} + I_{wx} u_c^{iv} - I_{wy} v_c^{iv} - C_w \Theta^{iv} - \frac{G I_d}{E} \Theta^{ii} = \frac{m(z)}{E}$$

…(66)

Attempts of uncoupling Equation (3-91) were also made for stability and free vibrations analysis of I-beams of variable cross section using the finite difference method. The coefficients in last equations become variable in this case; only numerical technique can be employed to solve the problem [13].

4 Numerical Examples

In order to show the efficiency of the proposed algorithm procedure and computation the properties of tall building, a H high-rise building, L high stories, include five identical w-shape steel columns and rectangular shear wall.

The material properties of concrete and steel are given in Figure(3-5). The values of sectional and sectorial properties obtained in the optimum analysis using the iterative technique suggested in this study are given in Table (1).

4.1 Building with Different Cross-Sections

Herein, a building shown in Figure (5), is analyzed, which has variable cross-sectional plane storey in height. The height of storey is (3m) and the first, second and third storey was X-shaped floor plane (Figure 5-a) while the other stories was L-shaped floor plane (Figure 5-b).

The mode shape and the behavior of this building with height (18m) have been analyzed by numerical procedure; the result is compared with the same building with one cross section (Figure 5-a). The result is shown in Figure (6).

The results in Figures (7, 8 and 9) indicate that the maximum value of translation is in (X, Y) direction and rotation at the top of building in case of one cross-sectional plane for storey. Whereas the maximum value of translation is in (X, Y) direction and rotation is at the third storey when the cross-sectional plane was varying .

4.2 Effect of building storey shapes

Figure (10), shows buildings with variable cross-sectional plan storey which have been analyzed. The overall building height is 18m, with storey height of 3m.

The mode shapes in X-translation direction and warping have been considered. Figure (11 and 12), shows that there is small differences in results can be seen between the cases X-shaped plan and H-shaped plan, compared with C-shaped plan and L-shaped plan for the translation in X-direction.

On the other hand, the results agree reasonably well with C-shaped plan, L-shaped plan and H-shaped plan.

4.3 Warping Effect or Inclusion

To study the effect of warping on free vibration behavior of high-rise buildings, a (C and H-shaped) building of Figures (10,a and d) are assumed to be of steel. They are assumed to be a linearly elastic material with the same properties. In general, a procedure has been presented for the formulation of the stiffness matrix and consistent mass matrix for an open cross-sectional plane in warp-restrained torsion, including the warping shear effect by coupling the twist displacement field and the warping shear displacement field.

The lateral response, when warping was considered that was using (7-DOFs per node) seems to be larger than the response of the structure with warping (6-DOFs per node), especially when the cross-section of the building increases, or as the structure height decreases.

The behavior is related to the increase in torsional and warping stiffness as cross-sectional dimensions increase, which results is larger torsional motions and separation of the lateral and torsional motions. These results are presented in Figure (13 and 14).

4.4 Column Shear Forces

L-shaped plan of the structure under the applied horizontal and vertical loads act on the peak point of the structure shown in Figure (10, c).

Similar correlation exists for the distribution of column shear forces around the plane framed as evidenced by Figure (15).

Figures (16 and 17), present the distribution of these forces over the height of the building. It is seen from the results the two cases are with or without warping. Therefore, the only shear force in Y-direction affects on warping deformation.

5 Ansys Result

The accuracy of the results obtained by the numerical procedure examined by comparing with data generated using the commercially available software ANSYS program. A 18m building is employed, Figure (4) show the plane layout of the building. A horizontal load is equal to 1.2 times area of plane and acting on the peak point of building and a vertical is equal to 1.2 times (0.5 *H*W), where H and W , height and width of building, respectively.

For comparison, the corresponding results given in software ANSYS program are also given in the Figure (18). The results agree reasonably well with those given in the present theory.

According to the results, differences (of about 15%) seem to be mainly due to fundamental mode between the calculations of present theory and ANSYS program.

6.Conclusions

An efficient mathematical theory for analysis of behavior of high-rise buildings with the inclusion of warping has been proposed in this study. Herein, the method incorporates the high-rise building with shear wall and bracing modeled as thin-walled beam elements in three dimensional analysis for displacements, natural frequencies and modes shape of tall building structures. From the present study the following conclusions have been drawn:

1. The shape of storey plane has great influence of the structural behavior of high rise steel buildings.
2. From studying different cases of loading in many directions, the largest value of natural frequency is obtained for uniform distributed load in the direction perpendicular to the shear wall.
3. Based on the theory of mathematical procedure, an analytical method of solution is proposed and general solution to the EIGN value problem is used to determining the natural frequencies and modes shape of either symmetric or un-symmetric tall building structures.
4. In general, the number of times nodal displacement fields are computed along the height of the building is not critical for floor slab displacements and natural frequencies. In the case of buildings under pure torsion, it has no influence on nodal displacements and member forces.
5. The presented theory has made use of sectorial properties of section which is quite useful in describing deformations and mode

shape when the plane cross-section no longer remains plane.

6. In this study the automated computation automated of sectional and sectorial properties in addition to the determination of the shear centre and mass centre of plan storey for building.

7. The present study has been applied to a tall building with variable cross-sections along the height of building and for different materials, thus considerably extending the range of applicability of accurate numerical formulations for this type of structure.

8. The numerical examples pertaining to free vibration analysis of high-rise building are presented. The results from the proposed analytical method are in good agreement with those from a software ANSYS program and results from theoretical research.

9. The presented study provides an efficient and practical free vibration approach for seismic response analysis of high-rise building with a large number of degree of freedom. Free vibration analysis of such a complex three dimensional model is more efficient than the conventional algorithms and high complexity of structural models.

10. Finally, the simplest method is presented in this study dealing with complex and tall building structures such as a cantilever column which contains different nodes each of which refers to the storey of tall building.

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Table (1): The sectional and sectorail properties for building in Figure (4)

No	**ώ(m^2)**	**S_{ax}(m^3)**	**S_{av}(m^3)**	**I_{ax}(m^4)**	**I_{ay}(m^4)**	**I_o(m^4)**	**C_x^2**	**C_y^2**	**I_d(m^4)**	**I_ω(m^6)**	**$I_{\omega x}$(m^5)**	**$I_{\omega y}$(m^5)**	**S_ω(m^4)**
1	1.67	-6.6384	2.649	0.537	0.0857	30.32 ρ_s L	0.026	4.2*10^-3	0.172	0.093	-0.223	0.089	0.056
2	1.64	-6.56	-2.303	0.537	0.0664	30.03 ρ_s L	0.0265	3.3*10^-3	0.169	0.09	-0.219	-0.0771	0.055
3	0.54	0	-0.762	894*10^-6	0.0664	1.26 ρ_s L		0.03	0.056	0.01	0	-0.026	0.018
4	1.64	6.56	-2.303	0.537	0.0664	30.03 ρ_s L	0.0265	3.3*10^-3	0.169	0.09	0.219	-0.0771	0.055
5	1.67	-6.6384	2.649	0.537	0.0857	30.32 ρ_s L	0.026	4.2*10^-3	0.172	0.093	0.223	0.089	0.056
6	0.62	-0.0775	-0.333	2.3*10^-3	0.0615	2.437 ρ_c L	0	0.43	0.386	2*10^-3	-5* 10^-3	-3.2* 10^-4	-1.95* 10^-4
∑		-0.0775	-0.403	2.151	0.4321	9.632*10^6L			3.45	0.38	-5* 10^-3	-2.2* 10^-3	-0.239

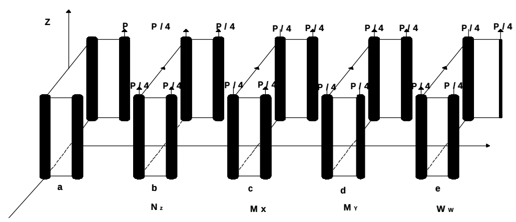

Figure (1) High-rise building channel cross-section[13].
a)Vertical load at corner; b)Symmetrical axial loading; c) Bending about X-axis; d)Bending about Y-axis; e)Self- equilibrate loads

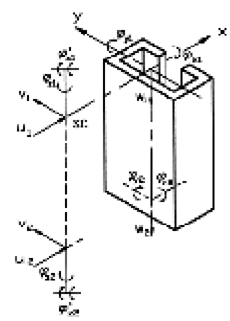

Figure (2) Generalized Displacement for Thin-Walled Element

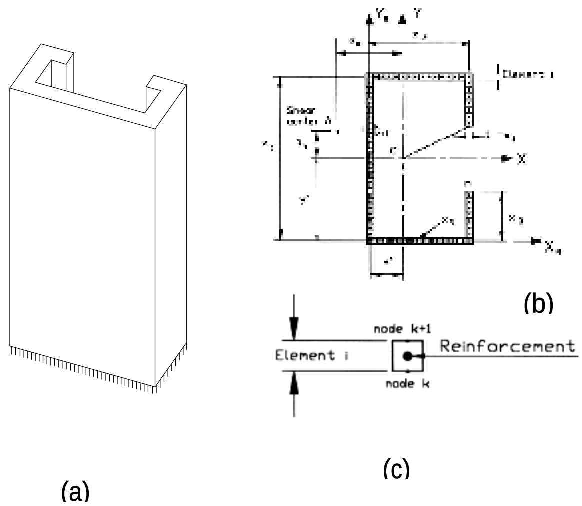

Figure (3) a) C-High rise building. b)Cross-section with principle axes. c) Small element of shear wall.

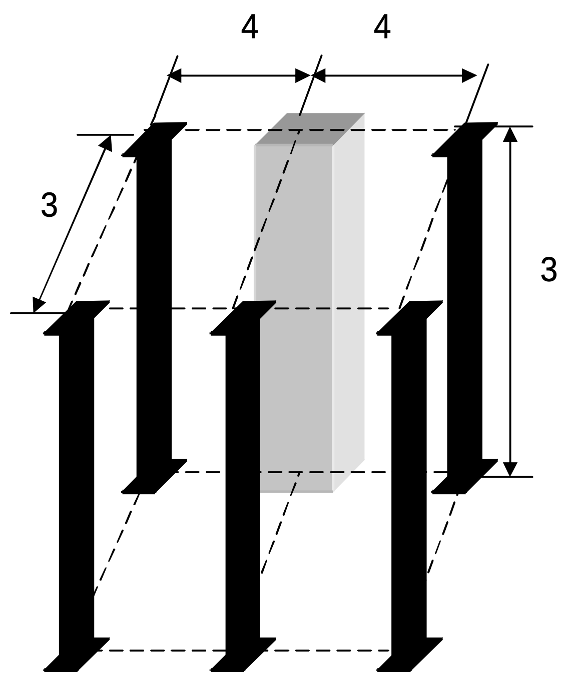

Figure (4) High-rise building prototype example

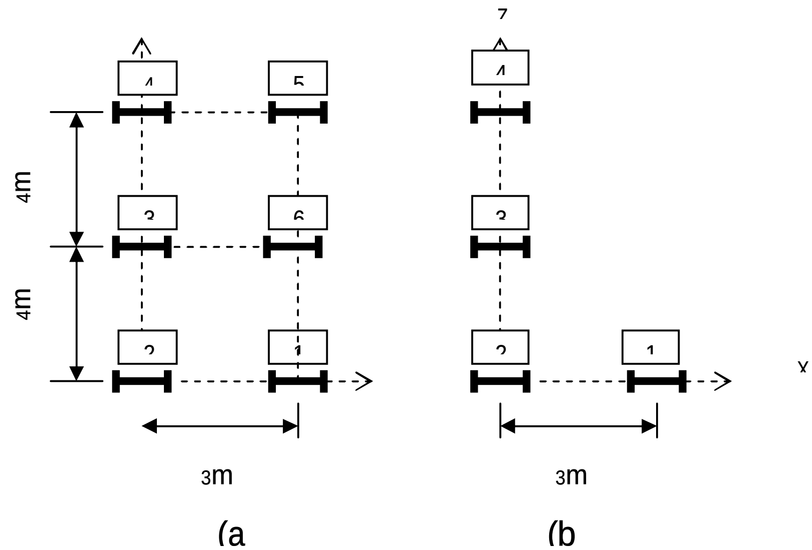

Figure (5) Plane layout of 18m height of building

a) X-shaped plane b)L-shaped plane

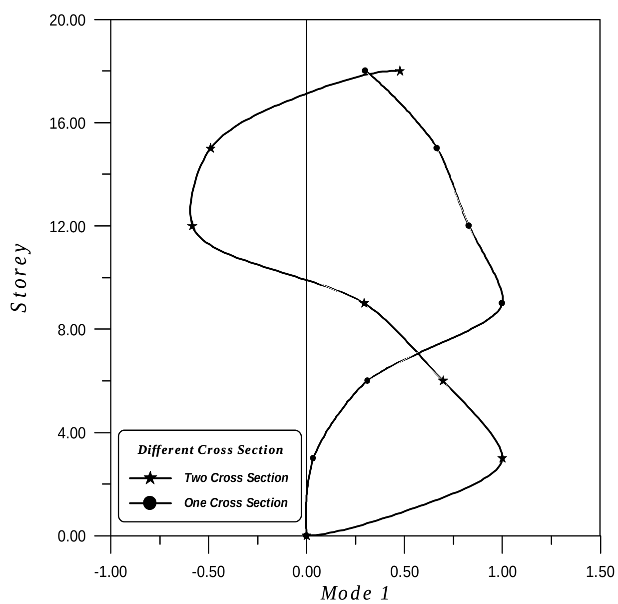

Figure (6) First mode shape of 18m height of building with variable cross-sectional plane of storey

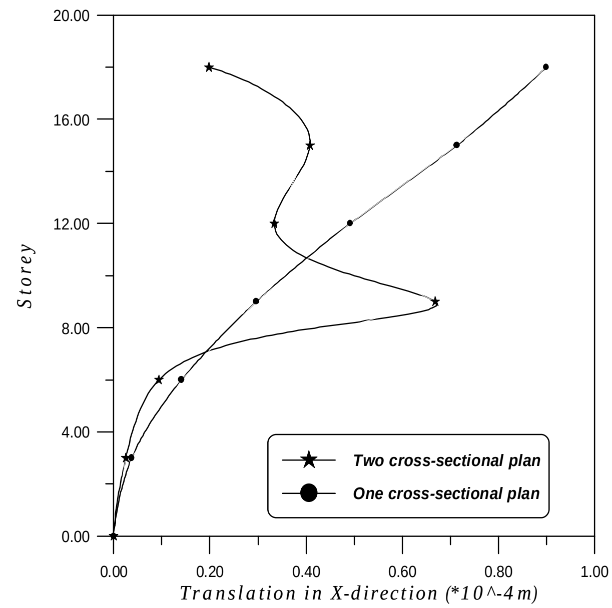

Figure (7) Translation in X- direction of 18m height of building with variable cross-sectional plane of storey

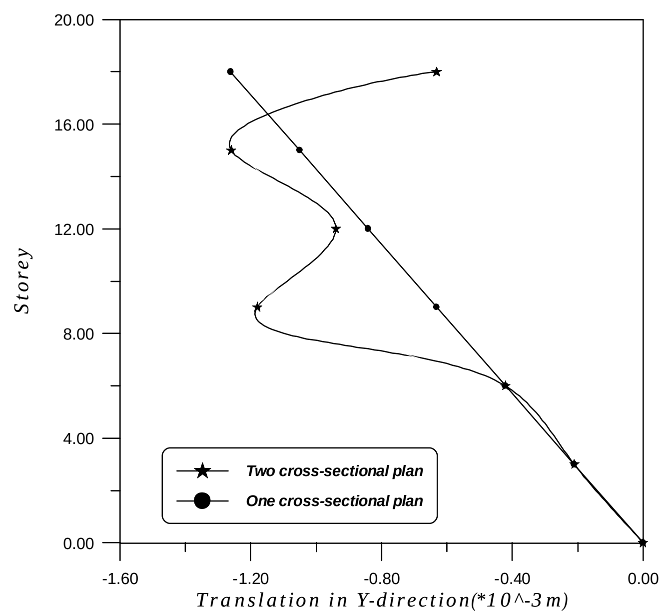

Figure (8) Translation in Y- direction of 18m height of building with variable cross-sectional plane of storey

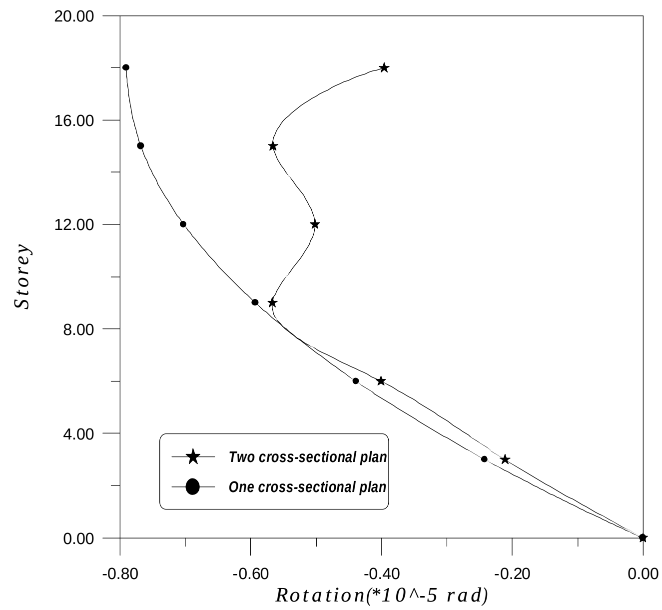

Figure (9) Rotation of 18m height of building with variable cross-sectional plane of storey

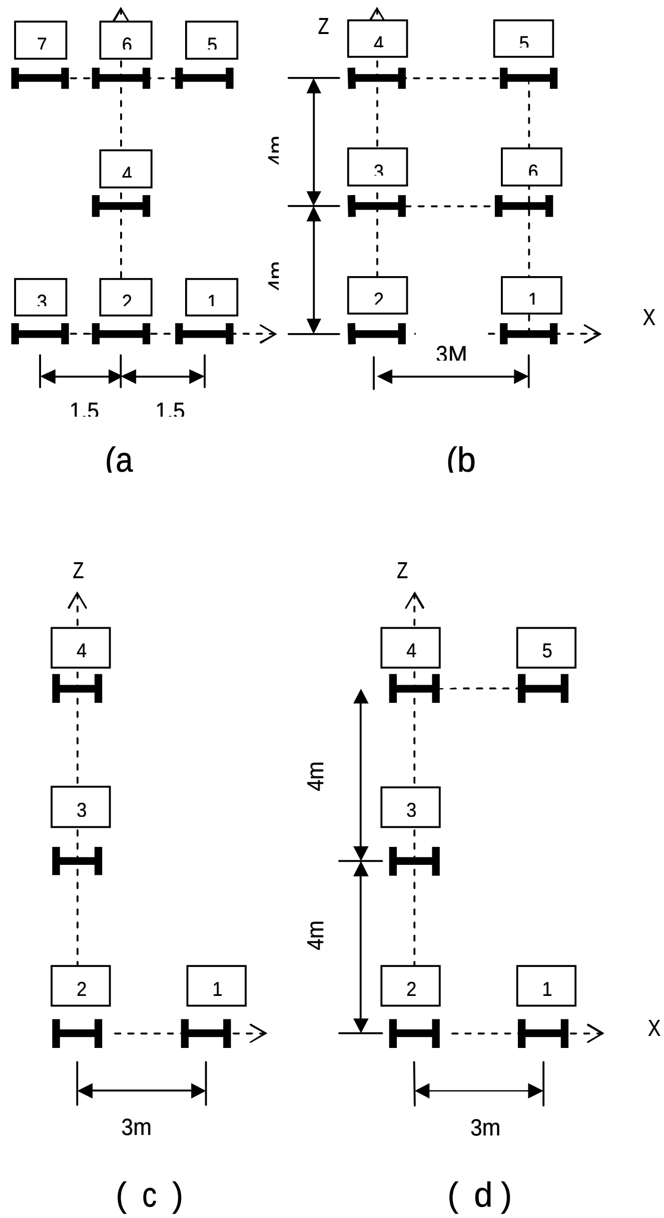

Figure (10) Plane layout of 18m height of building a) H-shaped plan, b) X-shaped plan c)L--shaped plan d)C-shaped plan

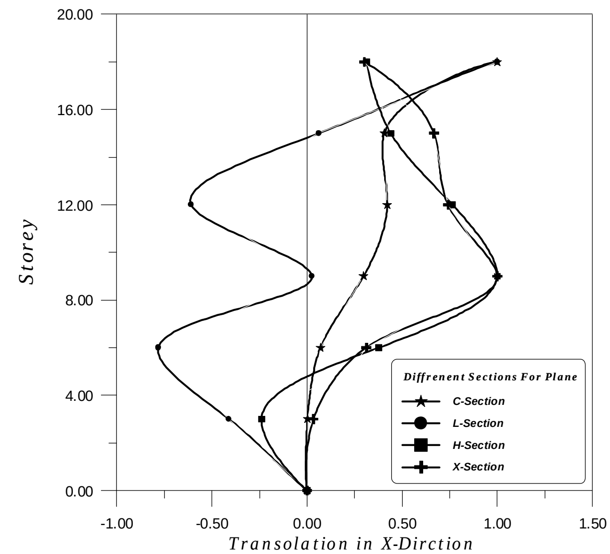

Figure (11) Mode shape for 18m height of building with variable cross-sectional of plane

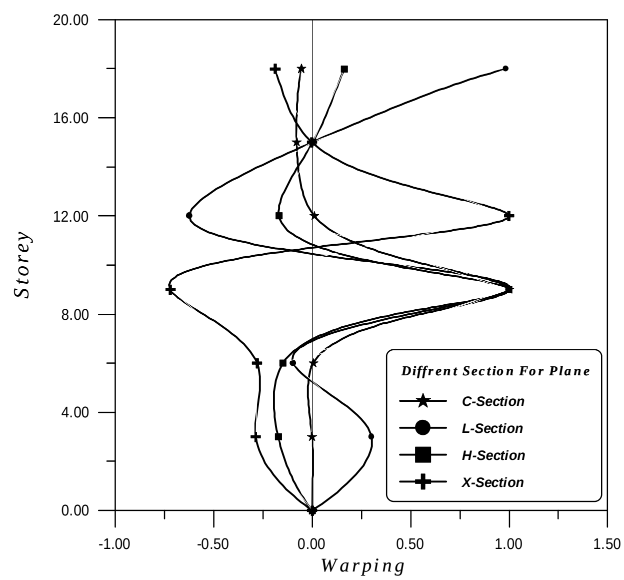

Figure (12) Mode shape for 18m height of variable building configuration

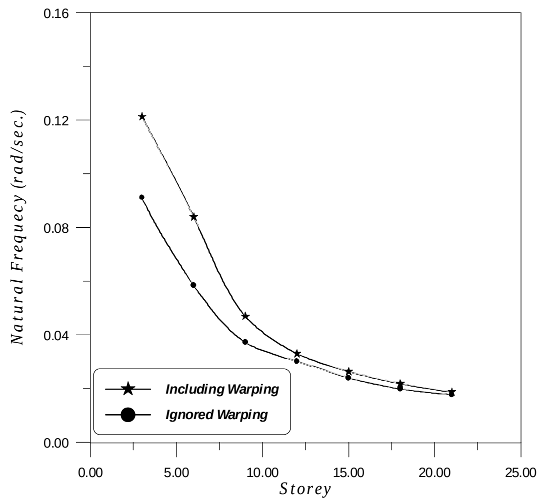

Figure (13) Natural frequency with height variation of C-shaped

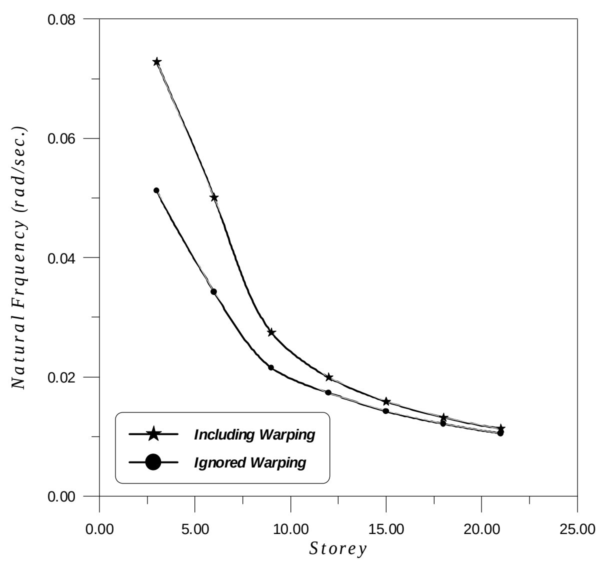

Figure (14) Natural frequency with height variation of H-shaped

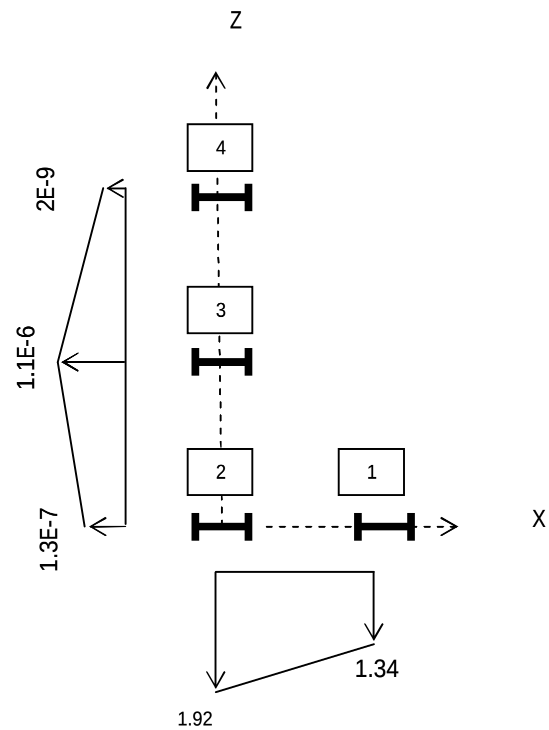

Figure (15) Distribution of column shear force (kN) at top level

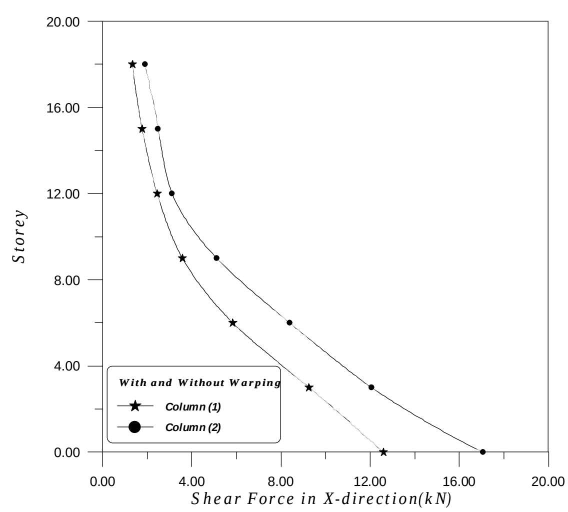

Figure (16) Column shear force (kN) in X-direction for columns (1&2) (L-shaped) building

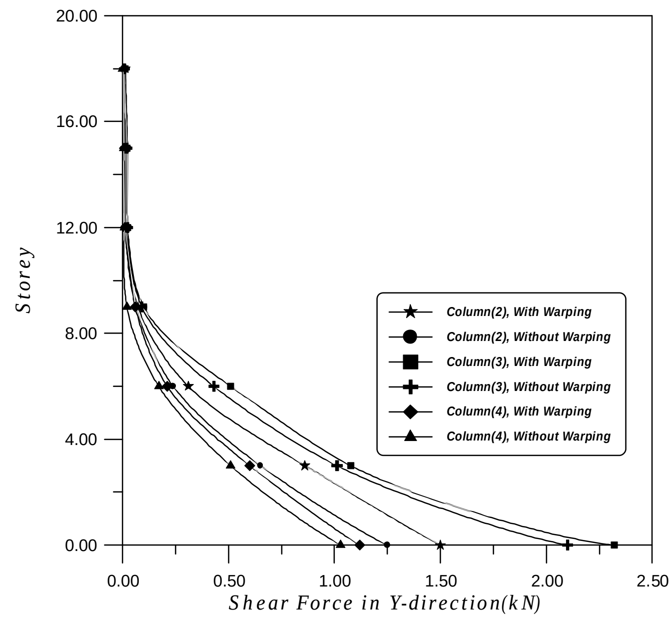

Figure (17) Column shear force (kN) in Y-direction for columns (1&2) (L-shaped) building.

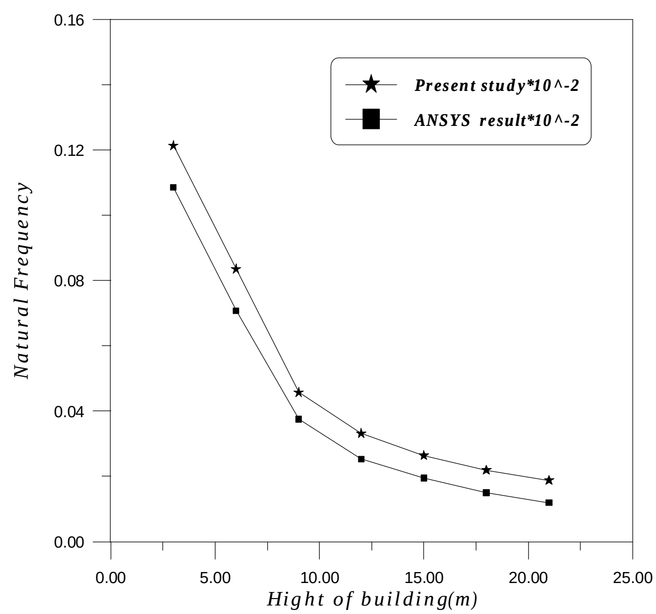

Figure (18) Natural frequencies with height building of 6-storey space structure