

## Modeling of Shear Strength in Rectangular Reinforced Concrete Beams

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### Abstract

In this paper a model for the shear capacity of rectangular reinforced concrete beams is proposed. This modeling depends on statistical analysis for 101 experimental results from literatures, as will be indicated. These results are pure shear results and shear resistance is predicted by concrete only (without shear reinforcement).

The ACI approach for predicting shear strength as the sum of diagonal cracking and 45° degree truss model predicts the shear strength of beams, with an average of experimental to predicted shear strength ratio of 1.4 with a coefficient of variation of 46.7%.

In this study, experimental pure shear test results as much as possible are revised to study the shear behavior and then modeling it using statistical analysis, to reduce the ratio and coefficient of variation. This model gives average ratio of 1.003 and coefficient of variation of 20.90%.

Also in this study, all the factors effecting shear capacity are introduced with separate modeling. These factors are, concrete compression strength ( $f_c'$ ), beam width ( $b$ ), size of aggregate ( $d_a$ ), shear span to depth ratio ( $a/d$ ), and percentage of longitudinal reinforcement.

Then assembly model for shear strength containing all these factors is presented and compared with experimental results.

### مقاومة القص في العتبات الخرسانية المستطيلة المقطع

#### الخلاصة

في هذه الدراسة تم تقديم نموذج رياضي لاجاد مقاومة القص للعتبات الخرسانية المستطيلة المقطع يعتمد هذا النموذج الرياضي على التحليل الاحصائي لـ (101) نتيجة فحص استخرجت من البحوث السابقة, كما ستوضح لاحقا, هذه النتائج هي للقص المجرد وان مقاومة القص تقتصر على الكونكريت فقط (لا يوجد تسليح قص).

معادلات المعهد الامريكي للخرسانة لاجاد مقاومة القص مكون من مجموع التشقق القطرية مع 45 درجة لنموذج الجملون, وكان معدل النتائج المختبرية مقسوما على مقاومة القص المستخرجة من ACI هي 1.4 مع معامل تغير 46% في هذه الدراسة تم جمع اكبر عدد ممكن من الفحوصات المختبرية للقص المجرد لدراسة تصرفها ومن ثم وضعها في معادلات لتقليل النسبة ومعامل التباين, وقد وجد النموذج الرياضي المقترح, يعطي نسبة معدل مقداره 1.003 ومعامل تغير مقداره 20.90%.

بالاضافة الى ذلك تم استعراض كل المعاملات المؤثرة على مقاومة القص للعتبات وعمل نموذج رياضي, والمعاملات هي, مقاومة الانضغاط, عرض العتبة, حجم الحصى, نسبة فضاء القص الى العمق الفعال, واخيرا نسبة حديد التسليح الطولي. واخيرا تم عمل نموذج رياضي يغطي كل هذه المعاملات ومقارنتها بنتائج الفحوصات العملية.

**Introduction**

Even of the behavior of reinforced concrete in shear has been studied for more than 100 years, the problem of determining the shear strength of reinforced concrete beams remains open to discussion. Thus the shear strengths predicted by different design codes for a particular beam section can vary by factors of more than 2, so in this study a formula (modeling) for shear strength of rectangular cross section beams are presented. This model contains all factors which are ,compressive strength of concrete( $f_c'$ ) , beam width (b) , size of aggregate (da) , shear span to depth ratio (a/d) , and percentage of longitudinal reinforcement ( $\rho$ ) depending on 101 pure shear test results obtained in(14) researches that deal with pure shear resistance (no shear reinforcement), Moody<sup>(2)</sup>, K.G. et al (1954); Taylor<sup>(3)</sup>, R. and Brewer, R. S. (1963); Mattock<sup>(4)</sup>, Alan H. (1969); Krefeld<sup>(5)</sup>, William J. and Thurston, Charles W. (1966); Gaston<sup>(6)</sup>, J. R. , Sles, C. P. and Newmark, N. M. (1952); Placas<sup>(7)</sup> and Regan (1971); Diaz de Cossio<sup>(8)</sup>, Roger and Siess, Chester P. (1960); Moody<sup>(9)</sup>, K. G. , et al. (1954); Mathey<sup>(10)</sup>, Robert G. and Watstein , David (1963); Ahmed<sup>(11)</sup> , Shuaib H. , Khaloo, A. R. (1986); Kani<sup>(12)</sup>, G. N. J. (1967), ;Kani<sup>(13)</sup>, G. N. J. (1966); Chana<sup>(14)</sup>, P. S. (1981); Elzanty<sup>(15)</sup>, Ashraf H., Nilson, Arthur H. and Slate, F. O. (1986). In 1962, joint ACI-ASCE Committee 326 published a report regarding the design and behavior of beams failed due to shear and the diagonal tension. To develop safe design recommendation, a database Of 194

beams tests without shear reinforcement was compiled. The database consisted of 130 laboratory specimens tested under single and two point load and 64 beams subjected to uniformly distributed loading. Based on those results, the following design equation was developed, which adopted by ACI 318M-05 code<sup>(1)</sup>.

$$V_c = [\sqrt{f_c'} + 120\rho_w(V_u.d/M_u)]bw.d/7 \dots\dots(1)$$

And the simplified formula

$$V_c = 1/6(\sqrt{f_c'.bw.d}) \dots\dots(2)$$

Where

$V_c$ = nominal shear strength of concrete

$f_c'$ = compressive strength of concrete

$\rho_w$ = web steel ratio  $A_s/bw.d$

$V_u$ = factored applied shear force

$M_u$ = factored applied moment

$bw$ = web width

$d$ = effective depth of the reinforcement

$A_s$ = area of tension reinforcement

To find the concrete shear resistance model, the effect of each factor will be discussed separately.

1- The Effect of compressive strength( $f_c$ ) on shear capacity:

Compressive strength, is evaluated under compression load using standard concrete cylinder at 28 days, now in this study we want to find the correlation formula between shear strength and compressive strength, for the same data, where  $f_c'$  varies from 15.37 to 72.8 using statistical analysis.

Model Definition(Tenth order Polynomial).

$$V_c = a_1*f_c'^{10} + b_1*f_c'^9 + c_1*f_c'^8 + d_1*f_c'^7 + e_1*f_c'^6 + f_1*f_c'^5 + g_1*f_c'^4 + h_1*f_c'^3 + i_1*f_c'^2 + j_1*f_c' + k_1 \dots\dots(1)$$

For 99% Confidence Intervals

Variable	Value
a <sub>1</sub>	2.78E-09
b <sub>1</sub>	-1.16E-06
c <sub>1</sub>	2.11E-04
d <sub>1</sub>	-2.22E-02
e <sub>1</sub>	1.49
f <sub>1</sub>	-66.54
g <sub>1</sub>	1998.65
h <sub>1</sub>	-39926.92
i <sub>1</sub>	507994.80
j <sub>1</sub>	-3720322.16
k <sub>1</sub>	11968863.51

Number of observations = 101

Solver type: Nonlinear

Residual Sum of Squares (Absolute) = 252348298.22

Residual Sum of Squares (Relative) = 252348298.22

Coefficient of Multiple Determination (R<sup>2</sup>) = 0.795

Proportion of Variance Explained = 79.57%

Adjusted coefficient of multiple determination (Ra<sup>2</sup>) = 0.753.

Residual tolerance = 0.000001

Sum of Residuals = -7.360E-05

Average Residual = -1.24E-06

Nonlinear iteration limit = 250

Fig1.shows the model plot for Tenth Order Polynomial,

Fig2.Shows the Residual Scatter Plot

Fig3.Shows Residual Normal Probability plot for presented model.

2-Effect of Shear Span to depth ratio(a/d) on shear strength.

Shear span(a) is the distance between point load and the support, has a symbol (a), and d is effective depth of steel reinforcement.

statistical Analysis.

Model Definition:(Tenth order polynomial).

Vc =

$$a_2*(a/d)^{10}+b_2*(a/d)^9+c_2*(a/d)^8+d_2*(a/d)^7+e_2*(a/d)^6+f_2*(a/d)^5+g_2*(a/d)^4+h_2*(a/d)^3+i_2*(a/d)^2+j_2*(a/d)+k_2 \dots\dots(2)$$

For 99% Confidence Intervals

Variable	Value
a <sub>2</sub>	1.32
b <sub>2</sub>	-57.35
c <sub>2</sub>	1053.74
d <sub>2</sub>	-10592.21
e <sub>2</sub>	62378.78
f <sub>2</sub>	-208029.14
g <sub>2</sub>	294677.06
h <sub>2</sub>	349124.0
i <sub>2</sub>	-2015017.46
j <sub>2</sub>	2885942.146
k <sub>2</sub>	-1384435.75

Residual Sum of Squares (Absolute) = 191126302.10

Residual Sum of Squares (Relative) = 191126302.10

Coefficient of Multiple Determination (R<sup>2</sup>) = 0.781

Proportion of Variance Explained = 78.17%

Adjusted coefficient of multiple determination (Ra<sup>2</sup>) = 0.72

Solver type: Nonlinear

Nonlinear iteration limit = 250

Residual tolerance = 0.0001

Sum of Residuals = 1.37E-04

Average Residual = 2.74E-06

Fig4:shows the model plot for vc vs.a/d

Fig5:shows Residual scattar plot.

Fig6:Shows Residual Normal probability plot

3- The Effect of Size of Aggregate (da) on shear strength:

Depending on database, the size of aggregate varies from 12.7mm to

25.4mm. This factor was not included in ACI code formula, and tried to be included in this study.

Equation ID:

Fourth order logarithm

Model Definition:

$V_c =$

$$a_3 \cdot da^4 + b_3 \cdot da^3 + c_3 \cdot da^2 + d_3 \cdot da + e_3 \dots (3)$$

For 99% Confidence Intervals

Variable Value

$a_3$	-24.28
$b_3$	1685.41
$c_3$	-42317.82
$d_3$	455232.94
$e_3$	-1719434.35

Solver type: Nonlinear

Nonlinear iteration limit = 250

Residual tolerance = 0.0001

Sum of Residuals = 4.89E-06

Average Residual = 1.13E-07

Residual Sum of Squares (Absolute) = 181801228.25

Residual Sum of Squares (Relative) = 181801228.25

Coefficient of Multiple Determination ( $R^2$ ) = 0.62

Proportion of Variance Explained = 62.20%

Adjusted coefficient of multiple determination ( $R_a^2$ ) = 0.58

Fig7: shows model plot for  $V_c$  vs.  $da$

Fig8: shows residual scattar plot

Fig9: shows normal probability plot

4- The Effect of Percentage of Longitudinal Reinforcement( $\rho$ ) on Shear Strength:

Percentage of longitudinal

reinforcement=  $As/b \cdot d$ . In

experimental results, the value of  $\rho$  varies from .0046 to .0503.

Model Definition:(Tenth order polynomial)

$$V_c = a_4 \cdot \rho^{10} + b_4 \cdot \rho^9 + c_4 \cdot \rho^8 + d_4 \cdot \rho^7 + e_4 \cdot \rho^6 + f_4 \cdot \rho^5 + g_4 \cdot \rho^4 + h_4 \cdot \rho^3 + i_4 \cdot \rho^2 + j_4 \cdot \rho + k_4 \dots (4)$$

For 99% Confidence Intervals

Variable Value

$a_4$	-4.72E+22
$b_4$	1.22E+22
$c_4$	-1.36E+21
$d_4$	8.63E+19
$e_4$	-3.42E+18
$f_4$	8.81E+16
$g_4$	-1.48E+15
$h_4$	1593905335.8
$i_4$	-10390806.033
$j_4$	364313227.0
$k_4$	-468538.12

Solver type: Nonlinear

Nonlinear iteration limit = 250

Residual tolerance = 0.00001

Sum of Residuals = -2.91E-05

Average Residual = -5.83E-07

Residual Sum of Squares (Absolute) = 153895532.15

Residual Sum of Squares (Relative) = 153895532.15

Coefficient of Multiple Determination ( $R^2$ ) = 0.791

Proportion of Variance Explained = 79.176%

Adjusted coefficient of multiple determination ( $R_a^2$ ) = 0.738

Fig10: shows model plot for  $V_c$  vs.  $\rho$ .

Fig11: shows residual scattar plot.

Fig12: residual normal probability plot.

Finally, formula for shear strength included all these factors is presented These factors are,  $b, f_c, da, a/d$  and  $\rho$ .

Model Definition:

$$V_c = \exp(a_5 \cdot b + b_5 \cdot (a/d) + c_5 \cdot da + d_5 \cdot f_c)$$

$$+e_5 * \rho + f_5) \dots(5)$$

For 99% Confidence Intervals

Variable	Value
a <sub>5</sub>	1.85E-03
b <sub>5</sub>	-6.30E-02
c <sub>5</sub>	1.71E-02
d <sub>5</sub>	6.36E-03
e <sub>5</sub>	5.72
f <sub>5</sub>	10.09

Number of observations = 101  
 Solver type: Nonlinear  
 Nonlinear iteration limit = 250  
 Residual tolerance = 0.0001  
 Residual Sum of Squares (Absolute) = 764821304.13

Also in this study we will introduce a simplified formula for shear strength including all significant factors, instead of using eq.5 as follows:

$$V_c = (b * a / d * \sqrt{f_c'})^{1.34} \dots\dots(6)$$

This simplified formula has a mean of 1.387 and C.O.V is 48.7% in comparison with ACI code 1.4 and 46.7% respectively.

Conclusions:

- 1- Depending on 101 experimental test results from 14 researches are revised present formula of shear strength including factors, compressive strength, shear span to depth ratio, Size of aggregate, beam width, and percentage of longitudinal reinforcement which is.  
 $V_c = \exp(a_5 * b + b_5 * (a/d) + c_5 * da + d_5 * f_c' + e_5 * \rho + f_5)$ .
- 2- The average of experimental to predicted shear strength ratio is 1.003, and Coefficient of variation is 20.90%

Residual Sum of Squares (Relative) = 764821304.13  
 Coefficient of Multiple Determination (R<sup>2</sup>) = 0.790  
 Proportion of Variance Explained = 79.09%  
 Adjusted coefficient of multiple determination (Ra<sup>2</sup>) = 0.779  
 By compares our model with ACI, we can notice the mean value 1.00296, and Coefficient of variation is 20.90% in compares with 1.4 and 46.7% respectively.

Fig13: model plot of shear strength included all factors  
 Fig14: residual scattar plot  
 Fig15: residual normal probability plot.  
 which gives us an indication of a good accuracy for the presented formula.  
 3- The effect of shear span to depth ratio has the model definition Tenth order polynomial.  
 4- The effect of size of aggregate on shear strength has the model definition Fourth order logarithm.  
 5- The effect of percentage of longitudinal reinforcement on shear strength has the model definition Tenth order polynomial.  
 6- The effect compressive strength on shear has the model definition Tenth order polynomial.  
 7- A simplified formula for shear strength including all significant factors is presented.  
 $V_c = (b * a / d * \sqrt{f_c'})^{1.34}$   
 which has a mean value of 1.387 and C.O.V is 48.7% in comparison with ACI code 1.4 and 46.7% respectively.

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Table 1- Components of database

References	No	b	a/d	Da	fc	$\rho$	Vc exp	Vc cal	Vc exp/Vc cal
Moody, K. G. et al (1954)	1	177.8	3.06	25.4	30.34	0.02163	60048	58934.37	0.981454
	2	177.8	3	25.4	21.16	0.01619	56267.2	54088.7	0.961283
	3	177.8	2.98	25.4	21.58	0.01624	60048	54317.53	0.904569
	4	177.8	2.96	25.4	19.23	0.016	55600	53505.02	0.962321
	5	177.8	2.95	25.4	16.75	0.01657	55600	52872.5	0.950944
Taylor, R. and Brewer, R. S. (1963)	6	186.69	3.79	19.05	27.99	0.0194	48323	49920.97	1.033069
	7	187.96	3.79	19.05	27.99	0.01928	46330.4	50004.31	1.079298
	8	187.45	3.79	19.05	37.43	0.0193	53304.8	53057.14	0.995354
	9	187.45	3.79	19.05	31.16	0.0193	49319.4	50981.2	1.033695
	10	187.45	3.79	19.05	31.16	0.0193	49319.4	50981.2	1.033695
	11	187.96	3.77	19.05	29.37	0.0123	46330.4	48530.36	1.047484
	12	187.96	3.77	19.05	37.44	0.01235	49817.6	51103.17	1.025806
Mattock, Alan H. (1969)	13	152.4	2.74	19.05	46.8	0.01033	54710.4	53565.67	0.979077
	14	152.4	2.74	19.05	18.54	0.031	56044.8	50368.6	0.89872
	15	152.4	5.14	19.05	16.13	0.031	40032	42631.77	1.064942
Krefeld, William J. and Thurston, Charles W. (1966)	16	152.4	3.84	25.4	30.06	0.0452	64051.2	61147.2	0.954661
	17	152.4	2.89	25.4	22.06	0.0267	60048	55499.37	0.92425
	18	152.4	2.86	25.4	19.92	0.00797	48483.2	49274.07	1.016312
	19	152.4	2.89	25.4	20.13	0.0134	45814.4	50801.86	1.108862
	20	152.4	2.89	25.4	20.68	0.0134	52041.6	50980.03	0.979602
	21	152.4	4.77	25.4	20.82	0.0198	44035.2	47009.63	1.067547
	22	152.4	4.81	25.4	22.82	0.0424	53376	54052.88	1.012681
	23	152.4	6.04	25.4	20.34	0.0335	44480	46787.23	1.051871
	24	152.4	7.2	25.4	21.24	0.02633	41811.2	41975.9	1.003939
	25	152.4	7.24	25.4	19.51	0.0335	39587.2	43147.01	1.089923
	26	152.4	7.3	25.4	19.1	0.0429	42256	45242.64	1.07068
	27	152.4	8.4	25.4	21.03	0.02633	36918.4	38863.1	1.052676
28	152.4	8.45	25.4	21.93	0.0335	41811.2	40596.16	0.97094	

	29	152.4	8.52	25.4	21.37	0.0429	40476.8	42500.45	1.049995
	30	152.4	3.62	25.4	20.13	0.0335	51152	54432.55	1.064133
	31	152.4	3.57	25.4	34.54	0.0198	55600	55334.82	0.995231
	32	152.4	3.6	25.4	29.2	0.0263	57824	55408.6	0.958228
	33	152.4	3.62	25.4	32.82	0.0335	56934.4	59012.08	1.036493
	34	152.4	3.65	25.4	34.4	0.0429	60048	62785.84	1.045594
	35	152.4	4.77	25.4	31.85	0.0198	53376	50428.95	0.944787
	36	152.4	4.8	25.4	30.47	0.02633	53820.8	51794.16	0.962345
	37	152.4	4.82	25.4	32.82	0.0335	54265.6	54709.03	1.008171
	38	152.4	4.86	25.4	34.13	0.0429	59158.4	58071	0.981619
	39	152.4	6	25.4	38.4	0.0263	52486.4	50494.86	0.962056
	40	152.4	6.03	25.4	37.44	0.0335	57379.2	52200.57	0.909747
	41	152.4	7.24	25.4	37.44	0.0335	53376	48363.67	0.906094
	42	152.4	7.3	25.4	33.78	0.0429	48926	49674.29	1.015294
	43	152.4	3.62	25.4	15.37	0.0335	50262.4	52807.96	1.050645
	44	152.4	7.3	25.4	36.26	0.0429	53376	50464.71	0.945457
	45	152.4	7.3	25.4	36.26	0.0429	53376	50464.71	0.945457
	46	152.4	6	25.4	35.71	0.0256	48483.2	49439.02	1.019714
	47	152.4	6	25.4	39.02	0.0256	52486.4	50491.76	0.961997
<b>Gaston, J. R. Slese, C. P. and Newmark, N. M. (1952)</b>	48	152.4	3.42	25.4	30.82	0.01395	53153.6	52756.69	0.992533
<b>Places and Regan (1971)</b>	49	152.4	3.36	19.05	26.2	0.041	52486.4	53853.53	1.026047
<b>Diaz de Cossio, Roger and Siess, Chester P. (1960)</b>	50	152.4	4.02	25.4	28	0.0335	53376	55802.69	1.045464
	51	152.4	6.04	25.4	27.92	0.0335	50929.6	49100.13	0.964078
<b>Moody, K. G. , et al (1954)</b>	52	152.4	3.41	25.4	36.68	0.0189	57824	56371.67	0.974884
	53	152.4	3.41	25.4	25.7	0.0189	52264	52566.14	1.005781
	54	152.4	3.41	25.4	30.8	0.0189	51152	54300.69	1.061555
	55	152.4	3.41	25.4	41.1	0.0189	53376	57980.28	1.086261
	56	152.4	3.41	25.4	23.9	0.0189	48928	51967.27	1.062117
	57	152.4	3.41	25.4	38.1	0.0189	60048	56883.54	0.947301



	58	152.4	3.41	25.4	20.2	0.0189	47148.8	50757.59	1.07654
	59	152.4	3.41	25.4	37.7	0.0189	55600	56738.88	1.020483
	60	152.4	3.41	25.4	37.3	0.0189	51152	56594.59	1.1064
Mathey, Robert G. and Watsein, David (1963)	61	203.2	3.78	25.4	26.3	0.0046	54488	52188.92	0.957806
	62	203.2	3.78	25.4	25.78	0.0046	49928.8	52016.45	1.041813
	63	152.4	4.36	19.05	20.3	0.02	41597.6	43179.87	1.038038
	64	152.4	4.9	19.05	17.87	0.02	39854.1	41092.88	1.031083
Ahmed, Shuaib H. Khaloo, A. R. and Proveda, A. (1986)	65	127	4	12.7	66.12	0.0395	57824	56578.65	0.978463
	66	127	4	12.7	66.12	0.0173	46704	49825.1	1.066827
	67	127	3	12.7	66.12	0.0173	48928	53070.09	1.084657
	68	127	2	12.7	66.12	0.0173	55600	56526.42	1.016662
	69	127	3	12.7	72.8	0.0503	68944	66892.53	0.970244
	70	127	2.3	12.7	72.8	0.0219	64051.2	59420.59	0.927705
	71	127	2.3	12.7	69.9	0.0324	56934.4	61948.38	1.088066
	72	152.4	2.4	19.05	27.23	0.026	51441.12	52852.68	1.02744
	73	154.43	2.66	19.05	26.61	0.0263	50151.2	52072.62	1.038313
	74	154.94	2.93	19.05	26.75	0.0264	51152	51316.55	1.003217
Kani, G. N. J. (1966)	75	152.4	1.48	19.05	28	0.00496	54710.4	49898.27	0.912044
	76	152.6	2.49	19.05	18	0.0077	43657.1	44641.5	1.022548
	77	152.6	2.48	19.05	18	0.0077	46170.2	44669.68	0.9675
	78	153.4	2.01	19.05	25.3	0.0076	48794.5	48246.79	0.988775
	79	152.4	2.5	19.05	26.2	0.0077	45302.8	46986.5	1.037165
	80	152.4	3	19.05	20.34	0.0185	48928	46658.21	0.95361
	81	155.4	2.98	19.05	16.27	0.0178	42700.8	45593.17	1.067736
	82	153.4	5.4	19.05	18	0.0178	40032	39423.92	0.98481
	83	151.13	2.48	19.05	18.13	0.0185	49550.7	47429.06	0.957182
	84	153.6	2.5	19.05	18.4	0.01814	51952.6	47570.41	0.91565
	85	152.4	2.5	19.05	29.786 4	0.018727	51374.4	51204.55	0.996694
	86	152.4	2.5	19.05	29.165 85	0.018727	54265.6	51002.68	0.939871
	87	152.4	4.5	19.05	24.546 2	0.018727	42923.2	43653.43	1.017013

	88	152.4	4.5	19.05	25.166 75	0.018727	46259.2	43826.22	0.947405
	89	155.19 4	3.5	19.05	26.063 1	0.018202	44924.8	47049.85	1.047302
	90	154.68 6	5	19.05	33.923 4	0.017852	48750.08	44864.89	0.920304
	91	153.41 6	2.44	19.05	34.612 9	0.018026	56712	52889.67	0.932601
	92	153.92 4	2.93	19.05	34.612 9	0.017966	51152	51310.23	1.003093
	93	153.16 2	3.94	19.05	34.612 9	0.018205	47260	48140.32	1.018627
	94	150.36 8	2.48	19.05	35.991 9	0.018351	53376	53020.1	0.993332
<b>Chana, P. S. (1981)</b>	95	150.11 4	3	14.986	45.507	0.016184	46383.74	50199.06	1.082256
	96	199.89 8	3	19.989 8	39.701 41	0.017968	54995.07	58394.09	1.061806
<b>Elzanty, Ashraf H., Nilson, Arthur H. (1986)</b>	97	199.89 8	3	19.989 8	39.701 41	0.017968	55995.87	58394.09	1.042829
	98	177.8	4	12.7	20.685	0.012016	44480	39775.21	0.894227
	99	177.8	4	12.7	39.991	0.012016	46259.2	44976.44	0.97227
	100	177.8	4	12.7	65.502 5	0.012016	58268.8	52906.99	0.907982
	101	177.8	6	12.7	63.434	0.012016	43590.4	46024.74	1.055846
								Mean	1.003
								Coe.of variation	20.90%

Where:

b in mm, a/d in mm/mm, da in mm, fc' in Mpa, ρ in mm<sup>2</sup>/mm<sup>2</sup>, Vc in N,

Vc exp=experimental shear strength value, Vc cal= calculated by presented shear strength model.

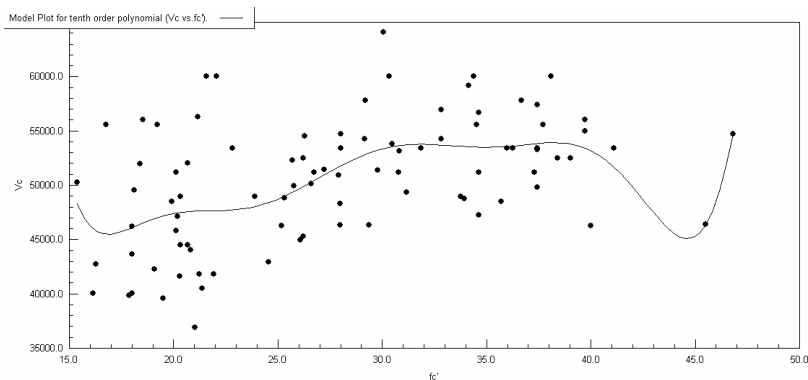


Fig1: Model Plot for Tenth Order polynomial Equation(Vc vs.fc)

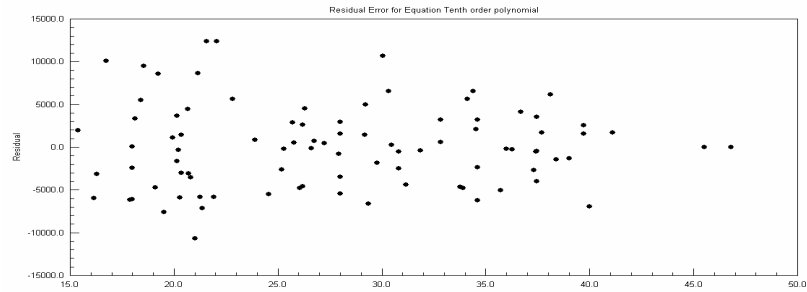


Fig2: Residual Scatter plot

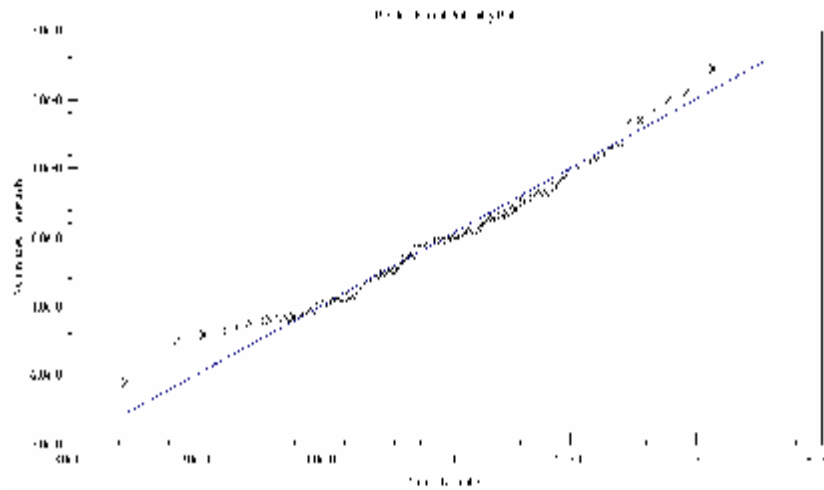


Fig3: Residual Normal Probability

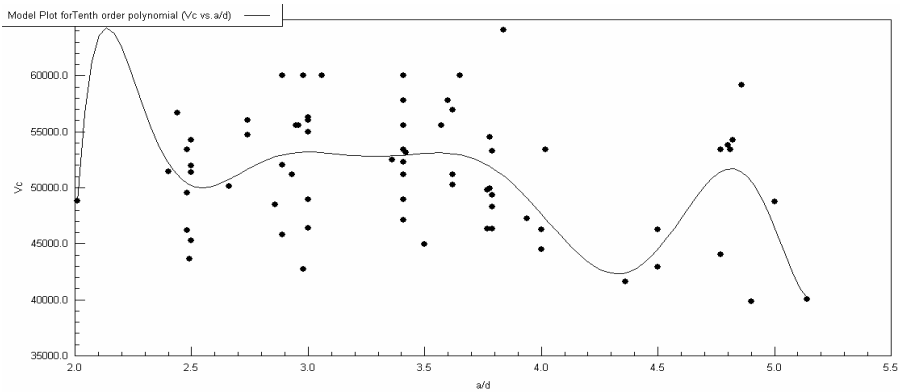


Fig4: Model Plot for Tenth Order Polynomial ( $V_c$  vs.  $a/d$ )

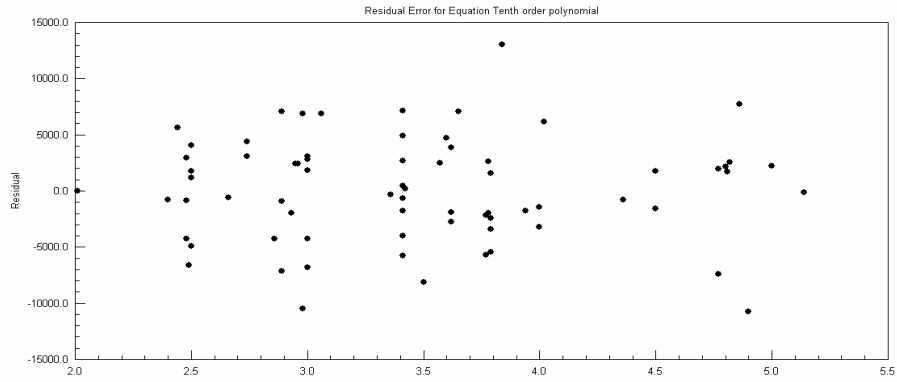


Fig5: Residual Scatter Plot

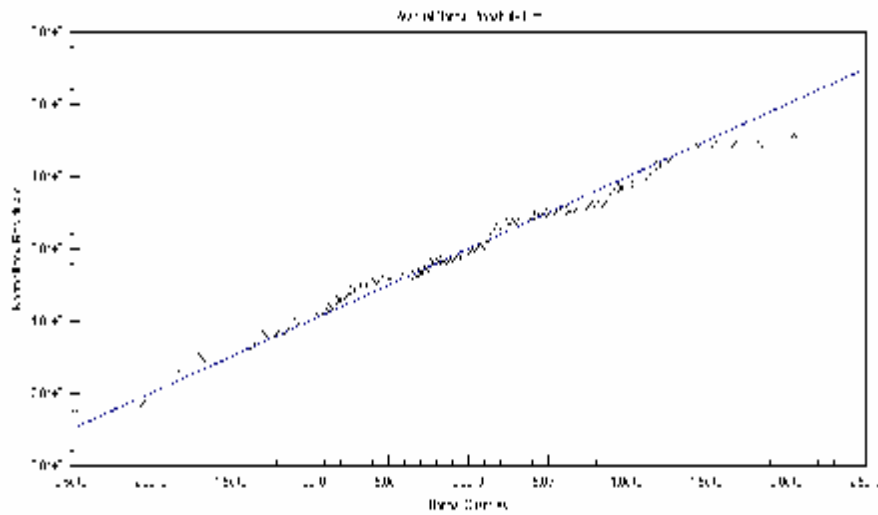


Fig6: Residual Normal Probability Plot

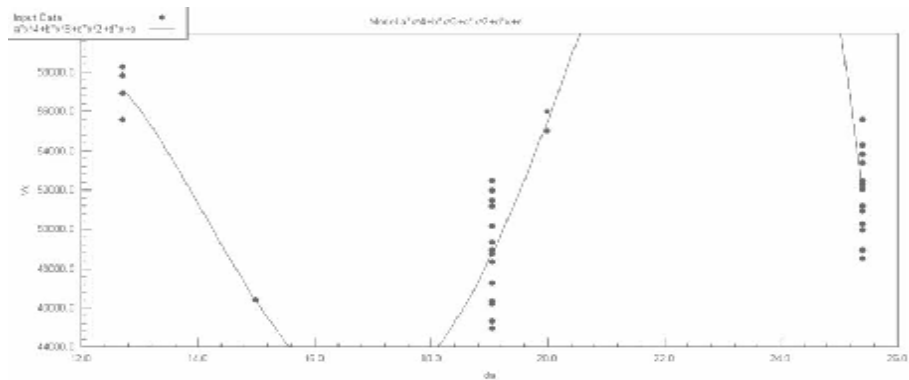


Fig7: Model Plot for Fourth order logarithm (Vc vs. da)

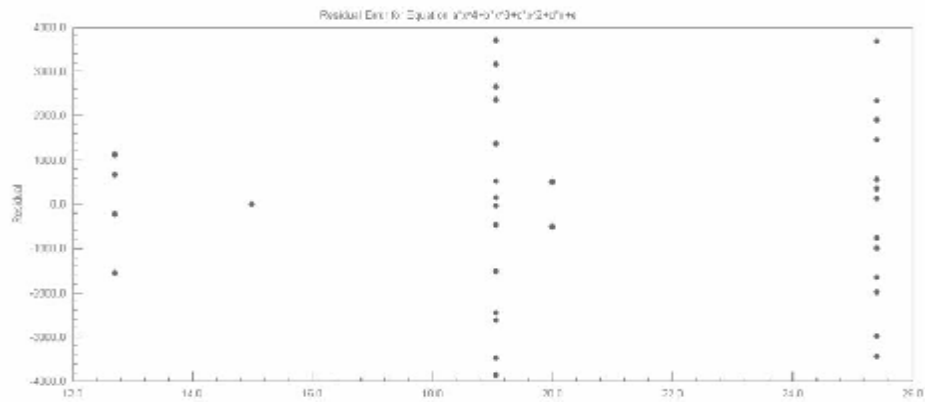


Fig8: Residual Scatter Plot

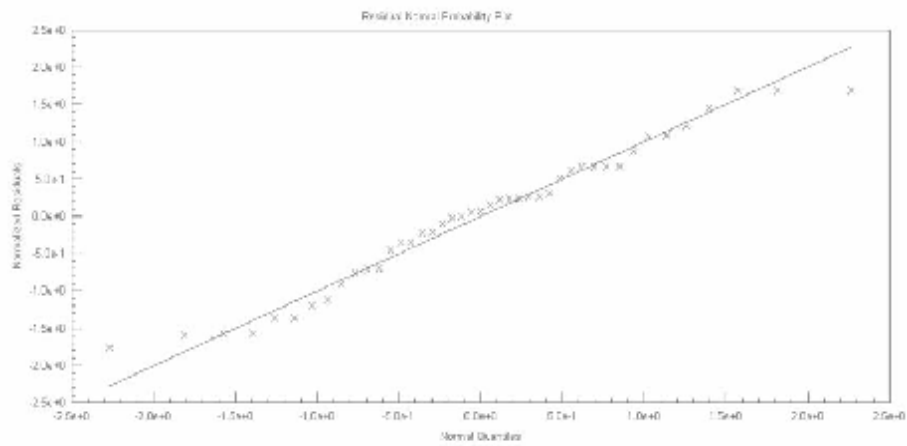


Fig.9: Residual Normal Probability Plot

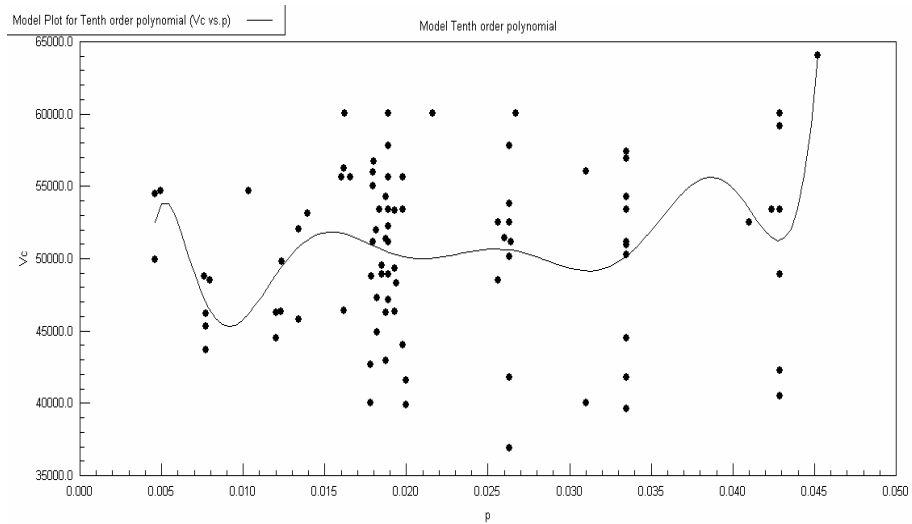


Fig.10: Model Plot for Tenth Order Polynomial ( $V_c$  vs.  $\rho$ )

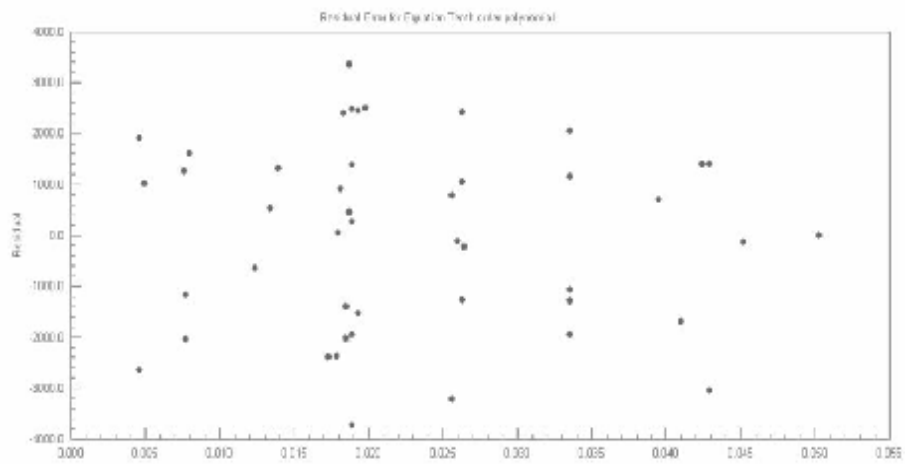


Fig.11: Residual Scatter Plot

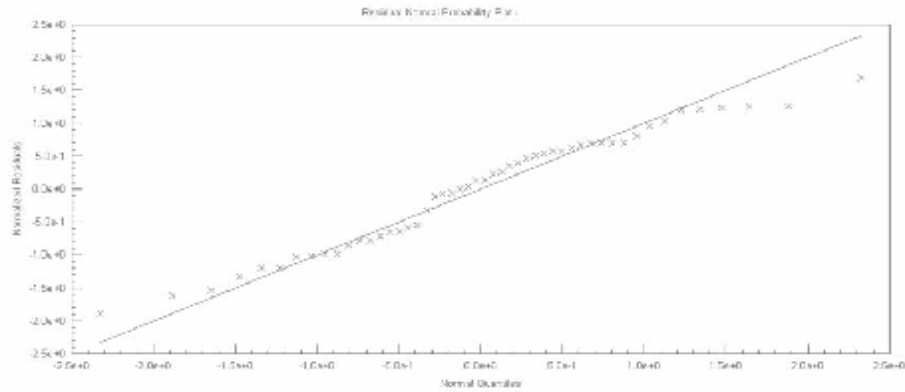


Fig.12: Residual Normal Probability Plot.

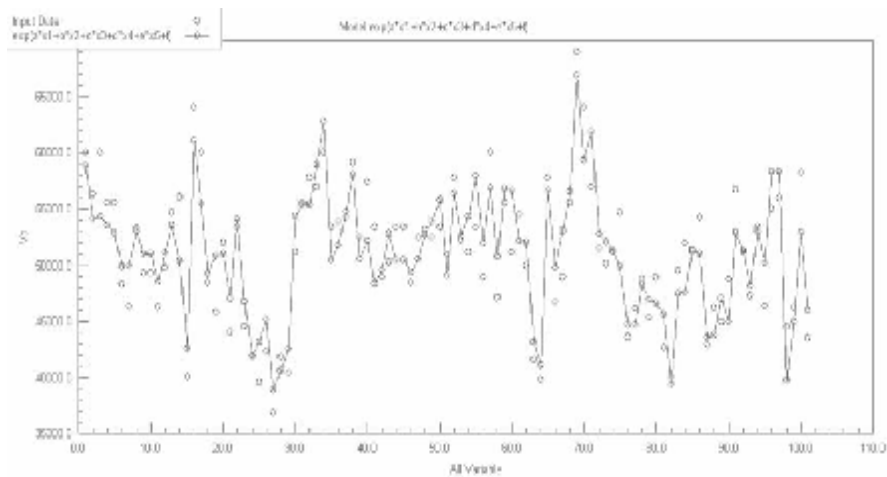


Fig.13: Model Plot of shear strength included all factors.

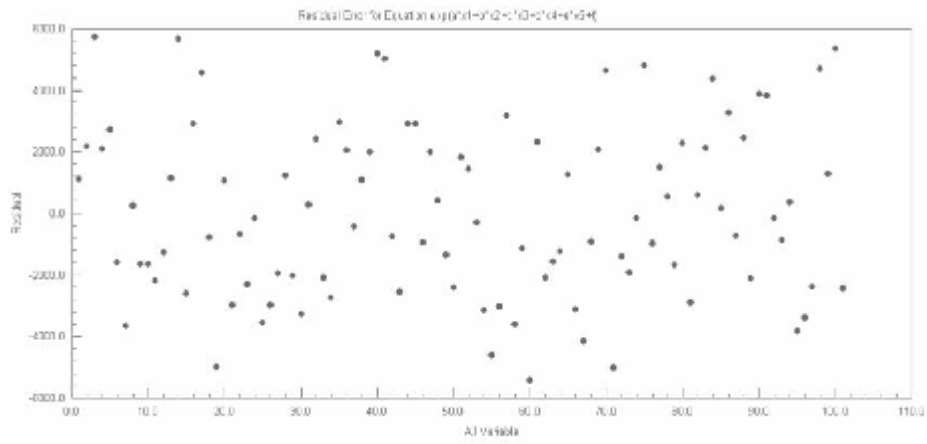


Fig.14: Residual Scatter Plot

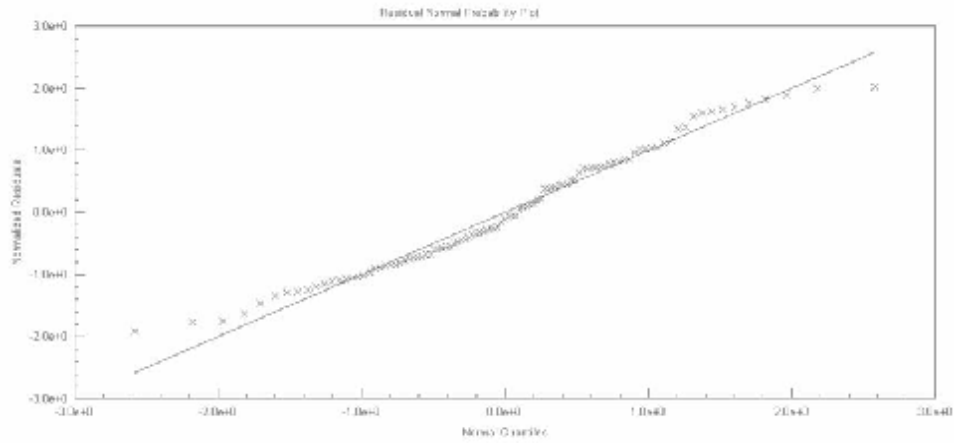


Fig.15: Residual Normal Probability Plot