Investigation the Redistribution Stresses in Fibre Composite Materials Due to Break in Non-Uniform Fibre

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ABSTRACT
In this research, the effect of break in a composite material containing one fibre with non-uniform cross section area was investigated depending on Nearest Neighboring Load Theory. Using Shear Lag Theory, a new equation described the redistribution phenomena was derived starting from the Beyerlein and Landis s equation that deals with the redistribution phenomena in a uniform fiber composite material. A mathematical model described the redistribution stresses phenomena in normal stress due to break in non-uniform fiber of composite material was made. Also, the finite elements model was built using ANSYS 11.0 software.A set of numerical calculating was done in order to study the parameters (Width of the matrix and diameter of the fibre)affecting on the normal stress values and to verify the new equation. The comparison between the ANSYS results and new equation results was done and a good agreement was found.

Keywords: Shear Lag Theory ,Composite material , Fibre with non-uniform, Redistribution stresses.

دراسة أعادة توزيع الأجهادات في المواد المتراكبة نتيجة الكسر في ليف متغير المقطع

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Nomenclatures

English Symbols		
Symbol	Description	Unit
E_f ,E_m	Modulus of Elasticity of fiber and matrix.	GPa.
A_f ,A_m	Cross section area of fiber and matrix.	m^2
W or Wm	Width of the matrix in composite material.	m
W^*_*	Displacement in single matrix break.	-
$\overline{W}$	Displacement in Fourier transformation in single matrix break.	-
D	Fiber diameter.	m
t	Specimen thickness.	m
L	Length.	m
G_m ,G_f	Modulus of Rigidity of matrix and fiber.	GPas.
$K_{i,j}$	Stiffness in finite element method.	-
U^f, U^m	Displacement of fiber and matrix in longitudinal direction.	m
$P^*_*( )$	Dimensionless stress functions due to single fiber break.	-
$Q^*_*( )$	Dimensionless stress functions due to single matrix break.	-
N_*	Shape function.	-
N	Number.	-
u	Total displacement	-
Greek Symbols		
Symbol	Description	Unit
ε_f , ε_m	Fiber and Matrix strain.	-
σ_f , σ_m	Fiber and matrix stress.	Pa.
ρ	Stiffness ratio.	-
χ , λ_1, λ_2 , and η	Function.	-
ξo	Adjustable parameter	-
ω^*	Weighting factor.	-

INTRODUCTION

In last years, the interesting in the failure mechanism in the composite materials were increased because of using the composite materials in several industrial applications, the variety of these materials and need to understand the failure criteria [1]. Therefore, several theories had been tried to explain the failure mechanism in composite materials. Shear Lag Theory was one of these theories [2,3]. Shear Lag Theory assumed that the fiber is the main part in unidirectional composite materials. When the fiber or matrix were broken, the load will be transferred from the broken fiber into the *nearest* intact fibers through the matrix. The transferred load will be distributed on the intact fibers. The share of each intact fiber depends on the distance between the intact fiber and the broken fiber. This theory was known as Nearest Neighboring Load Theory [2,3].

Cox [4] created the first mathematical model that describes the redistribution mechanism of the normal stresses in composite materials. Cox's model (Micro-mechanical model) was developed by Hedgepeth and Van Dyke [5] in order to calculate the stress concentration factor for the three dimensional composite materials. They, also, used this model and the superposition theory to calculate the normal stresses in the composite material containing several cracks.

The stresses transferred between the fiber and matrix was calculated by Nairn [6], which used, starting from Equation of Elasticity for symmetrical composite materials. He derived the general equations, which described the stress transfer mechanism and tested the assumptions that used in the Shear Lag Theory. He[7] made a sample of calculations for the composite materials containing single broken fibre and concluded that the *Shear* Lag Theory have good ability to predict the average axial stress which affects on the fibre and the total strain energy. Also, he concluded that the Shear Lag Theory failed to calculate the shear stress between the fibre and matrix.

Luay [8] studied theoretically the effects of fibre breaks and / or matrix breaks on the redistribution normal stresses. He discussed the Superposition Theory which was used to calculate the redistribution stresses in multi-breaks composite materials. He compared between the results that calculated by using Superposition Theory and that calculated by ANSYS software. He derived a new theoretical equation dealing with the redistribution stresses in the composite material that containing misalignment fibre.

In this paper, the effect of breaking the non-uniform fiber will be studied and a new theoretical equation that described the redistribution stresses will be derived and compared with ANSYS software.

THEORETICAL ANALYSIS

In order to study the breaking of non-uniform fibre on the redistribution mechanism, two models were used. The first model be developed from Hedgepeth and Van Dyke's model [5] (i.e. it used shear lag theory).While the second model used the finite elements method (ANSYS software) for studying the breaking effects. Therefore, the theoretical analysis will be divided into:

SHEAR LAG MODEL

Beyerlein and Landis [9] developed mathematical model that be considered the effect of matrix in carrying axial load in additional to the stresses transformation from fiber to fiber. Their model calculates the axial displacement and stress in the composite material that containing multi-breaks fiber and / or matrix. They assumed in their model:

(1)The composite material is one lamina that contained one raw of fiber only and it can be represented as Two Dimensional Plate. Also, they represented the fibre as One

Dimensional Spring Elements and the matrix as Two plane Rectangular Elements (see Figure(1)).

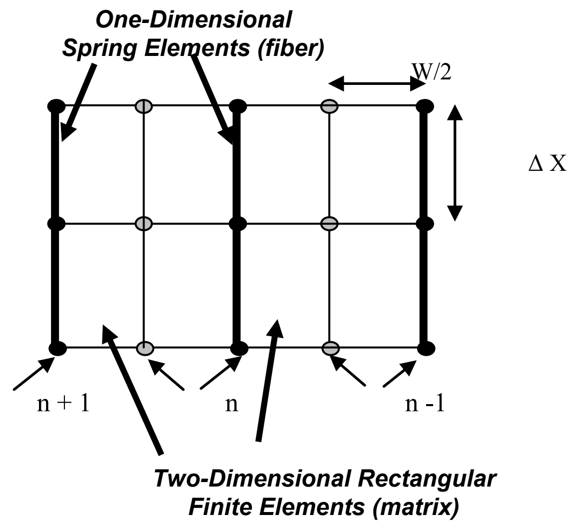


Figure (1): A representative finite element from the Shear lag model for a laminar fiber composite [8, 9].

(2)Thickness of composite material is constant and equals to the thickness of fiber and matrix.

(3)Neglecting the transverse displacement and stress.

(4)Applying the same value of axial strain on the fibre and matrix (see Fig.(2)).

A pair of ordinary differential equations, for each node, can be derived depending on the finite elements method and finite differences method (for more details see [8] and [9]), these equations can be written as:

$$\left[1+\frac{1}{3}\rho\right]\frac{d^2u_n^f}{d\xi^2}+\frac{1}{12}\rho\left(\frac{d^2u_n^m}{d\xi^2}+\frac{d^2u_{n-1}^m}{d\xi^2}\right)+2\left(u_n^m+u_{n-1}^m-2u_n^f\right)=0 \qquad \text{..... (1)}$$

$$\frac{1}{3}\rho\frac{d^2u_n^m}{d\xi^2}+\frac{1}{12}\rho\left(\frac{d^2u_n^f}{d\xi^2}+\frac{d^2u_{n+1}^f}{d\xi^2}\right)+2\left(u_n^f+u_{n+1}^f-2u_n^m\right)=0 \qquad \text{..... (2)}$$

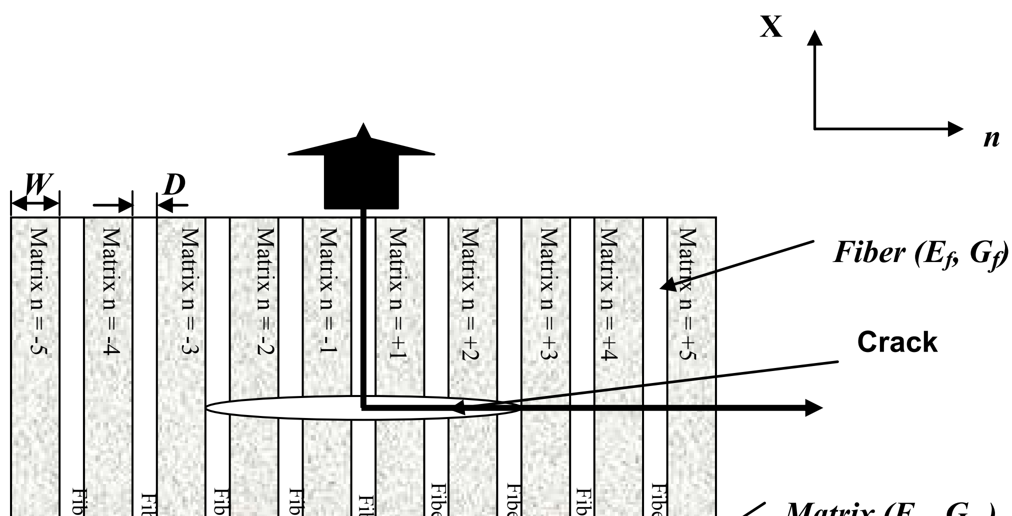


Figure (2): The Two Dimensions Unidirectional Fiber Composite Model.

These equations can be rewritten in the dimensionless form and can be solved for two cases of boundary conditions. The first case was the *Single Fibre Break* case and the second case was the *Single Matrix Break* case. The superposition theory was used to get general solution for the multi breaks case (for more details see [8] and [9]), these three solutions can be written as:

(1) Single Fibre Break Case:

$$P_n^f(\xi) = -\frac{1}{2\pi}\int_0^{\pi}\frac{\eta Cos(n\theta)}{\chi}\left[\lambda_1(\chi-1)\exp\left(-\eta\lambda_1|\xi|\right)+\lambda_2(\chi-1)\exp\left(-\eta\lambda_2|\xi|\right)\right]d\theta \quad \text{....(3)}$$

$$P_n^m(\xi) = \frac{(\rho+1)}{2\pi}\int_0^{\pi}\frac{\eta\left[Cos(n\theta)+Cos[(n+1)\theta]\right]}{\chi}\left[\lambda_1\exp\left(-\eta\lambda_1|\xi|\right)-\lambda_2\exp\left(-\eta\lambda_2|\xi|\right)\right]d\theta \quad \text{....(4)}$$

(2) Single Matrix Break Case:

$$Q_n^f(\xi) = \frac{\rho}{4\pi}\int_0^{\pi}\frac{\eta\left[Cos(n\theta)+Cos[(n-1)\theta]\right]}{\chi}\left[\lambda_1\exp\left(-\eta\lambda_1|\xi|\right)-\lambda_2\exp\left(-\eta\lambda_2|\xi|\right)\right]d\theta \quad \text{.. (5)}$$

$$Q_n^m(\xi) = -\frac{1}{2\pi}\int_0^{\pi}\frac{\eta Cos(n\theta)}{\chi}\left[\lambda_1(\chi+1)\exp\left(-\eta\lambda_1|\xi|\right)+\lambda_2(\chi-1)\exp\left(-\eta\lambda_2|\xi|\right)\right]d\theta \quad \text{.. (6)}$$

(3) Multi- Breaks Case:

$$\frac{\sigma_n^f(\xi)}{E_f\varepsilon} = \frac{\varepsilon_n^f(\xi)}{\varepsilon} = 1+\frac{1}{2}\sum_{j=1}^{N}\omega_j^f P_{n-n_j}^f\left(\xi-\xi_j\right)+\frac{1}{2}\sum_{j=1}^{M}\omega_j^m Q_{n-n_k}^f\left(\xi-\xi_k\right) \quad \text{........ (7)}$$

$$\frac{\sigma_n^m(\xi)}{E_m\varepsilon}=\frac{\varepsilon_n^m(\xi)}{\varepsilon}=1+\frac{1}{2}\sum_{j=1}^{N}\omega_j^f P_{n-n_j}^m\left(\xi-\xi_j\right)+\frac{1}{2}\sum_{j=1}^{M}\omega_j^m Q_{n-n_k}^m\left(\xi-\xi_k\right) \quad \text{........ (8)}$$

Where:

$$\lambda_1^2=\frac{\rho}{3}\left[2+Cos^2\left(\frac{\theta}{2}\right)\right]+1+\chi \ : \ \lambda_2^2=\frac{\rho}{3}\left[2+Cos^2\left(\frac{\theta}{2}\right)\right]+1-\chi \quad : \rho = E_mA_m / E_fA_f$$

$$\chi=\sqrt{\rho^2Cos^2\left(\frac{\theta}{2}\right)+2\rho Cos^2\left(\frac{\theta}{2}\right)+1}$$

$$: \qquad \eta=\frac{72}{12\rho\left(1+\frac{\rho}{3}\right)-\rho^2Cos^2\left(\frac{\theta}{2}\right)}$$

For calculating the values of the constants $\left(\omega_j^m\right)$and$\left(\omega_j^f\right)$, the procedure that used in Beyerlein and Landis [9] must be followed [8].

MODIFIED SHEAR LAG MODEL

For deriving equation that described the axial stresses in the composite material containing single non-uniform fiber break, the effects of varying of diameter of fiber and width of matrix must be studied (see Figure3)). In order to study the effect of the width of the matrix, we assume that there are two uniform composite materials. Each one of them has the same material of fiber and material of matrix and the same diameter of fiber (D). But, the width of the matrix in the first composite material was (W_S), while the width of the matrix in the second composite material was (W_L) (see Figure (4)). If these two composite materials have one broken fiber only, the axial stress, for each composite material, can be written as:

$$\left[P_n^f(\xi)\right]_{WS}=-\frac{1}{2\pi}\int_0^{\pi}\frac{\eta_s Cos(n\theta)}{\chi_s}\left[\lambda_{1s}(\chi_s-1)\exp\left(-\eta_s\lambda_{1s}|\xi|\right)+\lambda_{2s}(\chi_s-1)\exp\left(-\eta_s\lambda_{2s}|\xi|\right)\right]d\theta \quad \text{.. (9)}$$

$$\left[P_n^f(\xi)\right]_{WL}=-\frac{1}{2\pi}\int_0^{\pi}\frac{\eta_L Cos(n\theta)}{\chi}\left[\lambda_{1L}(\chi_L-1)\exp\left(-\eta_L\lambda_{1L}|\xi|\right)+\lambda_{2L}(\chi_L-1)\exp\left(-\eta_L\lambda_{2L}|\xi|\right)\right]d\theta \quad \text{...(10)}$$

$$\left[P_n^m(\xi)\right]_{WS}=\frac{(\rho_S+1)}{2\pi}\int_0^{\pi}\frac{\eta_S\left[Cos(n\theta)+Cos[(n+1)\theta]\right]}{\chi_S}\left[\lambda_{1S}\exp\left(-\eta_S\lambda_{1S}|\xi|\right)-\lambda_{2S}\exp\left(-\eta_S\lambda_{2S}|\xi|\right)\right]d\theta$$

.. (11)

$$\left[P_n^m(\xi)\right]_{WL} = \frac{(\rho_L + 1)}{2\pi} \int_0^{\pi} \frac{\eta_L\left[Cos(n\theta) + Cos[(n+1)\theta]\right]}{\chi_L}\left[\lambda_{1L}\exp(-\eta_L\lambda_{1L}|\xi|) - \lambda_{2L}\exp(-\eta_L\lambda_{2L}|\xi|)\right]d\theta$$

…(12)

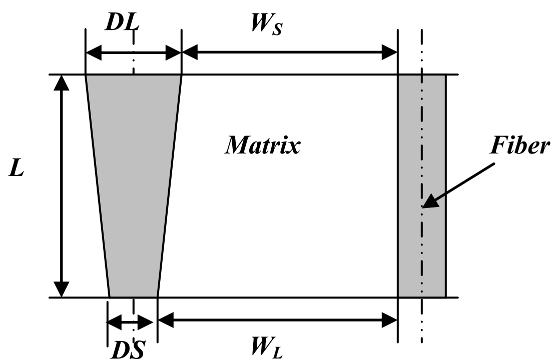


Figure (3): Non-Uniform Cross Section Phenomena.

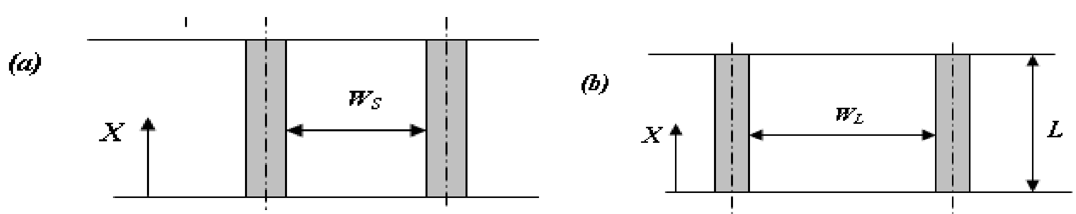


Figure (4): Divisions of Non-Uniform Section Phenomena.

The main difference between these two cases is the varying of the value of ($\rho = E_m A_m / E_f A_f$) due to the varying of the area of matrix (A_m), therefore, the value of axial stress will be change due to change in the values of $(\rho, \eta, \chi, \lambda_1, \lambda_2)$. According to this fact, general axial stress equation, that describes the varying of the matrix width from (W_S) to (W_L), can be written as:

$$\left(P_n^f(\xi)\right)_W = N_{WL}\left[P_n^f(\xi)\right]_L + N_{WS}\left[P_n^f(\xi)\right]_S \quad ......\ (13)$$

Where:

$$N_{WL} = \frac{(W_L - W_s) - W_i}{(W_L - W_s)} \quad ; N_{WS} = \frac{W_i}{(W_L - W_s)} \quad ; W_i = X_i\left(\frac{W_L - W_S}{L}\right)$$

In order to study the effect of the diameter of the fiber, we assume that there are two uniform composite materials. Each one of them has the same material of fiber and material of matrix and the same width of matrix (W). But, the diameter of the fiber in the first composite material was (D_S), while the diameter of the fiber in the second composite material was (D_L) (see Fig.(5)). If these two composite materials have one broken fiber only, the axial stress, for each composite material, can be written as:

$$\left[P_n^f(\xi)\right]_{DS} = -\frac{1}{2\pi}\int_0^{\pi}\frac{\eta_S Cos(n\theta)}{\chi_S}\left[\lambda_{1S}(\chi_S - 1)\exp\left(-\eta_S\lambda_{1S}|\xi|\right) + \lambda_{2S}(\chi_S - 1)\exp\left(-\eta_S\lambda_{2S}|\xi|\right)\right]d\theta \quad \text{....(14)}$$

$$\left[P_n^f(\xi)\right]_{DL} = -\frac{1}{2\pi}\int_0^{\pi}\frac{\eta_L Cos(n\theta)}{\chi_L}\left[\lambda_{1L}(\chi_L - 1)\exp\left(-\eta_L\lambda_{1L}|\xi|\right) + \lambda_{2L}(\chi_L - 1)\exp\left(-\eta_L\lambda_{2L}|\xi|\right)\right]d\theta \quad \text{.....(15)}$$

$$\left[P_n^m(\xi)\right]_{DS} = \frac{(\rho_S + 1)}{2\pi}\int_0^{\pi}\frac{\eta_S\left[Cos(n\theta) + Cos\left[(n+1)\theta\right]\right]}{\chi_S}\left[\lambda_{1S}\exp\left(-\eta_S\lambda_{1S}|\xi|\right) - \lambda_{2S}\exp\left(-\eta_S\lambda_{2S}|\xi|\right)\right]d\theta \quad \text{.....(16)}$$

$$\left[P_n^m(\xi)\right]_{DL} = \frac{(\rho_L + 1)}{2\pi}\int_0^{\pi}\frac{\eta_L\left[Cos(n\theta) + Cos\left[(n+1)\theta\right]\right]}{\chi_L}\left[\lambda_{1L}\exp\left(-\eta_L\lambda_{1L}|\xi|\right) - \lambda_{2L}\exp\left(-\eta_L\lambda_{2L}|\xi|\right)\right]d\theta \quad \text{.....(17)}$$

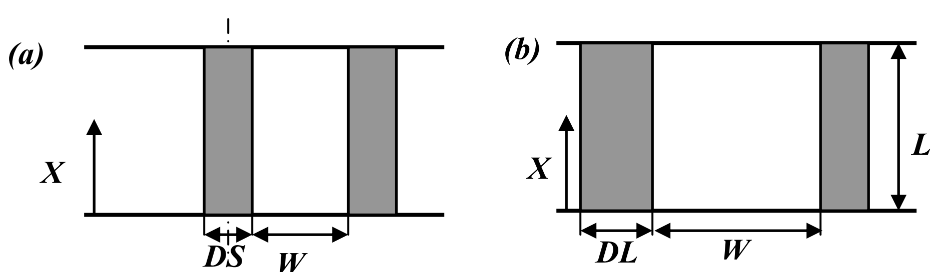
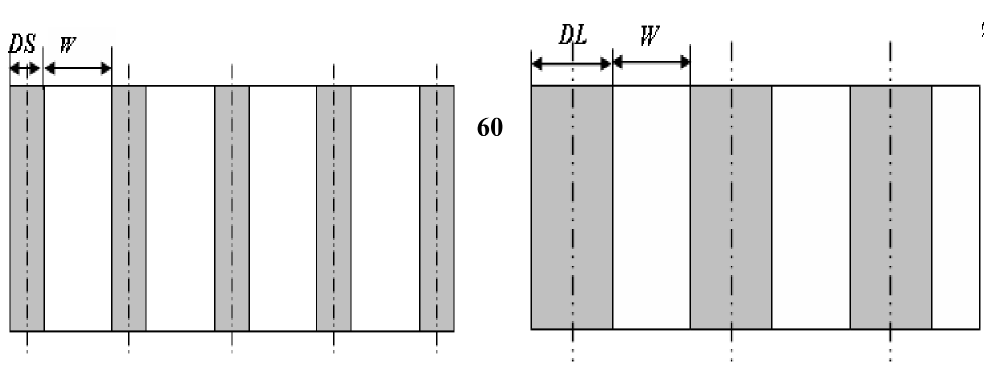


Figure (5.1): Divisions of Non-Uniform Section Phenomena



(a) ***(b)***

Figure (5.2): Fibers Arrangements in the Composite material

The main difference between these two cases is the varying of the value of ($\rho = E_m A_m / E_f A_f$) due to the varying of the area of fibre (A_f), therefore, the value of axial stress will be change due to change in the values of $(\rho, \eta, \chi, \lambda_1, \lambda_2)$. According to this fact, general axial stress equation, that describes the varying of the fibre diameter from (D_S) to (D_L), can be written as:

$$\left(P_n^f(\xi)\right)_D = N_{DL}\left[P_n^f(\xi)\right]_L + N_{DS}\left[P_n^f(\xi)\right]_S \quad \ldots\ldots (18)$$

Where:

$$N_{DL} = \frac{(D_L - D_s) - D_i}{(D_L - D_s)} \;;\; N_{DS} = \frac{D_i}{(D_L - D_s)} \;;\; D_i = X_i\left(\frac{D_L - D_S}{L}\right)$$

Now, general equation described the effects of the diameter of the fibre and the width of the matrix (i.e. the non-uniform cross section area of fibre) can be written as:

$$\left(P_n^f(\xi)\right)_{Total} = \left(N_{WL}\left[P_n^f(\xi)\right]_L + N_{WS}\left[P_n^f(\xi)\right]_S + N_{DL}\left[P_n^f(\xi)\right]_L + N_{DS}\left[P_n^f(\xi)\right]_S\right)/2 \quad \ldots. (19)$$

FINITE ELEMENTS MODEL (ANSYS MODEL)

A finite elements model was built using ANSYS software (ANSYS 11.0) [10]. This model assumed:

(1)The fibres and matrix in the composite material were represented as two dimensional elements called (PLANE82) (see [11],[12]).

(2)The nodes in the lines between each neighbouring fibre and matrix participate between them.

(3)The transverse displacements were neglected by deleting transverse degrees of freedom [10].

(4)The fibre and the matrix have the same thickness.
(5)The crack was represented as a very small rectangular space (i.e. without crack tip) in order to prevent the stress concentration effect and to keep the volume fraction at its value.

RESULTS AND CONCLUSIONS

In order to get a set of numerical results for non-uniform cross section fibre composite material, the following properties were used (Table (1)):

Table (1): The Properties of Fibre and Matrix in Composite Materials.

Material	Properties		
	Modulus of Elasticity (GPa)	Modulus of Rigidity (GPa)	Poisson Ratio
Epoxy	3.4	1.4	0.4
E- Glass	72.4	30	0.3

Also, the dimensions of the specimen composite material were:
Specimen Length = Length of Fibre = 60 mm.
Width of the matrix = 5 mm.
Largest Fibre diameter = 3 mm.
Smallest Fibre diameter = 2 mm.

Figure (6) shows arrangement of counter redistribution for normal stress in the Non-Uniform section area not including broken fiber, The normal stress in the Non-Uniform section area broken fiber and contour redistribution for displacement.

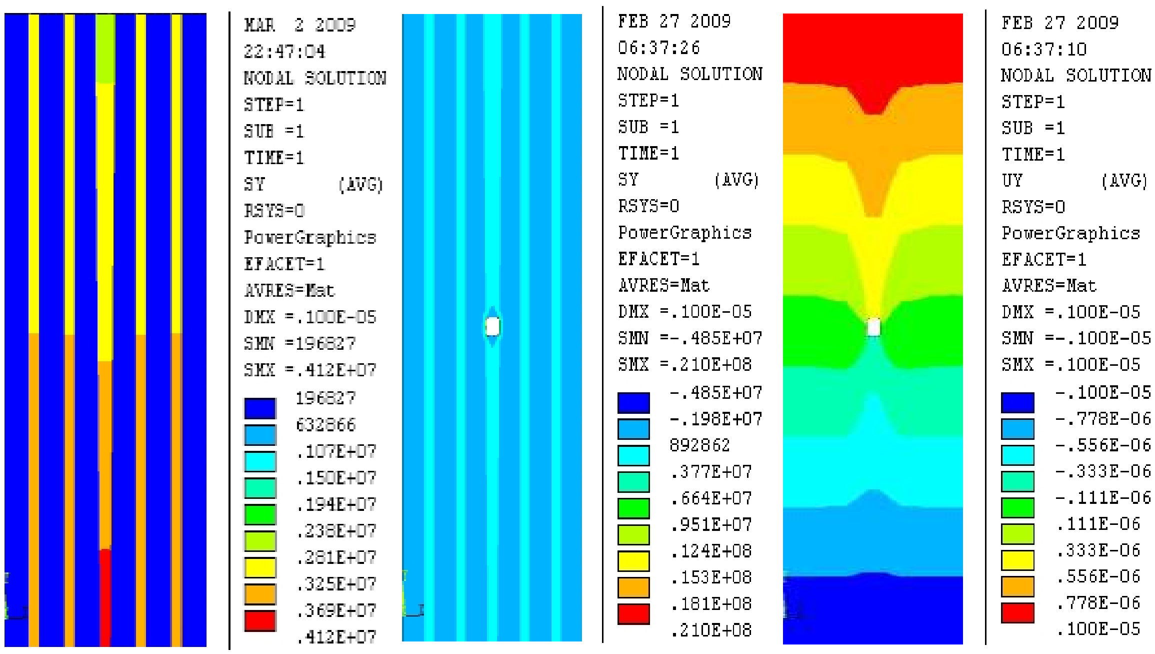
Figure (7) show the comparison between the normal stress, (σ_y), calculating by two model along the broken fiber (F_0). The normal stress will be decrease when the diameter of the fiber increases (or matrix width decrees). In the region has the highest proportion of fiber volumetric fraction (25%), while at the bottom of fiber volumetric fraction (12.5%) depending on the fiber variable diameter. From the figure, the shear lag result shows the effect of non-uniform fiber. Where the values of normal stress in the left side of crack are smaller than that in the right side.

Also, Figures (8-A,B and C) show the comparison between the normal stress, (σ_y), calculating by two models along the intact matrixes (M_1, M_2, M_3). The normal stress will be decrease when the diameter of the fiber increases (or matrix width decrees) continues to distribution of normal stress in other regularly as a result of the irregular fiber diameter (F_0) and its neighbouring matrix (M_1).

Figures (9-A and B) show the comparison of the normal stress, (σ_y), calculating by two models along the intact fibers (F_1, F_2). The normal stress will be decrease when the diameter of the fiber increases (or matrix width decrees).

For varying fiber diameter, the stress was distributed with different rate along the fibre because of:

1. In intact (uniforms) fibres and matrix, the stress was constant along them. But in broken fibre (varying fibre diameter) the stress along the fibre was changed due to the change of cross section area.
2. The stress transfers from the crack fibre to F_1 depending on the width of matrix and the ratio ($\rho = (E_m A_m)/(E_f A_f)$). Since these two variables were changed in case of broken and changeable fibre diameter, therefore the stress transferred will be changed too.



(a) (b) (c)

Figure (6): (a) Contour redistribution for normal stress not including broken fibre (N/m^2).(b) Contour redistribution normal stress in the including broken fibre (N/m^2). (c) Contour redistribution for displacement (m).

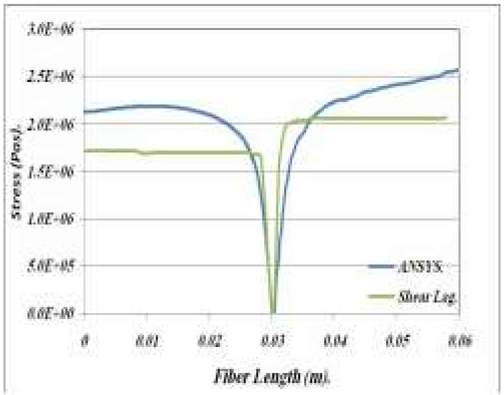


Figure.(7): Redistribution of Normal Stress along the Fiber Length for the Broken Fiber

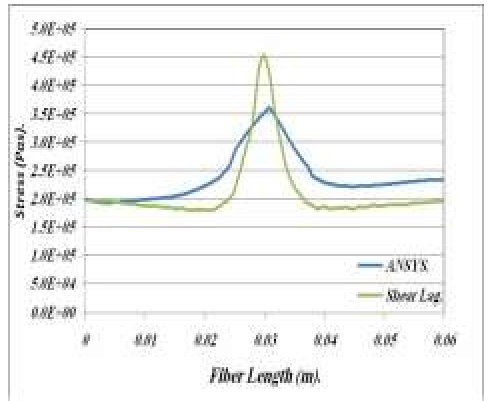


Figure.(8-A): Redistribution of Normal Stress along the Fiber Length for the First Nearest Neighboring Matrix

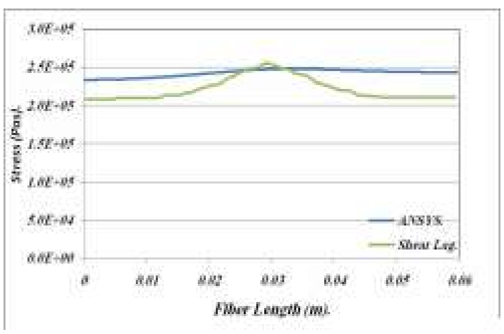


Figure.(8-B): Redistribution of Normal Stress along the Fiber Length for the Second Nearest Neighboring Matrix

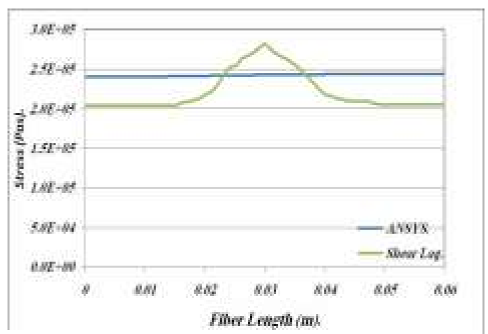


Figure.(8-C): Redistribution of Normal Stress along the Fiber Length for the Third Nearest Neighboring

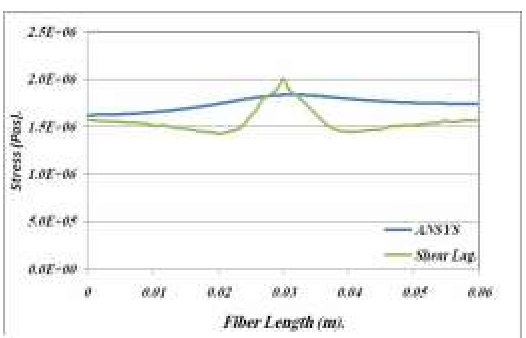


Figure.(9-A): Redistribution of Normal Stress along the Fiber Length for the First Nearest Neighboring Fiber

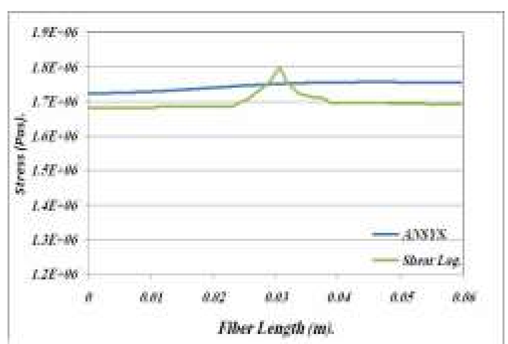


Figure.(9-B): Redistribution of Normal Stress along the Fiber Length for the Second

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