Kinematic Analysis of Semi-Flexible Robot

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ABSTRACT
In this work, a kinematic model of semi- flexible robot with two degrees of freedom based on the joint angles and arm’s deflections is presented with taking in consideration the small deflection parameter (exact model), and used experimental deformation results to make a comparison between the approximate model and the exact model. Due to the difficulties of using flexible robot in the real live, a two degrees semi-flexible robot was built; this robot will be used to get the experimental results for comparison. The comparison shows a small difference between the approximate model and the exact model, so with increasing the flexibility this difference will increase and in some applications of robot this difference will be significant and worth to take in consideration.

Keywords: semi-flexible robot, kinematic equation of flexible and semi- flexible robot.

INTRODUCTION
The kinematic analysis of a mechanical system means the deformation of the position, velocity and acceleration of the various mechanical elements forming the mechanism under consideration. The combination of position velocity and
acceleration of an element at a certain time is referred to henceforth as the state of this element [1]. In the case of open kinematic chain, a direct kinematic analysis determines the state of the end effector as function of the known state of the various joints. The direct kinematic analysis also includes the determination of the link’s position and orientation and their time derivatives, such information is essential for any subsequent dynamic analysis or when the position and orientation of a sensor mounted on a link are required for the data processing.

The references [2], [3] and [4] have supposed that the generalized deflection parameters are small, so a first order approximation can be applied to their trigonometric functions and product, and higher order equals zero.

In this work, the kinematic of robot will be analyzed using a homogeneous transformation without and with deflection and elongation.

POSITION AND ORIENTATION OF A RIGID BODY

The arm linkage of a manipulator can be modeled as a system of rigid bodies. The location of each single rigid body is completely described by its position and orientation [5]. The position can be represented by the coordinates of an arbitrary point fixed with respect to the rigid body. Figure (1) shows the coordinate frame O-xyz fixed to the ground and the point $O^b$ fixed to the rigid body.

The position of the coordinate frame ($O^b$) of the rigid body is represented with reference to the coordinate frame of the base $O_o$-xyz by:

$$X_o = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

(1)

The subscript ($_o$) shows that the vector is defined with reference to the coordinates frame of the base.

Also, the position of point $P$ on the rigid body can be represented with reference to the coordinate frame $O^b$ fixed to the rigid body by:

$$x^b = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

(2)

The subscript ($^b$) indicates that the vector is defined with reference to the body coordinate’s frame [6].

The three angles $\psi$, $\phi$ and $\theta$ determine the orientation of the coordinate frame uniquely about the axes $x$, $y$ and $z$, respectively and referred to as Euler angles. The Euler angles are independent in that each of them can vary arbitrarily [6].

The matrices $R(\psi)$, $R(\phi)$ and $R(\theta)$ represent the rotation about the axis $x$, $y$, $z$, respectively, also called yaw, pitch and roll, respectively, as shown in figure (2).
\[ R(\psi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \psi & -\sin \psi \\ 0 & \sin \psi & \cos \psi \end{bmatrix} \]  
\[ R(\phi) = \begin{bmatrix} \cos \phi & 0 & -\sin \phi \\ 0 & 1 & 0 \\ \sin \phi & 0 & \cos \phi \end{bmatrix} \]  
\[ R(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \]  

From figure (1), the relationship, that defines the coordinate transformation between the fixed frame and the coordinate of any point on the rigid body, will be:

\[ X = X_o + RX^b \]  

Equation (6) provides the desired coordinate transformation from the body coordinate \( X_b \) to the fixed coordinates \( X \). \( X_o \) and \( R \) represent the position and orientation of the rigid body or of the body’s coordinate frame relative to the fixed frame, respectively [6].

HOMOGENOUS TRANSFORMATION

Equation (6) can resemble the position and orientation in a conceptual form identified henceforth by the matrix \([A]\):

\[ A = \begin{bmatrix} R : X_o \\ \vdots : \vdots \\ 0 : 1 \end{bmatrix} \]  

The original vector \( X \) and \( X_o \) are augmented by adding 1 as the fourth element, so that the result is a 4x1 vector. Also, the rotation matrix \( R \) is extended to 4x4 matrix by combining it with 3x1 position vector \( X_o \) with three 0’s and 1 in the fourth raw. Equation (6) can then be written as:

\[ X = AX^b \]  

Thus

\[ \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} R : X_o \\ \vdots : \vdots \\ 0 : 1 \end{bmatrix} \begin{bmatrix} x^b \\ y^b \\ z^b \\ 1 \end{bmatrix} \]
The coordinate transformation given by equation (9) is referred to as the homogenous transformation equation [1].

**DENAVIT-HARTENBERG (DH) MATRIX**

Any robot can be kinematically described by giving the values of four quantities for each link. Two describe the link itself and two describe the link’s connection to a neighboring link. In usual case of a revolute joint, $\theta_i$ is called the joint variable, and the other three quantities would be fixed link parameters. For prismatic joints, $d_i$ is the joint variable and the other three quantities are fixe link parameters. The definition of mechanisms by means of these quantities is a convention, usually called the Denavit-Hartenber (DH) notation [7].

The DH displacement matrix for a rigid link is:

\[
A_i^{i-1} = \begin{bmatrix}
\cos \theta_i & -\sin \theta_i \cos a_i & \sin \theta_i \sin a_i & a_i \cos \theta_i \\
\sin \theta_i & \cos \theta_i \cos a_i & -\cos \theta_i \sin a_i & a_i \sin \theta_i \\
0 & \sin a_i & \cos a_i & d_i \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (10)

The four geometric quantities $\theta_i$, $a_i$, $d_i$ and $a_i$ represent the joint parameters defined as follow, as shown in figure (3):

- $\theta_i$ is the angle between the $x_{i-1}$ and the $x_i$ axis, obtained by screwing $x_{i-1}$ into $x_i$ around the $z_{i-1}$ axis. For a revolute joint, $\theta_i$ is a variable parameter.
- $d_i$ is the coordinate of the origin of $O_i$ frame on the $z_{i-1}$ axis, the distance between the origin of $O_{i-1}$ frame to the intersection of the $z_{i-1}$ axis with the $x_i$ axis, for a revolute joint, $d_i$ is a constant parameter.
- $a_i$ is the common normal distance between $z_{i-1}$ and $z_i$ axis, measured along the negative direction of $x_i$ from its origin, to where it intersects the $z_{i-1}$ axis (a constant parameter).
- $a_i$ is the angle between the $z_{i-1}$ axis and the $z_i$ axis, obtained by screwing $z_{i-1}$ into $z_i$ around the $x_i$ axis (a constant parameter) [1].

**KINEMATIC EQUATION OF SEMI-FLEXIBLE ROBOT (WITH ERROR)**

The link flexibility can cause elastic deformations of the structural members of the manipulator, resulting in large end-effector errors, especially in long reach manipulator systems. Hence as a result, the frames defined at the manipulator joints are displaced from their expected locations. So that, the using of kinematic equation of the rigid robot to position the manipulator end–effector, will place the manipulator in a different position than the desired one.

In figure (4) [8], the frame $O_i^{1}$ is the base frame, $O_i^{1}$ is the ideal location and $O_i^{a}$ is the actual location due to the deflection.

The transformation equation between the coordinates $O_i^{a}$ and $O_i^{1}$ consists of two parts, rotation and translation. According to the Euler angle principle, the rotation part
is the result of three sets of rotations which are roll, pitch and yaw about the axes z, y, x, respectively. The sequence of rotation is [8]:

\[ E_{Ra} = \text{Rot}(z, d\phi) \text{Rot}(y, d\beta) \text{Rot}(x, d\psi) \] (11)

That is a rotation of \( d\psi \) about the x axis, followed by a rotation \( d\beta \) about the y axis and finally a rotation of \( d\phi \) about the z axis.

\[
\text{Rot}(x, d\psi) = 
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos d\psi & -\sin d\psi & 0 \\
0 & \sin d\psi & \cos d\psi & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\] (12)

\[
\text{Rot}(y, d\beta) = 
\begin{bmatrix}
\cos d\beta & 0 & -\sin d\beta & 0 \\
0 & 1 & 0 & 0 \\
\sin d\beta & 0 & \cos d\beta & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\] (13)

\[
\text{Rot}(z, d\phi) = 
\begin{bmatrix}
\cos d\phi & -\sin d\phi & 0 & 0 \\
\sin d\phi & \cos d\phi & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\] (14)

\[ E_{Ra} = 
\begin{bmatrix}
a_{11} & b_{12} & c_{13} & 0 \\
a_{21} & b_{22} & c_{23} & 0 \\
a_{31} & b_{32} & c_{33} & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\] (15)

\[ a_{11} = \cos d\phi \cos d\beta \]
\[ a_{21} = \sin d\phi \cos d\beta \]
\[ a_{31} = -\sin d\beta \]
\[ b_{12} = \cos d\phi \sin d\beta \sin d\psi - \sin d\phi \cos d\psi \]
\[ b_{22} = \sin d\phi \sin d\beta \sin d\psi + \cos d\phi \cos d\beta \]
\[ b_{32} = \cos d\beta \sin d\psi \]
\[ c_{13} = \cos d\phi \sin d\beta \cos d\psi + \sin d\phi \sin d\psi \]
\[ c_{23} = \sin d\phi \sin d\beta \cos d\psi - \cos d\phi \sin d\psi \]
\[ c_{33} = \cos d\beta \cos d\psi \]

The equation (15) represents the rotation part of the transformation matrix. The translation matrix is:

\[
E_{Ta} = 
\begin{bmatrix}
1 & 0 & 0 & \delta x \\
0 & 1 & 0 & \delta y \\
0 & 0 & 1 & \delta z \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\] (16)
The transformation between coordinates $O_1^a$ and $O_1^I$ is become:

$$E_1^I = E_1^a$$

$$E_1^I = \begin{bmatrix} A_{11} & B_{12} & C_{13} & \delta x \\ A_{21} & B_{22} & C_{23} & \delta y \\ A_{31} & B_{32} & C_{33} & \delta z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$  (17)

$$E_1^I = \begin{bmatrix} \cos \delta \phi \cos \delta \beta \\ \sin \delta \phi \cos \delta \beta \\ -\sin \delta \beta \\ \sin \delta \phi \sin \delta \beta \cos \delta \psi + \cos \delta \phi \cos \delta \psi \\ \sin \delta \phi \sin \delta \beta \sin \delta \psi \\ \cos \delta \phi \cos \delta \beta \sin \delta \psi + \sin \delta \phi \sin \delta \psi \\ \cos \delta \phi \sin \delta \beta \cos \delta \psi - \cos \delta \phi \sin \delta \psi \\ \cos \delta \phi \cos \delta \beta \cos \delta \psi \end{bmatrix}$$  (18)

Here, the $\delta x$, $\delta y$, $\delta z$, $\delta \phi$, $\delta \beta$ and $\delta \psi$ are called generalized error parameters. Usually these generalized parameters are introduced on the basis of different approximations such as assumed modes, finite elements, or Ritz-Kantorovich expansions, with different implications on the model complexity and accuracy [9].

The references [2], [3], and [4] have supposed that the generalized deflection parameters are small, so a first order approximation can be applied to their trigonometric functions and product, and higher order equals zero.

$$\sin \delta \psi = \delta \psi, \quad \sin \delta \beta = \delta \beta, \quad \sin \delta \phi = \delta \phi,$$

$$\cos \delta \psi = \cos \delta \beta = \cos \delta \phi = 1$$

Based on this assumptions the matrix $E_1^I$ will be equal to:

$$E_{def1} = \begin{bmatrix} 1 & -\delta \phi & \delta \psi & \delta x \\ -\delta \phi & 1 & -\delta \psi & \delta y \\ -\delta \psi & \delta \phi & 1 & \delta z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$  (19)

In this work, the parameters deflection’s values are taken in consideration.

For a serial of links the kinematic equation covering the positions of this links with presence of flexible coordinates is:

$$T_{def} = A_1^a(q_1)E_1^{I1} A_2^a(q_2)E_2^{I2} \cdots A_n^{a-1}(q_n)E_n^{In}$$  (20)

$A_n^{a-1}$ is the homogenous transformation matrix in equation (10).

By using equation (20), the transformation relationship between the two frames $O_1^a$ and $O_{l-1}$ could be found.
T_{defl} may be expressed in simple form as a nonlinear function of \( q \) and \( \epsilon \) vectors:

\[
p_{defl} = f(q, \epsilon) \tag{21}
\]

Where:

\( p_{defl} \) is the set of Cartesian coordinates describing the position and orientation of the manipulator’s end-effector with respect to the inertial frame.

\[
q = [q_1 \ q_2 \ \cdots \ \cdots \ q_n]^T \tag{22}
\]

\[
\epsilon = [\epsilon_1 \ \epsilon_2 \ \cdots \ \cdots \ \epsilon_g]^T \tag{23}
\]

\[
\epsilon = [\delta x_m \ \delta y_m \ \delta z_m \ \phi_\rho_m \ \phi_\beta_m]^T \tag{24}
\]

Where:

\( \epsilon \) is a dimensional link deflection vector space, spanned to be the link deflections of the manipulator arm.

\( n \) is the number of joints.

\( g \) generally equals to 6 x m.

\( m \) is the number of flexible links.

The elements of the vector \( \epsilon \) cannot specify their values, since these elastic coordinates are dependent on the rigid coordinate’s \( q \) and on the applied forces \( F \).

For more generally:

\[
\epsilon = E(F, q) \tag{25}
\]

By substituting equation (24) into equation (23), \( p_{defl} \) will be:

\[
p_{defl} = f(q, E(F, q)) \tag{26}
\]

As mentioned in the previous section, the two degrees of freedom of the arm robot manipulator used in this study is confined to move within the vertical plane, and so is the deformation of each link. Hence, there is no rotation about the axes x and y, also there is no transformation in the z direction (the parameters \( \delta z \), \( \phi \rho \) and \( \phi \beta \) are equal zero). Figure (5) shows a single link coordinate system, the \( O_{i-1} \) represents the base local coordinate system, and the frame \( O_1^i \) is the local coordinate system assigned to link i in its undeformed position, while the \( O_2^i \) is the actual local coordinate system of link i, when it is under deformation.

The homogenous transformation matrix for this single link was derived as follow:
According to the motion of the links within the vertical plane, the homogenous transformation matrix $E^{a}_{l_i}$ between $O^{a}_{l_i}$ and $O^{l_i}$ becomes:

$$E^{a}_{l_i} = \begin{bmatrix}
\cos \phi & -\sin \phi & 0 & \delta x \\
\sin \phi & \cos \phi & 0 & \delta y \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}$$ (27)

The homogenous transformation matrix between the $O^{l_i}$ and $O^{l_{i-1}}$ is presented by the matrix $A^{l_{i-1}}_{l_i}$ in equation (10).

The homogenous transformation matrix for this a single link is found from the relations (10) and (27):

$$T^{i_{1a}}_{l_i} = A^{i_{1a}}_{l_i} E^{a}_{l_i}$$ (28)

$$T^{1a}_{l_i} = \begin{bmatrix}
T_{11} & T_{12} & T_{13} & T_{14} \\
T_{21} & T_{22} & T_{23} & T_{24} \\
T_{31} & T_{32} & T_{33} & T_{34} \\
0 & 0 & 0 & 1
\end{bmatrix}$$ (29)

As shown in figure (6), the total kinematic equation of semi-flexible robot of two degrees of freedom is:

$$T_{defl} = T^{0a}_{1a} T^{1a}_{2a}$$ (30)

The transformation matrix of robot in general form is [10]:

$$T_{2} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}$$ (31)

$T_{defl}$ can be written in the general form (equation (31)) as follow:
\[
\begin{align*}
n_x &= c_2[c_d c_d - s_d s_d c_2] - c_2 s_2[c_d c_d s_d + s_d c_d] + s_1 s_2[c_2 s_2 c_d s_d - c_2 c_d c_d] \\
&= c_2 s_2[c_d c_d c_1 + c_1 c_d c_2 s_d] + s_1 s_2 s_d s_d
\end{align*}
\]

(32)

\[
\begin{align*}
n_y &= c_2[c_d s_1 c_1 + c_1 c_d s_2] - s_2 s_2[s_1 c_d + c_1 c_d c_2] + c_2 s_2[c_1 c_d s_1 c_2] \\
&= c_2 s_2[c_d c_1 c_1 - s_d c_1 s_2 c_2] + s_1 s_2 s_d s_2
\end{align*}
\]

(33)

\[
\begin{align*}
n_z &= c_2[s_1 s_1 c_2 + s_2 c_1 c_2 s_d] + s_2[s_1 c_1 c_2 s_d - s_1 s_1 s_2 c_2] + c_1 c_2 s_d s_d
\end{align*}
\]

(34)

\[
\begin{align*}
o_x &= s_2 s_2[c_1 s_1 c_2 c_2 + c_2 c_2 c_2] - c_2 s_2[c_1 s_1 c_2 + s_1 s_2 c_2] + c_2 s_2[c_1 c_2 s_1 c_2] \\
&= c_2 s_2[c_1 c_1 c_1 - c_1 c_2 s_1 s_2] + s_1 s_2 c_1 s_2
\end{align*}
\]

(35)

\[
\begin{align*}
o_y &= c_2[c_1 c_1 c_2 c_2 - s_1 s_1 s_2] - s_2 c_2[s_1 c_1 c_2 + s_1 s_2 c_2] - c_2 s_2[c_1 c_2 s_1 c_2] + s_2 s_2[s_1 s_1 c_2 - c_1 c_2 c_2]
\end{align*}
\]

(36)

\[
\begin{align*}
o_z &= c_2[s_1 c_1 c_2 c_2 - s_1 s_1 s_2] + s_2[s_1 c_1 c_2 s_d - s_1 s_1 s_2 c_2] + c_1 c_2 c_1 c_2
\end{align*}
\]

(37)

\[
\begin{align*}
t_x &= s_2 c_1 c_2 c_2 - s_2 c_1 c_2 s_d + c_2 c_1 c_2 c_2 + s_1 c_2 c_1 c_2 s_d + s_1 s_2 c_1 c_2
\end{align*}
\]

(38)

\[
\begin{align*}
t_y &= s_2 c_1 c_2 c_2 + s_2 c_1 c_2 s_d + s_1 c_2 c_1 c_2 s_d = c_1 c_2 c_1 c_2 - c_2 s_1 c_2
\end{align*}
\]

(39)

\[
\begin{align*}
t_z &= s_2 s_1 s_1 s_2 c_2 - c_2 s_1 c_1 s_2 c_2 + c_1 c_2 c_2
\end{align*}
\]

(40)

\[
\begin{align*}
p_x &= c_2 c_1[c_1(\delta x_2 + a_2) - s_d(\delta y_2 c_2)] - c_2 s_2[c_1 c_2(\delta y_2) + s_d(\delta x_2 + a_2)] - s_1 c_2[c_1 s_1 s_1(\delta x_2 + a_2)] \\
&= c_1(\delta x_2 + a_2) - c_1 s_1(\delta y_2)
\end{align*}
\]

(41)

\[
\begin{align*}
p_y &= s_2 c_2[c_1(\delta x_2 + a_2) - s_d(\delta y_2 c_2)] - s_2 s_2[c_1 c_2(\delta y_2) + s_d(\delta x_2 + a_2)] + c_2 s_2[c_1 s_1 s_1(\delta x_2 + a_2)] \\
&= c_1(\delta y_2) - s_2 c_2(\delta x_2 + a_2)
\end{align*}
\]

(42)

\[
\begin{align*}
p_z &= c_2[s_1 s_1 s_1(\delta x_2 + a_2) + s_1 c_1(\delta y_2 c_2)] + s_2[s_1 c_1(\delta x_2 + a_2) - s_1 s_1 s_1(\delta y_2) + c_1 c_1(\delta y_2) - s_2 c_2(\delta x_2 + a_2)]
\end{align*}
\]

(43)
Where:
\[ c_i = \cos \theta_i \quad c_{di} = \cos d\phi_i \]
\[ s_i = \sin \theta_i \quad s_{di} = \sin d\phi_i \quad s_{ai} = \sin \alpha_i \]

The degree of freedom of the robot arm manipulator which is used in this work, is two degrees of freedom, and confined to move within the vertical plane with the kinematic parameters listed in the Table (1).

By substituting the values of parameters of the manipulator from the Table (1) in the equations (32) to (43) and with triangular operations, these equations will be as follow:

\[ n_x = [c_{12}][cd_{12}] - [s_{12}][sd_{12}] \quad (44) \]
\[ n_y = c_1c_2[cd_{12}] + s_1s_2[cd_{1-2}] + c_2s_1[cd_{1-2}] \quad (45) \]
\[ n_z = 0 \quad (46) \]
\[ a_x = s_1s_2[cd_{1-2}] - c_1c_2[cd_{12}] - c_1s_2[cd_{1-2}] - s_1c_2[cd_{12}] \quad (47) \]
\[ a_y = c_{12}[cd_{12}] - s_{12}[sd_{1-2}] \quad (48) \]
\[ a_z = 0 \quad (49) \]
\[ t_x = 0 \quad (50) \]
\[ t_y = 0 \quad (51) \]
\[ t_z = 1 \quad (52) \]
\[ p_x = c_1c_2[cd_1(\delta x_2 + a_2) - sd_1(\delta y_2)] - c_1s_2[cd_1(\delta x_2 + a_2) + sd_1(\delta x_2 + a_2)] - s_1c_2[cd_1(\delta x_2 + a_2) + cd_1(\delta y_2)] + s_1s_2[cd_1(\delta y_2) - cd_1(\delta x_2 + a_2)] + c_1(\delta x_1 + a_1) - s_1(\delta y_1) \quad (53) \]
\[ p_y = s_1c_2[cd_1(\delta x_2 + a_2) - sd_1(\delta y_2)] - s_1s_2[cd_1(\delta x_2 + a_2) + cd_1(\delta y_2)] + c_1c_2[cd_1(\delta x_2 + a_2) + cd_1(\delta y_2)] + c_1s_2[cd_1(\delta x_2 + a_2) - sd_1(\delta y_2)] + c_1(\delta y_1) + s_1(\delta x_1 + a_1) \quad (54) \]
\[ p_z = 0 \quad (55) \]

Where:
\[ c_{12} = \cos(\theta_1 + \theta_2) \]
\[ s_{12} = \sin(\theta_1 + \theta_2) \]
\[ cd_{12} = \cos(d\phi_1 + d\phi_2) \quad sd_{12} = \sin(d\phi_1 + d\phi_2) \quad (56) \]
\[ cd_{1-2} = \cos(d\phi_1 - d\phi_2) \]
\[ \sin(\phi_1 - \phi_2) \]

Equations (53) to (54) represent the end-effector position with presence of flexibility of robot.

With using the equation (19) instead of equation (18) to represent \( E^1_a \) in equation (20), the \( p_x \) and \( p_y \) will be as follow:

\[
p_x = c_1c_2[(\delta x_2 + a_2) - \delta x_2 + a_2] - c_1c_2[\delta y_2 + \delta y_2] - s_1c_2[\delta x_2 + a_2] + \delta y_2 + s_1s_2[\delta \phi_1 \delta y_2] + s_1(\delta x_2 + a_2) + \delta y_2 + s_1s_2[\delta \phi_1 \delta y_2] + s_1(\delta x_2 + a_2) + \delta y_2 + s_1s_2[\delta \phi_1 \delta y_2] + s_1(\delta x_2 + a_2)
\]

(57)

\[
p_y = s_1c_2[(\delta x_2 + a_2) - \delta x_2 + a_2] - s_1s_2[\delta x_2 + a_2] + \delta y_2 + s_1c_2[\delta x_2 + a_2] + \delta y_2 + s_1s_2[\delta x_2 + a_2] + \delta y_2 + s_1s_2[\delta x_2 + a_2] + \delta y_2 + s_1s_2[\delta x_2 + a_2] + \delta y_2 + s_1s_2[\delta x_2 + a_2]
\]

The set of equations (53) and (54) and the set of equations (57) and (58) will be called, the flexible kinematic equation and the approximate flexible kinematic equation, respectively.

**Experimental Work**

In this work, the test rig was represented by two degrees manipulator (two semi-flexible links) confined to move in a vertical plane; each link has its own control system for movement. These control systems (drivers for the actuators) were connected to the computer to control them by the main program which was built by the Visual Basic.net software. Also, the rig has measurement systems and data acquisition system (DAQ). Figure (7) shows the semi-flexible robot for the experimental data.

The experimental relations between the loads and deflection \( \delta y \) were found by applying different values of load and measuring the deflection at the tip for each load, and then by using the curve fitting, the relations were found.

To find the rotation of the tip of the beam \( \phi \) experimentally, the measuring of the deflections at two points along the beam was done, as shown in figure (8), at the points where the strain gauge No.3 and at the tip, and by applying these values of the deflections into the following equation [11]:

\[
\tan \phi = \frac{\Delta \delta y}{\Delta x}
\]

(59)

Where:

\( \Delta \delta y \) represents the difference between the two deflections.

\( \Delta x \) represents the distance between the two points where the deflections measured.

\( \delta x \) was experimentally found by extracting the strain from two sensors at the upper and lower surface at the same position of the strain gauge No.1 and No. 2 as shown in the figure (8). The difference value represents the extension of the beam in the longitudinal direction, as represented by the relation:

\[
\delta x = \varepsilon L
\]

(60)
These values of $\delta x$, $\delta y$ and $d\phi$ will be compensated in the kinematic equations (53), (54), (57) and (58) to be the deflection parameters.

The procedure that is worked on it is presented as follows:
1- Assembly the two links to work as a manipulator with their servo motor and DAQ system.
2- In this test, the sequence mode of the main program was activated. The range of angles for the first link was ($4^\circ$ to $29^\circ$ step $5^\circ$) , the second link was ($-35^\circ$ to $60^\circ$ step $5^\circ$). So, the first link is still at angle $4^\circ$, and the second link is still awhile at the start angle then moves $5^\circ$ to the next step, after completion the other steps, it returns back to the start angle, and the first joint moves $5^\circ$ to next step and so on.

This procedure was repeated for four values of load (0, 1, 2 and 3) kg hanging at the end of the link two. Figure (9) shows samples of positions during this test.

The samples of results of the parameters of error which were read in this test are shown in the figures from (10) to (13) which represent the change of values for $\delta y$, $\delta x$ and $d\phi$ at the tip with angles of the two links. The angles for the first link represent the angle of joint with addition $4^\circ$, while to the second link; the angles were taken relative to the dimension $a1$. From the figures (10) to (12) the deformation variables values of the first link decreased with increasing $\theta_2$ for each value of $\theta_1$, while figure (13) shows that; with increasing $\theta_2$ the values of the variables $\delta y_2$ and $d\phi_2$ decrease and $\delta x_2$ increases.

These results were used for the following:
To compare between the two sets of equations (53 and 54) and (57 and 58).

**The comparison between the two sets of flexible kinematic equations (53 to 54) and (57 to 58)**

The results, that were obtained in the part two, were used to calculate the px and py for the two sets of flexible kinematic equations (53) to (54) and (57) to (58) to compare between these two sets to explain why in this research the kinematic equation without approximations were used.

The figures (14) and (15) show the differences between the values of px and py of the two sets of equations. The difference were taken by subtracting the px and py of equations (53) and (54) from the px and py of equations (57) and (58) respectively.

The maximum value of differences in the x-direction (px) is (6.3E-8 m) as shown in the figure (14). While from figure (15) It can be easily to see that, the differences of the py in the y-direction, where the maximum value equals to (2.9E-8 m).

**CONCLUSIONS**

The difference between the two sets of equations is small, and it could be ignore in some applications while in the applications that need to high accuracy like micro applications it is not worth to ignore this difference. Also with increasing the flexibility of links this difference will be increase may be in significant values.
REFERENCES
Symbols:

<table>
<thead>
<tr>
<th>Sym.</th>
<th>Definition</th>
<th>Unit</th>
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<tbody>
<tr>
<td>a_i</td>
<td>Perpendicular distance between axis of shaft for two parallel joints</td>
<td>m</td>
</tr>
<tr>
<td>c_{12}</td>
<td>\cos(\theta_1 + \theta_2)</td>
<td></td>
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<td>c_{d12}</td>
<td>\cos(d\phi_1 + d\phi_2)</td>
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<tr>
<td>c_i</td>
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<tr>
<td>d_i</td>
<td>Prismatic joint displacement</td>
<td>m</td>
</tr>
<tr>
<td>d\phi</td>
<td>Rotation the tip of arm due to the bending</td>
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<tr>
<td>p_{defl}</td>
<td>Transformation matrix with error</td>
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<tr>
<td>q_i</td>
<td>Configuration variable for robot</td>
<td>m or Deg.</td>
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Table (1) The kinematic parameters of robot

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Figure (1) The position of point relative to coordinate frame.

Figure (2) Roll, Pitch and Yaw coordinates for a manipulator.

Figure (3) The Denavit-Hartenberg notation.
Figur (4) Frame displacement due to errors.

Figure (5) The coordinates system of a single semi-flexible link under deformation.

Figure (6) Coordinates systems of two links of semi-flexible robot arm.

Figure (7) The semi-flexible robot for experimental work.
Figure (8) Deflection curve of the beam

Figure (9) Samples of positions during experimental work.
Figure (10) \( \delta y \) vs. \( \theta_2 \) at \( \theta_1 = 4 \) Deg. Link-1

Figure (11) \( \delta x \) vs. \( \theta_2 \) at \( \theta_1 = 4 \) Deg. Link-1
Kinematic Analysis of Semi-Flexible Robot

Figure (12) $d\Phi$ Vs. 02 at 01=4 Deg. Link-1

Figure (13) $\delta y$, $\delta x$ and $d\Phi$ Vs. 02 Deg. Link-2
Figure (14) The difference between the px of the two equations (53) and (57)

Figure (15) The difference between the px of the two equations (54) and (58)