Effect of Moment of Inertia and Aerodynamics Parameters on Aerodynamic Coupling in Roll Mode

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ABSTRACT

The influence of moment of inertia and aerodynamic parameters on the aerodynamic coupling in rolling mode has been analyzed for Aircraft F-94A (case study) for different rolling rate in rolling mode, the equations of motion for aircraft has been analyzed to get the required equations of motion for aerodynamic coupling. The stability of these equations has been tested by Routh Discriminate.

The influence of moment of inertia and aerodynamic parameters on Routh Discriminate was clear, for example the wing span was the most positive influence on aerodynamic coupling stability.

LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e$</td>
<td>$gCho$</td>
</tr>
<tr>
<td>$ug.ft^2/sec$</td>
<td></td>
</tr>
<tr>
<td>$ug/ft.sec$</td>
<td></td>
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<tr>
<td>$ft/s$</td>
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<td>$ft/s$</td>
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INTRODUCTION

During the rolling maneuvers large angles of sideslip may occur as a result of kinematics coupling [1]. The vertical tail may produce large yawing moment that acts in the direction of roll. In such a case, it may not be possible to stop the flying body from rolling, although the lateral control is held against the roll direction. This is known as autorotation rolling. In this situation positive “G” would facilitate recovery [2]. As the angle of attack is increased to a positive value, kinematics coupling will be result in a moment that opposes the original direction of roll, thus alleviating the tendency for autorotation rolling [3].

The divergence experienced during rolling manufacture is complex because it involves not only inertia properties, but aerodynamic as well, [2]. Coupling results when a disturbance about one aircraft axis causes a disturbance created by an elevator deflection during straight and level flight, [4]. The resulting motion is restricted to pitching motion and no disturbance occurs in yaw or roll. An example of couple motion is the disturbance created by a rudder deflection [5]. The ensuing motion will be some combination of both yawing and rolling motion [6]. Although all lateral disturbance motion are coupled, the only motion that ever results in coupling problems large enough to threaten the structural integrity of the aircraft is
coupling as a result of rolling motion, [7]. F-94A aircraft was taken as case study, Figure (1), Table (1) [8]. The roll rate has been taken variable according to an equation in ref.[9].

MATHEMATICAL ANALYSIS OF ROLLING DIVERGENCE

The equation of motion for the airplane can be derived from Newton's second law of motion, which states that the summation of all external force acting on a body must be equal to the time rate of change of the momentum of the body, and the summation of the external moment of momentum (angular momentum). The time rates of change are all taken with respect to inertial space. These laws can be expressed by two vector equations.

The overall equation of motion, [2].

\[
\begin{align*}
\sum F_x &= \frac{m}{\rho s u_o} \ddot{u} - \left[ c_x(u_o, \alpha_o, \dot{q}_o) + \frac{1}{2} c_{x}\nu - \frac{1}{2} c_{r} u_o - c_T(M_o) \ddot{u} \right] + \left( \frac{2 m a_o}{\rho s c} - \frac{1}{2} c_{xq}\ddot{q} \right) + \frac{m g \cos \theta_o}{\rho s u_o^2} \ddot{\theta} + \frac{1}{2} c_{x a} \ddot{a} = m(\dot{u} + q v - r v) \\
\sum F_z &= \frac{2 m a_o}{\rho s u_o} \ddot{\dot{u}} - \left[ c_{z a} - 2 c_{x}(u_o, \alpha_o, \dot{q}_o, \delta_e) \right] \ddot{u} - \left( c_{z q} + \frac{4 m}{\rho s c} \dot{q} \right) \ddot{q} + \frac{2 m g}{\rho s u_o^2} \sin \theta_o \ddot{\theta} + \frac{2 m}{\rho s u_o} \ddot{a} - c_{x a} \ddot{a} = m(\dot{w} + p v - q u) \\
\sum M &= - \left( 2 c_m(u_o, \alpha_o, \dot{q}_o, \delta_e) + c_{m u} u_o \right) \ddot{u} + \frac{4 l_{yy}}{\rho s u_o^2} \dot{q} - c_{m q} \ddot{q} - c_{m a} \ddot{a} = l_{yy} \ddot{q} + (l_{xx} - l_{zz}) \dot{p} + l_{xz}(p^2 - r^2) \\
\end{align*}
\]

Pitching moment velocity:-

\[
\dot{u} = \ddot{\theta}
\]

\[
\begin{align*}
\sum F_y &= \frac{2 m}{\rho s u_o} \ddot{\dot{u}} - c_{y p} \ddot{p} - \left( c_{y p} + \frac{4 m a_o}{\rho s b} \right) \ddot{p} - \frac{2 m g}{\rho s u_o^2} \cos \theta_o \ddot{\theta} \ddot{\dot{u}} \quad \ldots (4) \\
\sum \ell &= - c_{t p} \ddot{p} - \frac{4 l_{xx}}{\rho s u_o^2 b^2} \ddot{p} - c_{t p} \ddot{p} - \frac{4 l_{xx}}{\rho s u_o^2 b^2} \ddot{p} - c_{t r} \ddot{r} = l_{xx} \dot{p} - (l_{yy} - l_{zz}) \dot{q} - l_{xz}(\dot{r} - \dot{q}) \\
\sum N &= - c_{n p} \ddot{p} - \frac{4 l_{xx}}{\rho s u_o^2 b^2} \ddot{p} - c_{n p} \ddot{p} + \frac{4 l_{xx}}{\rho s u_o^2 b^2} \ddot{p} - c_{n r} \ddot{r} = l_{zz} \dot{p} - (l_{yy} - l_{xx}) \dot{q} \quad \ldots (6)
\end{align*}
\]

Rolling Velocity:-

\[
\frac{2 u_o}{b} \ddot{p} = \ddot{\dot{u}} - \sin \theta_o \ddot{\dot{u}}
\]

Yawing Velocity:-

\[
\frac{2 u_o}{b} \ddot{r} = \cos \theta_o \ddot{\dot{u}}
\]

The approach for solving the autorotation rolling equations was derived based on some necessary assumption to fit into the present analysis of autorotation rolling [1].

I. Velocity remains constant during the roll maneuver \( \dot{u} = 0 \), \( u = u_o \).
2. The rate roll rate is constant $\dot{\phi} = 0$, so that $p = 0$.
3. $v, w, q, r$ are small therefore their products are negligible.
4. Engine gyroscopic effect is negligible.
5. Rudder and elevator are fixed in their initial trim position.
6. Aerodynamic coefficients are negligible with the exception of $c_{m_a}, c_{m_p}, c_{n_p}$ and $c_{n_r}$.
7. Small angle assumption on $\alpha$ and $\beta$.

When these assumptions are applied to the six equations of motion the following results are obtained:

\[ \sum F_x = 0 \] \hspace{1cm} (7)

\[ \sum F_y = m u_o (\dot{\beta} + r - p_o \alpha) = 0 \] \hspace{1cm} (8)

\[ (\dot{\beta} + r - p_o \alpha) = 0 \hspace{1cm} \text{from assumption} \]

\[ \sum F_z = m u_o (\dot{\alpha} + p_o \beta - q) = 0 \] \hspace{1cm} (9)

(Both lift and side force will average zero throughout a roll)

\[ \sum \ell = -(r^* + q p_o) l_{xz} = 0 \] \hspace{1cm} (10)

This is a reasonable condition because one considers the motion to be principally a steady state roll because such a situation the aileron moment and damping in roll exactly oppose one another.

\[ \sum M = \dot{q} I_{yy} + p_o r (l_{xx} - l_{zz}) + p_o^2 l_{xz} = \frac{1}{2} \rho u_o^2 s c c_{m_a} \alpha + c_{m_q} q \frac{c}{2 u_o} \] \hspace{1cm} (11)

\[ \sum N = \dot{r} l_{zz} + p_o q (l_{yy} - l_{xx}) = \frac{1}{2} \rho u_o s b \left[ c_{n_p} \beta + c_{n_r} \frac{b}{u_o} \right] \] \hspace{1cm} (12)

Rewriting the equations in nature form:

\[ \dot{\alpha} + p_o \beta - q = 0 \] \hspace{1cm} (13)

\[ \dot{\beta} + r - p_o \alpha = 0 \] \hspace{1cm} (14)

\[ \frac{1}{2} \rho u_o^2 s c c_{m_a} \alpha - \dot{q} l_{yy} - p_o r (l_{xx} - l_{zz}) + \frac{\rho u_o s c^2}{4} c_{m_q} q = p_o^2 l_{xz} \] \hspace{1cm} (15)

\[ \frac{1}{2} \rho u_o^2 s b c_{n_p} \beta - \dot{r} l_{zz} - p_o q (l_{yy} - l_{xx}) + \frac{\rho u_o s b^2}{4} c_{n_r} r = 0 \] \hspace{1cm} (16)

Note that there are four equations in four unknowns ($\alpha, \beta, q, and r$). Particular solution to these equations exists because the pitching moment equation is not homogenous. However, the investigation of the particular solution holds only for design interest. On the other hand the homogenous solution represents motion...
which is indicative of stable or unstable coupling. Accordingly, the equations are Laplace transformed and coefficient matrix determinant becomes.

\[
\begin{vmatrix}
  s & p_o & -1 & 0 \\
  -p_o & s & 0 & 1 \\
  1 \frac{\rho u_0^2 s \tilde{c}}{l_{yy}} c_{m_a} & 0 & s - \frac{\rho u_0^2 s \tilde{c} c_{m_q}}{4 l_{yy}} & p_o \left( l_{xx} - l_{zz} \right) \\
  0 & \frac{-1}{2} & \left( l_{yy} - l_{xx} \right) & s - \frac{\rho u_0^2 s \tilde{c} c_{n_r}}{4 l_{zz}}
\end{vmatrix}
\]

The determinant must be expanded to solve for the characteristic equation:

\[AS^4 + BS^3 + CS^2 + DS + E = 0\]

The equation must be tested for stability in several methods such as Routh Discriminate [6] which conditioned for stability:

\[BCD - B^2E - AD^2 \geq 0\]

The stability derivatives formation which is given below, [2] was helpful in this analysis for determining:

\[c_{m_a} = c_{La_a} \left( \bar{x} - \bar{x}_{\eta} \right) \quad \ldots(17)\]

\[c_{m_q} = -2 \xi \eta \bar{b} \left( \frac{\partial \tilde{c}}{\partial a} \right) \frac{l_{zz}}{l_{yy}} \bar{c} \quad \ldots(18)\]

\[c_{n_r} = -2 \left( \frac{\partial \tilde{c}}{\partial a} \right) \frac{l_{yy}}{l_{zz}} \bar{c} \quad \ldots(19)\]

\[c_{n_{\beta}} = c_{La_{\beta}} \left( 1 - \frac{d\alpha}{d\beta} \right) \bar{V}_F + c_{n_{\beta fus}} \quad \ldots(20)\]

**RESULTS AND DISCUSSION**

The parameters exits in autorotation characteristic equation were select as effected parameters, which tested with different roll rate \(p_o = 10, 20, 30\) rad/sec.

From fig.(2,3 and 4) which present the aerodynamic parameters have positive effect toward the cross coupling, when they increase the stability of aerodynamic cross coupling increases too due to increases in lift.

The height or density have great effect toward the stability of aerodynamic cross coupling and it is clearly shown in fig.(5) as the height increases(density decreases) the stability of aerodynamic cross coupling decreases due to lift decreases.

Fig.(6,7and 8) show the effect of moment of inertia in \(x,y,z\) axis and all have negative effect toward the stability of aerodynamic cross coupling because large moment of inertia causes uncontrollable pitching moment.

The static longitudinal stability \(c_{m_a}\) little effect toward the stability fig. (9) because of lift increases, but the damping in pitch \(c_{m_q}\) have great negative effect toward the aerodynamic cross coupling stability fig.(10), which increases as the distance between horizontal stabilizer and airplane he aerodynamic coupling which becomes uncontrollable.

Directional stability \(c_{n_{\beta}}\) have a positive effect toward the stability of aerodynamic cross coupling fig.(11), but \(c_{n_r}\) which present the rolling stability have great positive effect toward the aerodynamic cross coupling stability.
fig.(12)), which increases as the distance between vertical stabilizer and airplane he aerodynamic coupling which becomes uncontrollable.

**CONCLUSIONS**

1. X-plane no effect plane.
2. The increasing of the wing area \( S \) leading to increase the stability.
3. High height aerodynamic coupling have more stable than low height.
4. The wing area \( S \) and directional stability \( \text{cn} \) have the most powerful parameters.
5. The weight distribution is very important for avoiding the aerodynamic coupling.

![Figure (1) Views of Supersonic Aircraft F-94A (Case Study)](image-url)
Effect of Moment of Inertia and Aerodynamics Parameters on Aerodynamic Coupling in Roll Mode

### Aerodynamic Data

<table>
<thead>
<tr>
<th>Wing Span (ft)</th>
<th>Wing Area ((\text{ft}^2))</th>
<th>Wing Mean Chord (ft)</th>
<th>Aspect Ratio</th>
<th>Wing Sweep Angle (deg)</th>
<th>Taper Ratio</th>
<th>Airfoil Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>130</td>
<td>2400 ((\text{ft}^2))</td>
<td>20.2</td>
<td>7.04</td>
<td>24</td>
<td>0.2</td>
<td>NACA 65A004.8</td>
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### Stability Derivatives

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<thead>
<tr>
<th>(C_{m\alpha})</th>
<th>(C_{m\theta})</th>
<th>(C_{n\phi})</th>
<th>(C_{n\beta})</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.619/rad</td>
<td>-11.4/rad</td>
<td>-0.107/rad</td>
<td>+0.096/rad</td>
</tr>
</tbody>
</table>

### Other Data

<table>
<thead>
<tr>
<th>(I_{xx}) (SI-ft²)</th>
<th>(I_{yy}) (SI-ft²)</th>
<th>(I_{zz}) (SI-ft²)</th>
<th>(I_{xz}) (SI-ft²)</th>
<th>Max Speed (ft/sec)</th>
<th>Mass (slug)</th>
<th>Density (\text{M}=0.8) Alt=20000 ft (SI/ft³)</th>
<th>Engine Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.55 \times 10^8</td>
<td>2</td>
<td>4</td>
<td>0</td>
<td>440</td>
<td>5900</td>
<td>0.001267</td>
<td>J.33A-33</td>
</tr>
</tbody>
</table>

**Figure (2) Effect of Wing Span on Air Aerodynamic Coupling**
Effect of Moment of Inertia and Aerodynamics Parameters on Aerodynamic Coupling in Roll Mode

Figure (3) Effect of Wing Mean Chord on Aircraft Aerodynamic Coupling
Effect of Moment of Inertia and Aerodynamics Parameters on Aerodynamic Coupling in Roll Mode

Figure (4) Effect of Wing Area on Aircraft Aerodynamic Coupling
Effect of Moment of Inertia and Aerodynamics Parameters on Aerodynamic Coupling in Roll Mode

Figure (5) Effect of Air density on Aircraft Aerodynamic Coupling
Figure (6) Effect of X-Axis moment of Inertia on Aircraft Aerodynamic Coupling
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Figure (7) Effect of Y-Axis moment of Inertia on Aircraft Aerodynamic Coupling
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**Figure (8) Effect of Z-Axis moment of Inertia on Aircraft Aerodynamic Coupling**
Figure (9) Effect of Static Longitudinal Stability on Aircraft Aerodynamic Coupling
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Figure (10) Effect of Damping in Pitch on Aerodynamic Coupling
Figure (11) Effect of Directional stability on Aerodynamic Coupling
Effect of Moment of Inertia and Aerodynamics Parameters on Aerodynamic Coupling in Roll Mode

Figure (12) Effect of Rolling Stability on Aerodynamic Coupling
REFERENCES
[1] Dr. Chuan-Tau and Dr. Jan Roskam