

Evaluation Study of Alexander And Abramowicz Absorption Energy Models Using Thin Metallic Tubes

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Abstract

To estimate the models of Alexander and Abramowicz Aluminum alloy AISI 5052 tubes specimens are used under impact loading and room temperature. Two types of specimens are used the circular and square cross – section tubes with equal wall cross – sectional area. The main conclusions that can be drawn from this work are as follows :

- This study showed that the models of Alexander and Aromawicz are valid for the aluminum alloy.
- .The circular section specimens absorbed energy greater than the square cross section specimens.

دراسة تقييم نماذج الكسندر و برامكس لامتصاص الطاقة باستخدام الأنابيب المعدنية الرقيقة

الخلاصة

لتقييم نماذج الكسندر وبرمكس تم استخدام عينات سبيكة الالمينيوم 5052 الأنبوبية تحت تحميل الصدمة ودرجة حرارة الغرفة. العينات الأنبوبية كانت على شكل دائرية المقطع و مربعة المقطع. توصلت هذه الدراسة إلى الاستنتاجات المهمة التالية :
_ هذه الدراسة توصلت إلى أن نماذج الكسندر و برمكس صحيحة لسبيكة الألمنيوم .
العينات ذات المقاطع الدائرية تمتص طاقة أكبر من العينات ذات المقاطع المربعة.

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<u>Sym</u>	<u>Meaning</u>	<u>Unit</u>
R	radius of circular tube	mm
D	diameter of circular tube	mm
c	length of square side	mm
h	Wall thickness	mm
L	length	mm
P_m	Theoretical static crushing load	KN
^s _M P	Experimental static crushing load	KN
M_y	Plastic bending moment	N.m
σ_y	yield stress	N/m ²
σ_u	Ultimate tensile stress	Mpa
^d _m P	Theoretical mean dynamic crushing load	KN
V	Impact velocity of striking mass	m/s
^d _y σ	Dynamic yield stress	N/m ²
ε^o	Mean strain rate	-
Z	Material constant	S ⁻¹
P	Material constant	S ⁻¹
E	Elastic modulus	Gpa
G	Shear modulus	Gpa
μ	Poissons ratio	-
wt	Weight percentage	N
t	Time	Sec
H	Height(distance between two stations)	mm
g	Acceleration of gravity	m/s ²
E_s	Energy absorption (statically)	J
En	Predicted theoretical energy absorption (statically)	J
δ	Reduction in axial	mm

	length in static experimental test	
δ_f	Reduction in axial length in dynamic experimental test	mm
K.E	Intial kinetic energy	J
A	Wall cross – sectional area	m ²
A₁	Cross – sectional area	m ²
η	Structural effectiveness	-
Ø	Solidity ratio	-
X	Constant	-
B	Constant	-
β	Constant	-
η	Constant	-
C	Constant	-
γ	Constant	-

1.1 Introduction:

Impact refers to the collision of two masses with initial velocity. In some cases it is desirable to achieve a known impact design ;for example ,this is the case in the design of coining , stamping ,and forming presses. In other cases, impact occurs because of excessive deflections ,or because of clearances between parts, and in these cases it is desirable to minimize the effects.

Shock is a more general term which is used to describe any suddenly applied force or disturbance . Thus the study of shock , depending upon whether only static's is used in the analysis or both static and dynamic are used. After impact energy absorbed by two impact masses [1,2]

1.2The Energy Absorption

Method:

Many mechanical devices and elements were designed to absorb impact energy. These devices can be

used to protect the vehicles and air crafts at high speed [3]. Energy absorption can be classified into:

- a) Energy absorbing by friction such as brakes of the cars and trains.
- b) Energy absorbing by deformation of metallic components.

This method depends on:

1. elastic Deformation: Such as metal springs which are used in vehicles.

2. Plastic Deformation: Such as wires, bars , frames and tubes. The load deflection characteristics and the energy – absorbing capacity differ from one component to the next in manner which

2. Experimentale Work:

2.1 Introduction:

A series of 24 axial crushing tests were conducted on circular and square sectioned aluminum tubes specimens loaded either statically or dynamically. There are two type of testing namely quasi-static test and dynamic test.

2.2 Material:

5052 Aluminum alloy was used in the present study. This metal is widely used in aircraft structures and in many particle of automobiles.

2.2.1 Mechanical Properties:

Table(1) shows the mechanical properties of the metal used. The data is the average of three readings.

2.2.2 Chemical Composition:

Table(2) illustrate the chemical composition of Aluminum alloy 5052 in weight percentage.

2.3 Specimen Preparation:

A seamless circular and square sectioned aluminum alloy tubes formed by deep drawing process were used. These tubes were cut to equal length by cutter machine. The thickness of circular tubes was reduced by machining them from

(1.51 mm) to (1.21 mm) to be equal the thickness of the square tubes. Then annealing was performed of all specimens (circular and square) at 250 C^o for one hour and cooled slowly in the furnace to room temperature inside the furnace [4].

2.3.1 Specimens:

Fig (1) shows the shape and dimensions of the two types of specimens used in this work [5].

2.4 Quasi – Static Test:

The circular and square tubes were crushed using universal testing machine. All specimens tested at constant cross-head speed 5mm/min with strain-gauged-load capacity (50 Kg). The reduction in axial length of specimens progressed until failure. The failure is defined to be complete buckling of specimens.[5]

2.5 Dynamic Test:

All the dynamic test were carried out by using a drop weight of (29 Kg) impact on the specimens from different heights (1-3 m). The details of the impact test rig can be described elsewhere.[6]

2.6 The Velocity Measurement system:

A timer was used to measure velocity of the drop mass in dynamic test. It consist of two stations , the first station is the start movement and the other is the stop movement. Each station has two terminals of wire the terminals of start movement were placed under the moving flat cylinder head at any height. When the flat cylinder head passes, it will contact the first two wires which will start the counting. A similar event will stop the counting. Time (t) is taken by the flat cylinder head pass from the first to the second station . Then the velocity of the falling mass can be calculated by the flowing equation:

$$V = \frac{H}{t} \quad \dots\dots(1)$$

The theoretical velocity is calculated by the following equation:

$$V = \sqrt{2gH} \quad \dots\dots(2)$$

g = 9.81 m/s² (acceleration gravity)

3.Theoretical Models:

3.1Circular Tube:

3.1.1Quasi- static and dynamic condition of circular tube:

I-a Alexander model:

The mean crushing load under quasi-static condition is illustrated by the following equation[3]:-

$$\frac{P_m}{M_y} = K \left(\frac{2R}{h} \right)^{\frac{1}{2}} \quad \dots\dots(3)$$

(assumed of the fully plastic bending moment per unit width),which can be described by the following equation , using the mises criterion[3]:

$$M_y = \frac{2}{\sqrt{3}} S_y \frac{h^2}{4} \quad \dots\dots (4)$$

I-b) Abramowicz and Jones model:

The mean load under quasi-static conditions is given by the following formula:-

$$\frac{P_m}{M_y} = \frac{24734(2R/h)^{\frac{1}{2}} + 11685}{0.85 - 0.568(h/2R)^{\frac{1}{2}}} \quad \dots\dots(5)$$

The mean dynamic crumpling load of symmetric crushing mode was calculated by Abramowicz and Jones by the following equation[9]:

$$\frac{P_m^d}{M_y} = \left(\frac{24.734 \left(\frac{2R}{h} \right)^{\frac{1}{2}} + 11.658}{0.86 - 0.568 \left(\frac{h}{2R} \right)^{\frac{1}{2}}} \right)$$

$$* \left(1 + \left(\frac{0.25 V}{6500 R \left(0.86 - 0.568 \left(\frac{h}{2R} \right)^{\frac{1}{2}} \right)} \right)^{\frac{1}{4}} \right) \quad \dots\dots(6)$$

The derivation of equation (5) and (6)is given elsewhere[8]:

I-C)The Present Empirical model:

The collapse load under quasi-static condition is function of plastic bending moment and mean diameter to thickness ratio as [3].The constant **A** and **α** were obtained based on experimental results.

$$\frac{P_m}{M_y} = A \left(\frac{2R}{h} \right)^{\alpha} \quad \dots\dots\dots(7)$$

where A=45.43 and α = 0.45 .

$$\frac{P_m}{M_y} = 45.43 \left(\frac{2R}{h} \right)^{0.45} \quad \dots\dots\dots(8)$$

Under the dynamic condition ,the material shows a sensitivity to the strain rate and the simple empirical expression was found by cowper-symonds [9].which is widely used to assess material strain rate effects in structures as follows :

$$\frac{S_y^d}{S_y} = 1 + \left(\frac{e^o}{Z} \right)^{\frac{1}{P}} \quad \dots\dots\dots(9)$$

Where (Z) and (P) are constant depended on the material used (Z=6500 s⁻¹) and (P=4) for aluminum alloys [9,10].

While under dynamic state. It is assumed that the deformation mode remains unchanged for dynamic crushing so that the associated load

(P_m^d) is estimated [11]. From equation (8 and 9).

$$\frac{P_m^d}{P_m} = 1 + \left(\frac{e^o}{6500} \right)^{\frac{1}{4}} \dots\dots\dots(10)$$

The strain rate in axially buckled of circular tube is a function of impact velocity of striking mass to mean radius ratio [9] as follows:

$$e^o = X \frac{V}{R} \dots\dots\dots(11)$$

From equation (8,10 and 11) the following equation can be produced

$$\frac{P_m^d}{M_y} = 45.43 \left(\frac{2R}{h} \right)^{0.45} \left(1 + \left(\frac{XV}{6500R} \right)^{\frac{1}{4}} \right) \dots\dots\dots(12)$$

Where (X=1.1) constant evaluated from the correlation of the experimental data

$$\frac{P_m^d}{M_y} = 45.43 \left(\frac{2R}{h} \right)^{0.45} \left(1 + \left(\frac{1.1V}{6500R} \right)^{\frac{1}{4}} \right) \dots\dots\dots(13)$$

II. Non - Symmetric (diamond) Collapse Mode:

The mean crushing force under quasi - static and dynamic conditions can be observed by the following models equations:

II.b) Abramowicz and Jones Model:

The mean buckling load under quasi-static condition can be observed by the following equation:

$$\frac{P_m}{M_y} = 86.14 \left(\frac{2R}{h} \right)^{\frac{1}{3}} \dots\dots\dots(14)$$

The mean dynamic crushing load for non-symmetric crushing mode was calculated by Abramowicz and Jones [12] as follows:

$$\frac{P_m}{M_y} = 86.14 \left(\frac{2R}{h} \right)^{\frac{1}{3}} \left(1 + \left(\frac{0.37V}{6500R} \right)^{\frac{1}{4}} \right) \dots\dots\dots(15)$$

II-C)The Present empirical model:

The crushing load under static condition is a function of plastic bending moment and mean diameter to thickness ratio as is [11]:

$$\frac{P_m}{M_y} = B \left(\frac{2R}{h} \right)^b \dots\dots\dots(16)$$

where B = 87.71 and $b = \frac{1}{3}$ (empirical data) used in this work . Thus

$$\frac{P_m}{M_y} = 87.71 \left(\frac{2R}{h} \right)^{\frac{1}{3}} \dots\dots\dots(17)$$

The mean dynamic crushing load for non - symmetric crushing mode can be obtained

from equation (10, 11 and 17) as the following :

$$\frac{P_m^d}{M_y} = 87.71 \left(\frac{2R}{h} \right)^{\frac{1}{3}} \left(1 + \left(\frac{XV}{6500R} \right)^{\frac{1}{4}} \right) \dots\dots\dots(18)$$

Where X = 0.025 material constant can be found from present work., the equation of mean dynamic crushing load is:-

$$\frac{P_m^d}{M_y} = 87.71 \left(\frac{2R}{h} \right)^{\frac{1}{3}} \left(1 + \left(\frac{0.025V}{6500R} \right)^{\frac{1}{4}} \right) \dots\dots\dots(19)$$

3.3 Square Tube:

3.3.1 Quasi-static and dynamic condition of square tube:

I. Symmetric Collapse Mode:

The mean collapse force under quasi-static and dynamic condition can be illustrated by the following modes with equation:-

I-a) Wierzbicki and Abramowicz Model:

The mean crushing load under quasi-static condition can be described by the following formula:-

$$\frac{P_m}{M_y} = 38.12 \left(\frac{c}{h} \right)^{\frac{1}{3}} \dots\dots(20)$$

$$M_y = S_y \frac{h^2}{4} \dots\dots(21)$$

c: mean length of square length ,mm

I.b) Abramowicz and Jones Model:

The mean buckling load under quasi-static condition can be observed by the following equation:

$$\frac{P_m}{M_y} = 52.22 \left(\frac{c}{h} \right)^{\frac{1}{3}} \dots\dots(22)$$

The mean dynamic crushing load of symmetric crushing mode was calculated by Abramowicz and Jones by the following equation :

$$\frac{P_m^d}{M_y} = 52.22 \left(\frac{c}{h} \right)^{\frac{1}{3}} \left(1 + \left(\frac{0.33V}{6500c} \right)^{\frac{1}{4}} \right) \dots\dots(23)$$

I-C)The Present empirical model:

The mean crushing load under quasi-static condition is function of plastic bending moment and mean length of square side to thickness ratio as follow[13]:

$$\frac{P_m}{M_y} = C \left(\frac{c}{h} \right)^g \dots\dots(24)$$

Where (C = 56.25) and (γ = 0.3) material constant evaluated from the experimental data, Thus

$$\frac{P_m}{M_y} = 60.29 \left(\frac{c}{h} \right)^{0.3} \dots\dots(25)$$

Where the dynamic condition of square tube, the strain rate is a function of impact velocity of striking mass to mean length of square side ratio [5]thus,

$$e^o = F \frac{V}{c} \dots\dots(26)$$

from equation (10 ,24 and 25) can be produced the following equation:-

$$\frac{P_m^d}{M_y} = 60.29 \left(\frac{c}{h} \right)^{0.3} \left(1 + \left(\frac{FV}{6500c} \right)^{\frac{1}{4}} \right) \dots\dots(27)$$

Where F = 2.6 material constant evaluated from the present work. Thus

$$\frac{P_m^d}{M_y} = 60.29 \left(\frac{c}{h} \right)^{0.3} \left(1 + \left(\frac{2.6V}{6500c} \right)^{\frac{1}{4}} \right) \dots\dots(28)$$

II. Asymmetric Collapse Mode:

The mean crumpling force under quasi-static and dynamic condition can be illustrated by the following modes with equation:-

II-a) Wierzbicki and Abramowicz Model:

The mean buckling load under quasi-static condition can be described by the following formula[13] :-

$$\frac{P_m}{M_y} = 38.12 \left(\frac{c}{h} \right)^{\frac{1}{3}} + 2.92 \left(\frac{c}{h} \right)^{\frac{2}{3}} + 2 \dots\dots(29)$$

II.b) Abramowicz and Jones Model:

The mean collapse load under quasi-static condition as follows:

$$\frac{P_m}{M_y} = 43.61 \left(\frac{c}{h}\right)^{\frac{1}{3}} + 3.79 \left(\frac{c}{h}\right)^{\frac{2}{3}} + 2.6 \dots\dots\dots(30)$$

The mean dynamic crushing load of asymmetric crushing mode was found by the following equation :

$$\frac{P_m^d}{M_y} = \left(43.61 \left(\frac{c}{h}\right)^{\frac{1}{3}} + 3.79 \left(\frac{c}{h}\right)^{\frac{2}{3}} + 2.6 \right) * \left(1 + \left(\frac{0.49V}{6500c}\right)^{\frac{1}{4}} \right) \dots\dots\dots(31)$$

II-C)The Present Empirical model:

The mean crushing load under quasi - static condition can be described by the following equation:

$$\frac{P_m}{M_y} = x \left(\frac{c}{h}\right)^{\tau} \dots\dots\dots(32)$$

where (x = 58) and (τ = 1/3) material constant evaluated from present work:

$$\frac{P_m}{M_y} = 58 \left(\frac{c}{h}\right)^{\frac{1}{3}} \dots\dots\dots(33)$$

The mean dynamic crushing load from non - symmetric crushing mode can be obtained from equations (10 , 25 and32).

$$\frac{P_m^d}{M_y} = 58 \left(\frac{c}{h}\right)^{\frac{1}{3}} \left(1 + \left(\frac{FV}{6500c}\right)^{\frac{1}{4}} \right) \dots\dots\dots(34)$$

and for the present work (F) can be obtained from the experimental work to be (2).Thus,

$$\frac{P_m^d}{M_y} = 58 \left(\frac{c}{h}\right)^{\frac{1}{3}} \left(1 + \left(\frac{2V}{6500c}\right)^{\frac{1}{4}} \right) \dots\dots\dots(35)$$

4.EXPEIMENTAL REULTS:

4.1 Circular tube:

4.1.1 Quasi- static Results:

Table (3) illustrates the static compression data of circular tubes at cross head speed of (5 mm / min),and strain rate(8.3*10⁻⁴ s⁻¹).

The energy absorbed by the specimen is calculated from the load deflection diagram which is shown in Fig (2) and Fig (3),two modes of deformation are distinguished for circle tube is **Concertina or Axisymmetric deformation.**

The other mode of deformation is denoted by the mixed mode (**Concertina and diamond**) behavior.

4.1.2 Dynamic Results :

An experimental data from the dynamic test on circular tubes is presented in table (4). δ_f is the final reduction in axial length of the test specimen and the average dynamic force (P_m^d) which is defined as the initial kinetic (K.E)

Three modes of deformation re illustrated in dynamic test on circular tubes in table (4).There are **Concertina** (Axisymmetric) , **mode**(non-Axisymmetric), and **mixed mode** (concertina and diamond).

4.2 Square – Static Results:

4.2.1 Quasi- static Results:

Table (5) shows the compression results of square tubes under same condition

The energy absorbed by the specimen is calculated from the load

deflection curve which observed in Fig (4.) and Fig (5) , two modes of deformation are illustrated in table (5) ,**symmetric and asymmetric**

4.1.2 Dynamic Results :

A experimental data from the dynamic test on circular tubes is illustrated in table (4). δ_f is the final reduction in axial length of the a test specimens and the average dynamic force (P_m^d) which is defined as the initial kinetic (K.E).

Three modes of deformation re illustrated in dynamic test on circular tubes in table (4).

,**symmetric** deformation , the other modes of deformation are **asymmetric**, and **mixed mode** (symmetric and asymmetric) collapse deformation.

5. Conclusion:

1.The absorbing energy of circular cross –section specimens is better than the square cross –section specimens.

2.The static and dynamic load for circular cross –section is greater than square one. 3

3. The absorbing energy mainly depends on the manner of deformation and shape of the cross – section.

4.The application of Alexander and Abramowicz models is valid for (AL 5052), alloy.

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Table(1) The mechanical properties of Aluminum Alloy 5052

Designation	σ_y Mpa	σ_u Mpa	ϵ %	Hardness (Kg/mm ²)		G (Gpa)	E (Gpa)	μ
				BRINELL	VICKER			
Experimental	101	198	27.5	45	63	27	70	0.29
AISI Standard	90	193	30	45	62	30	71	0.29

Table(2) chemical composition of Aluminum alloy 5052

AL	Fe	Si	Cu	Mn	Mg	Cr	pb	Ni	TI	Zn
96.7	0.312	0.118	0.045	0.038	2.21	0.178	0.003	0.007	0.024	0.03

Table(3) Static compression properties of circular tubes

Specimen NO Circular tube	E_s (J)	δ (mm)	$P_m^s = E_s \delta$ (kN)	Mode of deformation
111	806.53	78.5	10.27	Concertina
112	1009.3	79	12.77	Mixed mode (Concertina and diamond)

**Table (4).Dynamic test results of 5052 Aluminum alloy circular tubes under
(29) Kg dropping mass**

Specimen NO	V (m/s)	K.E (J)	δ_f (mm)	$P_m^d = K.E / \delta_f$ (kj)	Mode of deformation
11	7.6	848.57	56	15.15	diamond
12	7.23	7759.2	52.1	14.58	diamond
13	7.06	723.38	50.2	14.4	diamond
14	6.76	664.18	40.1	39.4	concertina and diam
15	6.48	610.55	39.4	15.5	concertina and diam
16	6.26	569.67	38	15	concertina and diam
17	5.945	512.47	34	15.1	concertina
18	5.513	440.7	29.3	15	concertina
19	5.198	391.77	27.7	14.4	concertina
110	4.87	343.89	25.2	13.65	concertina

Table (5) Monotonic compression data on square tubes

SpecimenNO	E_s (J)	δ (mm)	$P_m^s = E_s / \delta$ (kN)	Mode of deformation
R11	522.03	79	6.6	Symmetric
R12	537.2	79	6.8	a Symmetric

**Table (6).Dynamic test results of 5052 Aluminum alloy square tubes under
(29) Kg dropping mass**

Specimen NO	V (m/s)	K.E (J)	δ_f (mm)	$P_m^d = K.E / \delta_f$ (kN)	Mode of deformation
R1	7.6	848.57	80	15.15	Symmetric
R2	7.23	7759.2	78	14.58	Symmetric
R3	7.06	723.38	76	14.4	Symmetric
R4	6.76	664.18	69.8	39.4	symmetric & asymmetric
R5	6.48	610.55	60.3	15.5	symmetric & asymmetric
R6	6.26	569.67	55.2	15	symmetric & asymmetric
R7	5.945	512.47	49.2	15.1	Symmetric
R8	5.513	440.7	48	15	Symmetric
R9	5.198	391.77	40.3	14.4	Symmetric
R10	4.87	343.89	37.1	13.65	Symmetric

Table (7) Comparison the energy absorption between circular and square tube statically

Circular tubes		Square tubes		Energy absorption percentage%
E_s (J)	Mode of deformation	E_s (J)	Mode of deformation	
806.537	Concertina	522.0	Symmetric	35
1009.369	Mixed mode	537,2	asymmetric	46.78

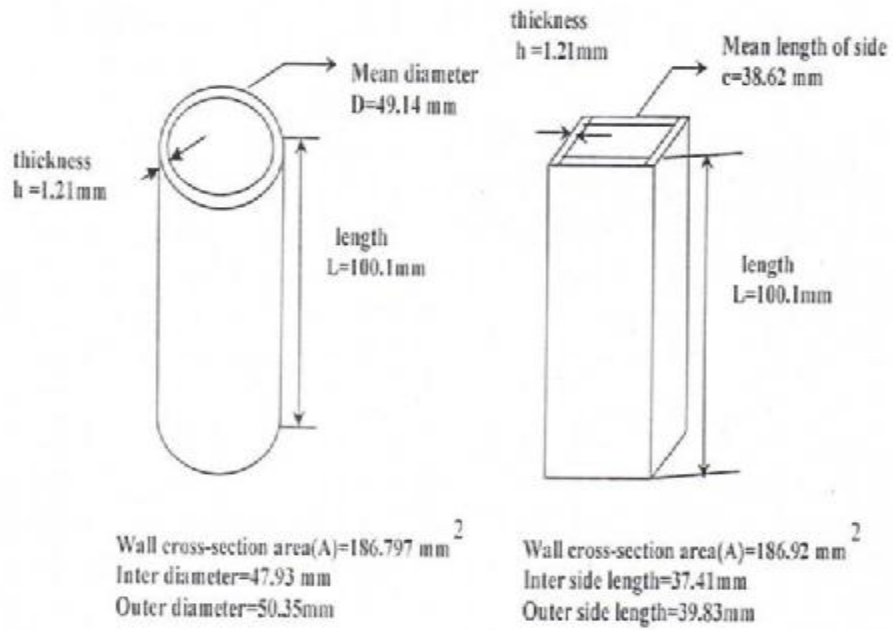


Fig. (1) Geometry of specimens

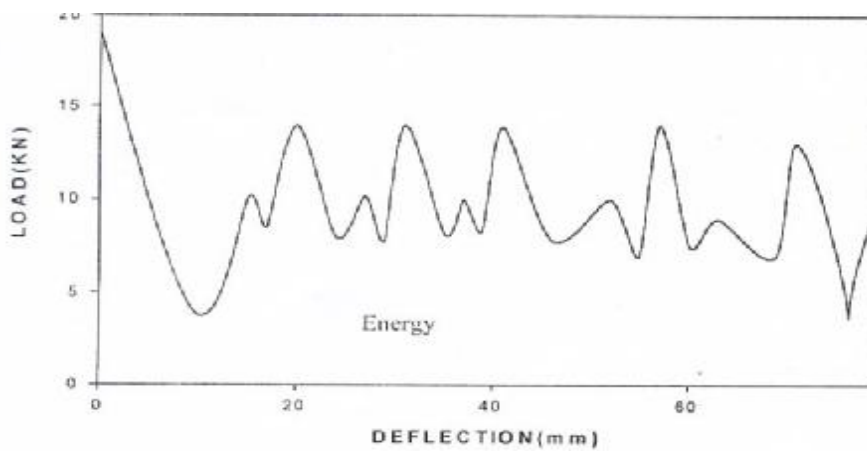


Fig (2) Static axial load versus crushing distance for test specimen No.111 in table 3

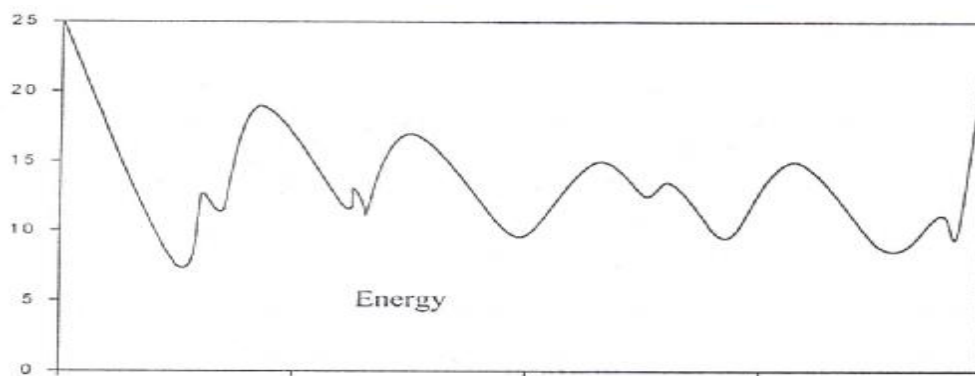


Fig (3) Static axial load versus crushing distance for test specimen No.11 in table 3

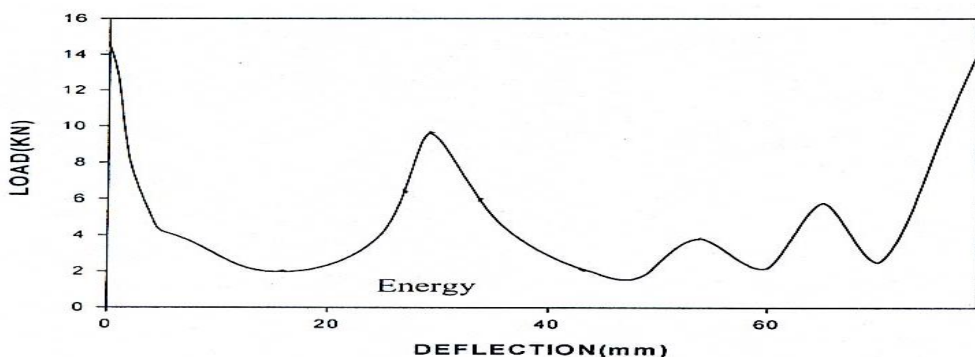


Fig (4) static axial load versus crushing distance for test specimen
No.R11 in table 5

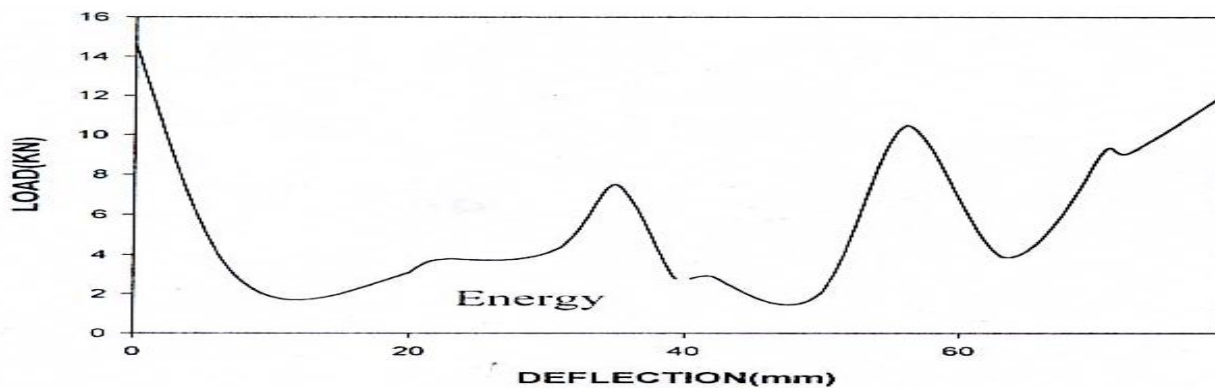


Fig (5) static axial load versus crushing distance for test specimen No.R12 in table 5