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Chattering Attenuation of Sliding Mode Controller Using Genetic Algorithm and Fuzzy Logic Techniques

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Abstract

Sliding Mode Controller design provides a systematic approach to the problem of maintaining stability and consistent performance in the face of modeling imprecision. The major drawback that sliding mode control suffers from is the chattering phenomenon, which is a zigzag motion along the sliding surface caused by the high frequency motion on the sliding surface. This phenomenon is an undesirable property since it excites unmodeled dynamics and results in tear and wears in the mechanical systems. In this work several methods are proposed to reduce the chattering. One of these methods is to use the boundary layer solution to smooth the hard switching signal. This solution is compared to another one represented by involving the intelligent systems to enhance the performance of the sliding mode controller system like involving the Genetic Algorithm (GA) and the fuzzy tuning technique. GA has proved its efficient ability to attenuate chattering and reduce the hitting time compared to other methods.

Keywords: Sliding Mode Control (SMC), Chattering Reduction, Genetic Algorithm , Fuzzy Logic Techniques.

توهين التذبذب لمسيطر النمط الانزلاقي بأستخدام تقنيات خوارزمية الجينات والمنطق الضبابي

الخلاصة

يمنح تصميم مسيطر النمط الأنزلاقي طريقةحل منظمة لمشكلة الحفاظ على الاستقرارية والأداء الثابت ضد الغموض وقلة الدقة في التمثيل الرياضي للنظام.العائق الوحيد الذي يعاني منه مسيطر النمط الأنزلاقي هو ظاهرة التذبذب والتي هي عباره عن حركة متعرجة على طول سطح الأنزلاق ذات تردد عالي على نفس السطح, تعتبر هذه الظاهرة غير مرغوبة ممكن ان تتسبب بأثارة ديناميكيات النظام غير الممثّلة رياضيا وتنتج عنها مشاكل كثيرة في الأنظمة الميكانيكية.في هذا العمل تم تقديم عدة طرق لتقليل التذبذب , واحدة من هذه الطرق هي استخدام طريقة طبقة الحدّ كحلّ من أجل صقل أشارة التحويل المتذبذبة. تمت مقارنة هذا الحل مع حل أخر متمثل بأستخدام الأنظمة الذكية من اجل تحسين أداء انظمة مسيطر النمط الأنزلاقي مثل استخدام خوارزمية الجينات وتقنية الموالفة الضبابية. اثبتت الخوارزمية الجينية كفاءتها في توهين التذبذب وتقليل الوقت اللازم لحالات النظام لتصل الى سطح الانزلاق.

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List of Abbreviation

AFSMC	Adaptive Fuzzy Sliding Mode Control
DOF	Degree Of Freedom
DTC	Direct Torque Protocol
FLC	Fuzzy Logic Controller
FSMC	Fuzzy sliding mode control
GA	Genetic Algorithm
PID	Proportional Integral Derivative
RP	Representative Point
Sat(.)	Saturation Function
Sig(.)	Sigmoid Function
SMC	Sliding Mode Control
TCP	Transmission Control Protocol
TP	Tensor Product
VSC	Variable Structure Control
VSCS	Variable Structure Control System
VSS	Variable Structure System

List of Symbols

$\mathbf{A}$	The system matrix
A^l, B^l	Fuzzy sets
B	Continuous function vector
abs(.)	Absolute value
CH	The value of the chattering
$\mathbf{c}^T$	The vector of the sliding surface coefficient
cf	Scalar constant
g_1(HT)	The part concerning the hitting time in the fitting function,
g_2(CH)	The part concerning the chattering in the fitting function
HT	The value of the hitting time
k_{Gm}	The discontinuous control gain of the genetic based modified SMC applied to the non-linear system
k_s	Switching gain
$\mathbf{k}_{eq}$	The equivalent control gain
k_h	The discontinuous control gain
L	Parameter set
l	The length of the pendulum
m	The mass of the pendulum
M	System order
mcol	Mutated columns
mrow	Mutated rows
n	The number of fuzzy rules
N_{keep}	The number of chromosomes that are kept for mating
N_{pop}	The number of population inside a chromosome
N_{var}	The number of elements inside a chromosome
P	One of the chromosome element
p_d	The element of parent2 chromosome
p_m	The element of parent1 chromosome
$R^{M \times N}$	The state range of $M \times N$

	dimensions
R^N	The state range of N dimensions
s	The sliding surface
s_1, s_2	The coefficients of the sliding surface
$sign(.)$	The sign function
Sin(.)	Sine function
$\dot{s}$	The switching function derivative
t	Time
u	The applied control as an input
$u_c(t)$	The continuous control part of SM action applied to the pendulum system
$u_d(t)$	The discontinuous control part of the SMC action to the pendulum system
$u_{eq}(t)$	The equivalent control part of the SMC applied to the linear system
u_f	Fuzzified control action
$u_h(t)$	The discontinuous control part of the SMC
X	State vector
$\mathbf{x}(0)$	the initial condition vector
α_i, β_i	Sets of control parameters.
δ_1, δ_2	Membership function that is used to express the grade of goodness of the fitness function of each performance
$\mu_A(\cdot)$	Membership functions

Introduction

The Sliding Mode Controller (SMC) is a particular type of Variable Structure Controller (VSC), which is defined as a system whose physical structure is changed intentionally during the time in accordance with a preset structure control law. The instants at which the changing of the structure occurs are determined by the current state of the system [1]. Sliding mode was discovered at the beginning of the sixties. For the needs of military aeronautics, and even before the term of robustness was used, control engineers were looking for control laws insensitive to the variations in the system to be controlled.

Standard sliding mode controllers are characterized by high frequency switching of control which causes a problem in practical application some thing called “chattering effect” which is characterized by the states repeatedly crossing rather than remaining on the surface. The fast dynamics which were neglected in the system model are often excited by the switching of sliding mode controllers, another type of chattering is called a discritization chatter that occurs in microcontrollers[2,3].

Several methods were proposed to reduce chattering, like modifying the discontinuous control action such that instead of forcing the states to lie on the sliding surface they are forced to remain within a small boundary layer about the surface by using a saturation rather than the sign discontinuous function.

Or using the dead zone function instead of the sign discontinuous function will turn off the control entirely allowing the system to coast. The saturation can provide a smoother behavior than the dead zone [2]. The intelligent systems can be utilized also to overcome the problem of chattering such as GA which can be used to choose the appropriate SMC discontinuous part's gain to reduce the problem of chattering in SMC. The result controller is known as Genetic based sliding mode controller.

fuzzy logic techniques can also be used to reduce the zigzag motion. The smooth control action feature of FLC can be used to overcome the disadvantages of SMC systems. This is achieved by the merging of the FLC with the variable structure of the SMC to form a Fuzzy Sliding Mode Control (FSMC). In this hybrid control system, the strength of the SMC lies in its ability to account for modeling imprecision and external disturbances while the FLC reduces chattering [4].

2- Sliding mode control design and problem description

The sliding mode control is designed for the following linear second order system[5]:

$$\dot{x} = Ax + Bu \qquad \text{....(1)}$$

In the sliding mode control design, the first step is the determination of the switching functions. Assume the sliding surface is given by[5]:

$$s(\mathbf{x}) = s_1 x_1 + s_2 x_2 = C\,x \qquad \text{...(2)}$$

,M is $c \in R^{M\times N}$, and $x \in R^{N}$ where the number of inputs

$\mathbf{c}^T = [\, s_1 \;\; s_2]$ is the vector of the sliding surface coefficient.

The coefficient s_2 is set to one since there will be an order reduction then the setting of the last coefficient to (1) will not cause a loss of enerality [5].

The second step in the sliding mode control design is the choice of the control law. In general, the control law can be considered separately by the two control terms (uh and ueq) and is represented by[6]:

$$u(t) = u_h(t) + u_{eq}(t) \qquad \text{....(3)}$$

with,

$$u(t) = \mathbf{k}_{eq}\mathbf{x}(t) + k_h \operatorname{sign}(\mathbf{s}) \qquad \text{.....(4)}$$

When the state is on the sliding surface, the purpose of the equivalent control is to keep the state staying on the sliding surface so it can be derived from setting the time derivative of s, $\dot{s}$ equal to zero, that is[6]:

$$u_{eq} = u\big|_{\dot{s}=0}$$

From equation (1) and (2)

$$\dot{s}(t) = c^T \dot{x}(t) \qquad \text{.... (5)}$$

$$\dot{s}(t) = c^T\left(Ax(t) + Bu_{eq}(t)\right) = 0 \qquad \text{..(6)}$$

i.e.

$$u_{eq}(t) = (c^T B)^{-1} c^T A\, \mathbf{x}(t) \qquad \text{... (7)}$$

Then, $k_{eq} = (c^T B)^{-1} c^T A$ . ...(8)

3-Chattering Reduction Methods

Three methods were utilized to attenuate chattering as follows:

A- Boundary layer solution

The boundary layer solution seeks to avoid control discontinuities and switching action in the control loop. The discontinues control law is replaced by a saturation function which approximates the sign(s) term in a boundary layer of the sliding manifold s(x)=0 as an illustrative example. Consider a simple linear saturation function[7]:

$$\mathrm{sat}(a)=\begin{cases} \mathrm{sign}(a) & \text{if } \mathrm{abs}(a) \geq 1 \\ a & \text{if } \mathrm{abs}(a) < 1 \end{cases} \quad \ldots(9)$$

The discontinuous control part of the SMC signal will be as follows [14].

$$u(t)=\begin{cases} k\,\mathrm{sign}(s(x)/\varphi) \text{ for} & |s(x)/\varphi| \geq 1 \\ k\left(\frac{s(x)}{\varphi}\right) & |s(x)/\varphi| < 1 \end{cases} \quad \ldots.\ (10)$$

B- Discontinuous gain parameter's selection using GA

A parameter selection algorithm is proposed by GAs to select the gain parameters so that the controlled system can achieve a good overall performance in the slide mode control design. It is desirable to have the fast reaching velocity into the switching hyperplane during the reaching phase and herein slide to the origin with little chattering phenomena. The selection of SMC parameter through this algorithm is used to conquer the difficulty that how to simultaneously consider the hitting time and the chattering in the selection of the gain parameters. genetic based sliding mode control method is suggested so that the parameters of switching gain are self-generated by means of GAs based on the direction of a proposed fitness function. In order to select the set of controlparameters **$L=(\alpha_1,\beta_1,\alpha_2,\beta_2,\ldots,\alpha_n\beta_n)$** by using GA, first, first L will be selected as a parameter set, then a fitness function will be chosen so that GAs can be used to search for a better solution in the parameter space. If a function is defined, the search direction of GA will depend on the requirement of fitness function. It is a key role on the defined fitness function so that the controlled system can achieve a desired performance[6].

The gain parameters of the sliding mode control will be optimally chosen to reduce the hitting time and the chattering of the controlled system. The fitness function that will be used as follows[6]:

$$\text{Fitness function} = g_1(HT)*g_2(CH) \quad \ldots(11)$$

That is

$$g_1=\frac{1}{1+\frac{HT}{\delta 1}} \ and\ g_2=\frac{1}{1+\frac{CH}{\delta 2}} \quad ..\ (12)$$

where HT is the value of the hitting time denoted by time required for the states to hit the surface :which is measured in this work, until s(HT)=0 and CH is the value of the chattering. $\delta_1 and \delta_2$ are membership functions that are used to express the grade of goodness of each performance and are chosen by tuning. In this work it is chosen to be 0.5 and 4.5 respectively.

By taking the merit of the genetic algorithm, two major performance measures (the hitting time and chattering) of the controlled system's response in the slide mode control design can be considered simultaneously in the proposed fitness function so that the selected controller by GAs has the ability to consider the hitting time and the chattering of the controlled system simultaneously The selection problem will become an optimization problem as follows[6]:

$$MAX_{L\in p}\{fitness\ function_i(L)\}$$

where *L* is a string which represents a point located in the search space *P*. while the fitness function increases as greatly as possible; the global performance of the controlled system corresponding to the string will work as well as possible[6].

Concerning the gain parameters of the discontinuous control part, it is appropriately chosen as follows

$$k_h{}^i = \begin{cases} \alpha_i & if\ sx_i > 0 \\ 0 & if\ sx_i = 0 \\ \beta^i & if\ sx_i < 0 \end{cases}$$

The procedure for selecting the gain parameter will be as follows[6]:

1. Start with a randomly generated population of *n* *(No.of chromosome)* *l−gene* *(length of chromosome)*chromosomes (candidate solutions to a problem).
2. Calculate the fitness $f$it(x) of each chromosome in the population.
3. Repeat the following steps until *n* offspring have been created:

a. Select a pair of parent chromosomes from the current population. The probability of selection is being an increasing function of fitness. Selection is done "with replacement," meaning that the same chromosome can be selected more than once to become a parent.

b. With probability *p*c (the "crossover probability" or "crossover rate"), cross over the pair at a randomly chosen point (chosen with uniform probability) to form two offspring. If no crossover takes place, form two offspring that are exact copies of their respective parents. (Note that here the crossover rate is defined to be the probability that two parents will cross over in a single point. There are also "multi−point crossover" versions of the GA in which the crossover rate for a pair of parents is the number of points at which a crossover takes place.)

c. Mutate the two offspring at each locus with probability *p*m (the mutation probability or mutation rate), and place the resulting chromosomes in the new population. If *n* is odd, one new population member can be discarded at random.

4. Replace the current population with the new population.
5. Go to step 2.

C- Fuzzy tuning scheme

A fuzzy set is a generalization of the classical notion of a set. Whilst the characteristic function of a classical set can take values of either 0 or 1, which means that an object either belongs to or does not belong to a given set, the characteristic function called "(***membership* function)** in fuzzy set theory[8].

C-1-Fuzzy SMC

The sliding surface *s*(*t*) forms the input space of the fuzzy implications of the major switching rule. Its switching gain is written in the form of fuzzy rule, given by[9]:

If $s(t)$ is A^l, **Then** K_s is B^l

where l=1,..,n and n is the number of rules. With fuzzy implications, K_s is then transformed to an adjustable parameter and hence the fuzzy inference mechanism can be used as estimation mechanism for the adaptive control the inclusion of such fuzzy scheme has thus accounted for bounded uncertainties in the system. The use of a fuzzy scheme for determining the values of Ks improves the performance by improving the damping ratio of the control system. Ideally, it works in a such way that when $s(t)$ is far away from the sliding surface, the control gain has a higher value and when $s(t)$ is near to the sliding surface, the gain is adjusted to a smaller value. Hence, a soft computing approach is adopted here for such a soft-switched FSMC system. A one-dimensioned input space is usually adequate determining the switching rules and is being adopted here to reduce the computational demand of the FLC. Or a fuzzy sliding surface is introduced to develop a sliding mode controller directly, The IF-THEN rules of fuzzy sliding mode Controller can be described in a general form to describe these fuzzy rules:

The fuzzy rules applied are:

If sx_1 is NB then k_h^1 is β_1

If sx_1 is ZERO then k_h^1 is ZERO

If sx_1 is PB then k_h^1 is $\propto_1$

If sx_2 is NB then k_h^2 is β_2

If sx_2 is ZERO then k_h^2 is ZERO

If sx_2 is PB then k_h^2 is $\propto_2$

where *NB*, *NM*, *ZERO*, *PM*, *PB* are linguistic terms of antecedent fuzzy set, they mean negative big, negative medium, zero, positive medium, and positivebig,respectively,

$\propto_1, \propto_2, \beta_1, \beta_2$ are the SMC parameters to be chosen appropriately though FLC.

4- Simulation Results

In this section, the performance of the proposed method is illustrated by applying it to deal with the following system:

$$\dot{x}(t) = \mathbf{A}x(t) + \mathbf{B}u(t)$$

Where $\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$, $\boldsymbol{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$,

$$\mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

The sliding surface is chosen as

$s = \mathbf{c}^T\mathbf{x}(t) = [1\ 1]\mathbf{x}(t)$

from (8) we have,

$$k_{eq}^T = [-1\ \ -1]$$

Using the conventional SMC

let $k_h = -5$

Then the total control signal will be as follows

$u(t) = (x_1 + x_2) - 5\text{sign}(s(t))$

Obviously a considerable amount of chattering appears in the phase plane of Fig.(1) which is the drawback of SMC, the control signal of Fig.(4) presents high frequency switching which needs certain improvement. Fig.(3) the system state x_2 suffers from an offset and a high switching control action and The states are forced to reach the equilibrium point but unfortunately they bound the equilibrium point until they reach it asymptotically ($t \rightarrow \infty$).

4-2 Using boundary layer solution

By replacing the discontinuity sign function of the SMC structure by the saturation function which makes a boundary layer around the sliding surface such that chattering phenomenon is reduced, the control signal of the modified SMC will be as follows:

$$u(t) = \mathbf{k}_{eq}\boldsymbol{x}(t) + k_h \,\mathrm{sat}\,(\mathbf{s}/1)$$

$$\mathbf{k}_{eq}=[1\ 1],\ k_h= -5$$

As seen from Fig. (5) the boundary layer approach has succeeded in reducing satisfactorily the chattering phenomenon and the sliding motion is fairly accepted. The control signal of Fig. (8) is improved since the high frequency switching is removed and the control signal is smooth. The states trajectories are directly guided towards the equilibrium point within 6 seconds as appeared in figures (6)and (7).

4-3 using GA for optimal gain parameter's selection of the discontinuous SMC control signal

The required work is to find an appropriate combination of gain parameters by the proposed method so that a better performance with a small hitting time and a small chattering can be achieved. For the comparison, the searching spaces of the gain parameters $\alpha 1$,$\beta 1$,$\alpha 2$ and $\beta 2$ by GAs are all limited to [-10,10] according to the values chosen in equation (30). The sampling period is 0.01 sec, $\delta 1 = 0.5$,$\delta 2 = 4.5$, The control signal is given by:

$$u(t)= -\left(k_{eq}^{T} + k_{h}^{T}\right)x$$

$$u(t)=\left(1 - k_{h}^{1}\right)x_1 + \left(1 - k_{h}^{2}\right)x_2$$

The population size is chosen to be 40 , the generations are chosen to be 20, the crossover probability is 0.6 and the mutation probability is 0.05. GA programmed by using m-file matlab. The parameters to be searched are $\propto^{i}$ and β^{i} which means four parameters.

The optimal values of the gains of the discontinuous controller (k_{h}^{i}) were found by using GA to be

$$\propto_1 = 9.96, \propto_2 = 10, \beta_1 = 10 \ and\ \beta_2 = 9.79$$

The number of crossover is 11 and the number of mutation is 1. As seen from Fig. (9) the phase trajectory sharply reaches the sliding surface within 0.559 seconds and the chattering is removed, from Fig. (10)and Fig. (11) the states have no offset, but x_2 response shows an overshoot in its response finally Fig. (12) Shows the control action is fairly smooth with no transient period.

4-4 Fuzzy tuning technique

Replacing the sign function of the discontinuous SMC control signal by the above mentioned rules shows that the chattering phenomenon of controlled system is suppressed in the sliding mode fuzzy controller as shown in Fig. (13), but the hitting time becomes longer. This result is due to the smoothness of control force in the sliding mode fuzzy controller. The smooth control force decreases the sudden change in the sliding surface, but provides a smaller force to speed the state to the sliding surface as shown in Fig(16). As shown the states in fig's(14) and (15) reached the equilibrium point within (5.4) seconds, with no offset , x_2 response shows an overshoot smaller than the overshoot shown in x_2 response of the GA based SMC.

As a comparison, table (1) shows the differences between the hitting time required for each method to reach the sliding surface.

5- Analysis of the Simulation Results

Initially the conventional SMC is applied to selected case study, we wish to have a fast reaching velocity to the sliding surface in the reaching phase and herein slide to the origin with little chattering phenomena in the sliding phase but Obviously a considerable amount of chattering appears in the phase plane of Fig.(1) which is the draw back of SMC and the control signal of Fig.(4) presents high frequency switching which needs certain improvement. The states are forced to reach the equilibrium point but unfortunately they bound the equilibrium point until they reach it asymptotically ($t\rightarrow \infty$).. The main objective is to propose an efficient method to choose an appropriate parameter then the problems appearing in it are solved by the three methods mentioned above.

The modified sliding mode controller invites an idea to restrict the width of boundary layer φ , and uses a continuous function to smooth the control action The $sat(s(x)/\varphi)$ is substituted for the $\mathrm{sgn}(s(x))$ in SMC structure. Therefore, the problem of the discontinuousness of u was solved, and the chattering phenomena eliminated. As seen from Fig.(5)the boundary layer approach succeeded to reduce satisfactorily the chattering phenomenon and the sliding motion is fairly accepted. The control signal of Fig.(8) is improved since the high frequency switching is removed and the control signal is smooth. The states trajectories are directly guided towards the equilibrium point within 6 seconds.

By using the GA to select the appropriate discontinuous SMC gain parameters, we chose the hitting time and the chattering of the controlled system's response as the performance measures for selecting the parameters. The proposed fitness function is defined in such a way that the selected parameters can drive the state to hit the sliding surface fast and then keep the state slide along the surface with less chattering. As seen from fig.(9) the phase trajectory sharply reaches the sliding surface within 0.559 seconds and the chattering is removed, hence the state trajectories reaches the equilibrium point faster using GA based SMC than the modified SMC.

By using the fuzzy rules instead of the discontinuous function of the SMC structure, the resulting phase plane shows no chattering, the states reach their equilibrium point faster than the modified SMC (the boundary layer solution) but slower than the GA based SMC, the control action is quite smooth with transient. As shown in Fig.(13) the phase plane of the FSMC controlled linear system was obviously succeed to have no chattering after entering the sliding phase, the states as shown in fig's(14) and (15) reached the equilibrium point within 5.4 seconds, and from Fig.(16)the FSMC action obviously smooth.

6-Conclusions

Three methods are used for attenuating chattering from the conventional SMC, the first method involves replacing the discontinuous function of SMC with a saturation function, the second method involves utilizing GA to find the optimal value of the discontinuous gain, in the third method the sign function is replaced in the conventional SMC by the appropriate fuzzy rules, these three methods are applied to linear second order system.

The conclusion that have been came through is that, the parameter selection using GA is the best method to deal with chattering problem and to drive the state to reach the sliding phase within minimum amount of time. The advantage of the GAs is that they don't need extra professional knowledge or mathematics analysis. During the execution of the GAs, only the fitness function of the strings is evaluated.

References

[1]: G. Spiazzi, P. mattavelli, L. Rosset to and L. Malesani, "Application of Sliding Mode Control to Switch-Mode Power Supplies", Journal of Circuits , Systems and Computers (JCSC), VOL. (5) , NO.(3), September, pp[337-354], 1995.

[2]: T. Kealy, "PI Controller and Sliding Mode Controllers for Coupled Tanks", Dublin City University ,Ireland, April, 2001.

[3]: Christopher Edwards and Sarah K. Spurgeon "Sliding Mode Control :Theory and Appliction", Taylor and Francis Ltd., London ,U.K, 1998.

[4]:H. Liu, K. Kouami and T. Hagiwara,"Semi-Active Fuzzy Sliding Mode Control of Full Vehicle and Suspensions", Journal of Vibration and Control, Vol.(11) ,No.(8),pp[1025-1042],Japan ,2005.

[5]:Govert Monsees, " Discrete Time Sliding Mode Controllers ", PH.D.Thesis, Delft University of Technology Delft, The Netherland, 2002.

[6]:Ching-Chang Wong and Shih-Yu Chang, "Parameter Selection in the Sliding Mode Control Design Using Genetic Algorithms".

Table (1) Comparison table concerning the hitting time.

The solution method for the linear system	The hitting time /sec.	hitting time %
The boundary layer solution	6	12.5
Genetic algorithm	0.559	3.12
Fuzzy tuning mechanism	5.4	60

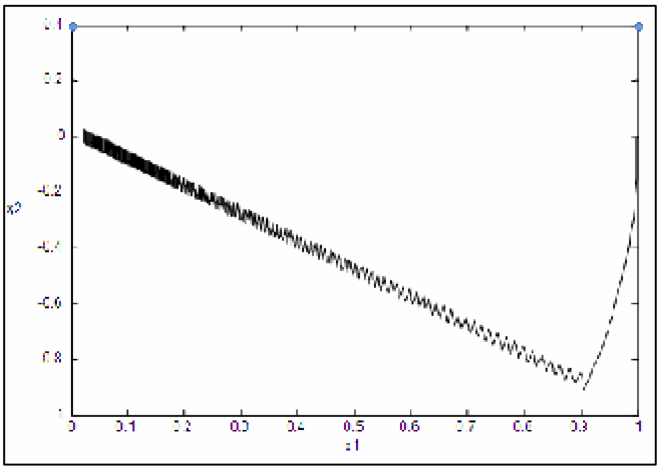
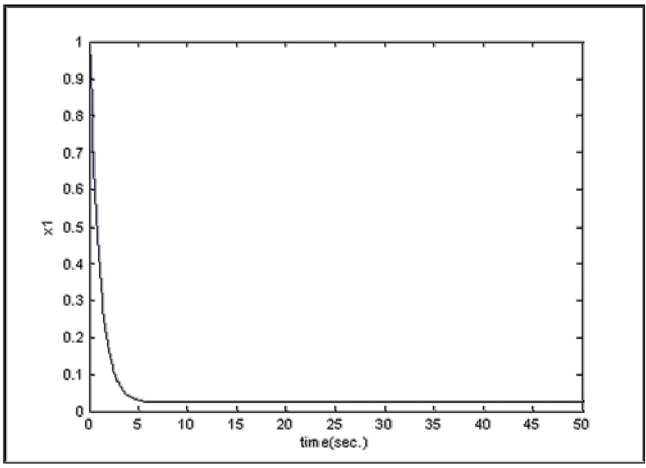


Figure (1) Phase plane of the conventional SMC controlled system



. Figure (2) Time response of x1 using conventional SMC

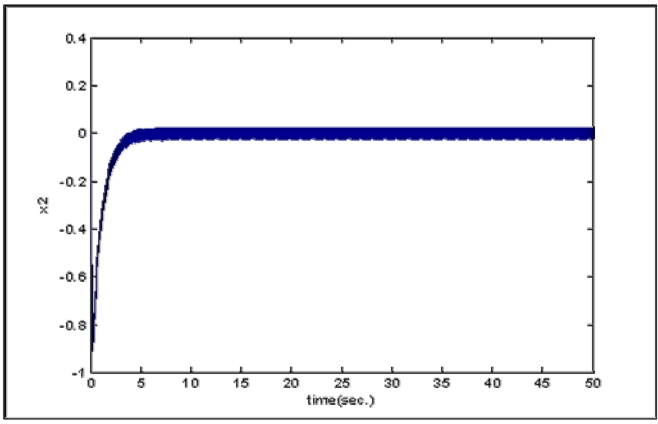


Figure (3) Time Response of x2 using Conventional SMC.

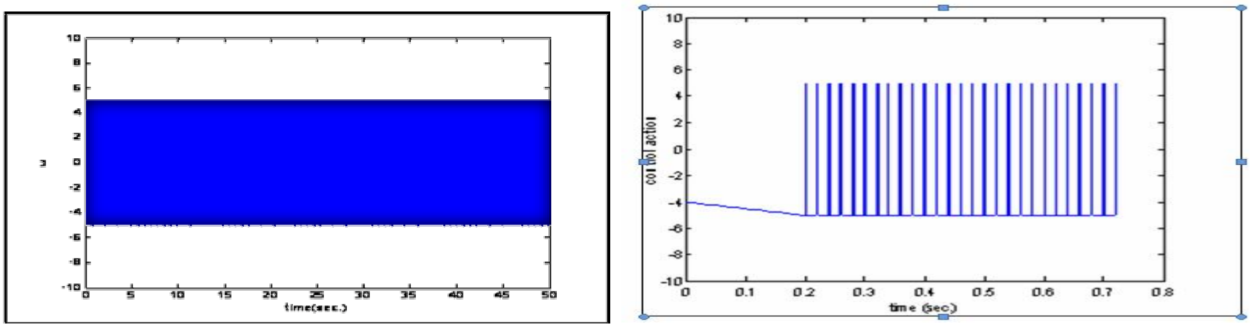


Figure (4) a):Conventional SMC signal b) enlarged section

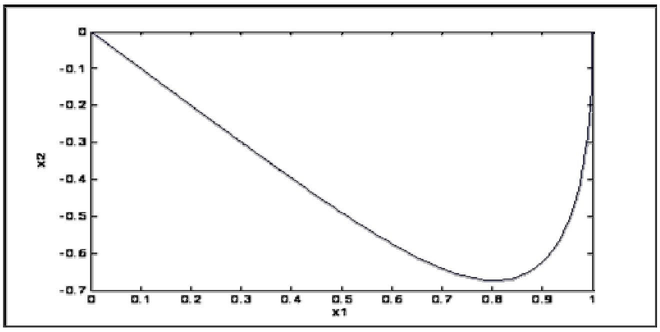


Figure (5) phase plane of the boundary layer

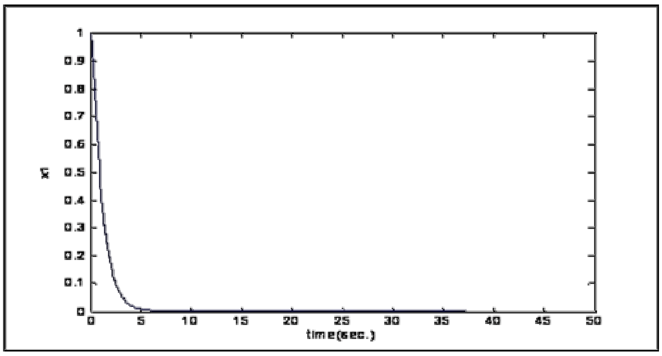


Figure (6) Time response of X_1 for the boundary layer

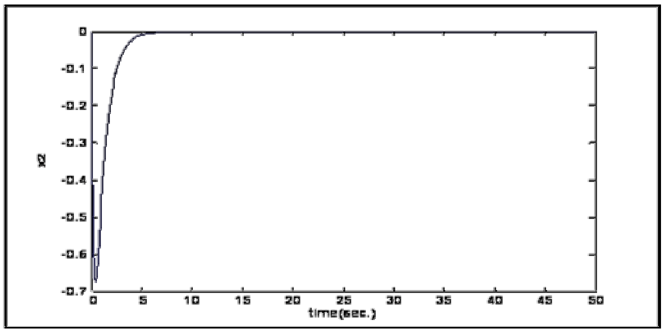


Figure (7) Time response of X_2 for the boundary

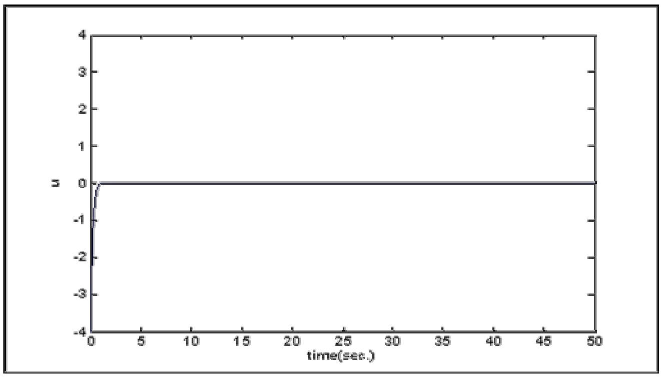


Figure (8) Time response of modified SMC signal.

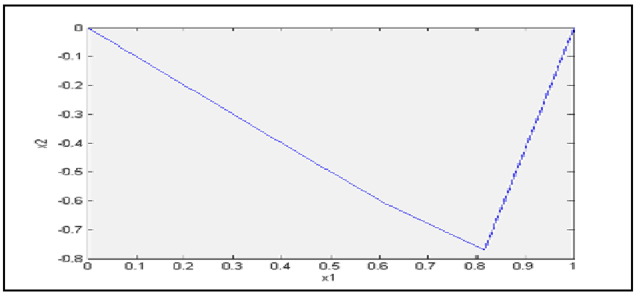
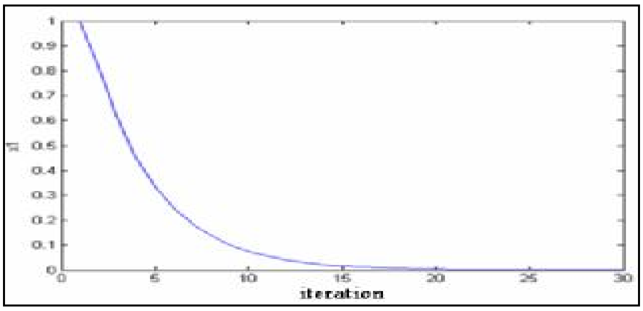


Figure (9) Phase plane of the genetic based SMC



Figure(10):Time response of X_1 of the genetic based SMC

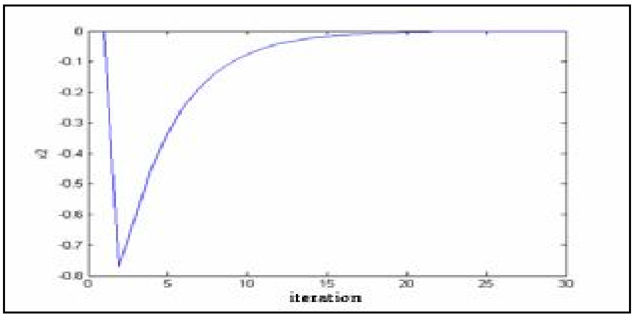


Figure (11) Time response of X_2 of the genetic based SMC

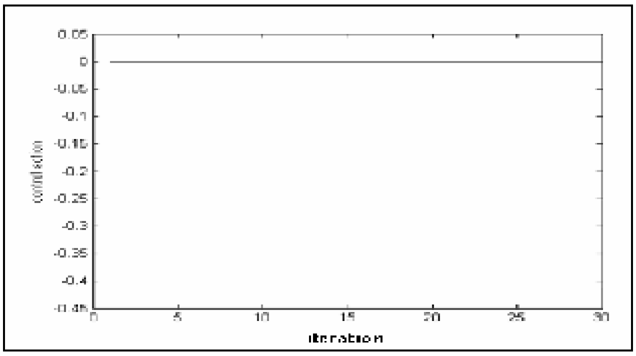


Figure (12) Time response of genetic based SMC signal.

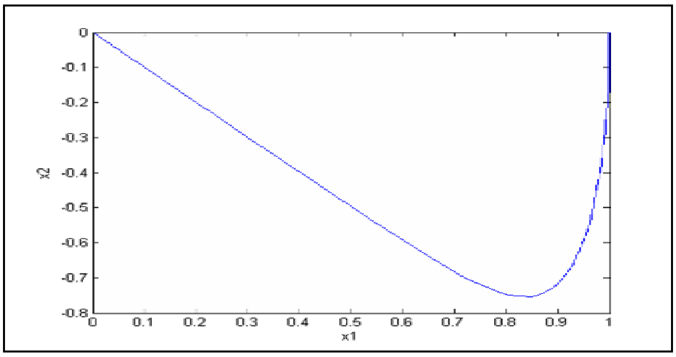


Figure (13) Phase plane of the FSMC controlled

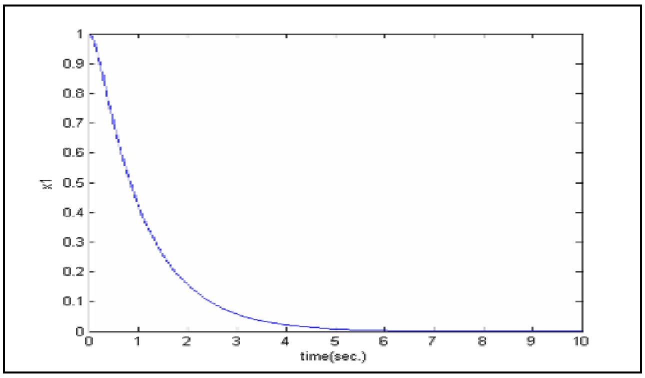


Figure (14) Time response of $X_1$of the FSMC

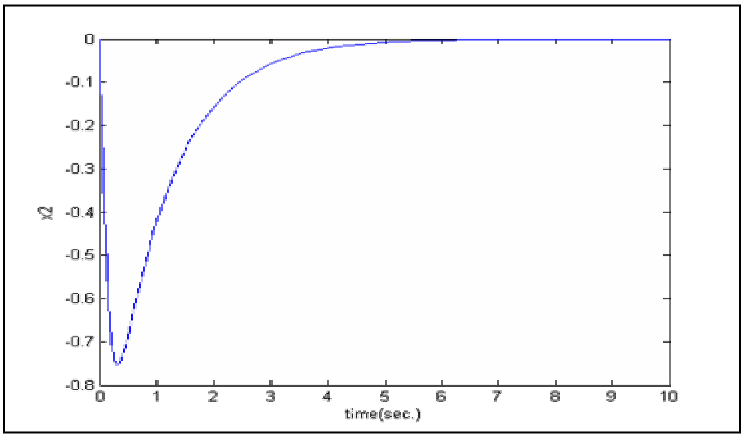


Figure (15) Time response of $X_2$of the FSMC

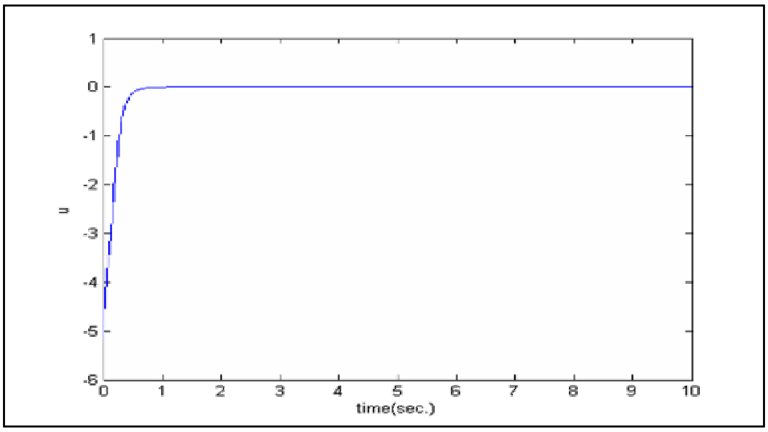


Figure (16) Time response of FSMC signal.