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# Evaluation of Critical Stress Intensity Factor ($K_{ic}$) for Plates Using New Crack Extension Technique

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## ABSTRACT

The technique of crack extension is applied to the computation of critical stress intensity factor in linear elastic fracture mechanics for cracked plates in tension for different crack configuration (i.e. central crack, edge crack, and double edge crack). The new technique uses the Brown approximate solutions for stress intensity factors and the Westergaard analytical solutions for stress and displacement near a crack tip in finite plate to calculate crack extension during each load step and then calculating the critical stress intensity factor using an incremental procedure. A matlab program was developed for the purpose of this work, which proved to be a good tool for the computation of critical stress intensity factors for cracked plates. The results were in good agreement with results of other methods available in the literature.

## حساب معامل تركيز الاجهاد الحرج للصفائح المتشققة باستخدام تقنية جديدة لتمدد الشق

### الخلاصة

تقنية تمدد الشق طبقت لغرض حساب معامل شدة الاجهاد الحرج في ميكانيكية الكسر الخطي لصفائح متشققة تحت احمال شد لوضعية شقوق مختلفة ( شق متمركز، شق جانبي، و صفيحة ذات شقيين جانبيين). التقنية الجديدة تستخدم حلول براون التقريبية لايجاد معاملات تركيز الاجهاد وحلول وسترجارد التحليلية للاجهاد والمسافة قرب راس الشق في صفيحة محددة، المتوفرة في المصادر لحساب تمدد الشق لكل خطوة تحميل ومن ثم حساب معامل التركيز الحرج باستخدام طريقة تصاعدية. طور برنامج ماتلاب لهذا الغرض . واثبتت انها اداة جيدة لحساب معامل شدة الاجهاد الحرج للصفائح المتشققة واثبتت النتائج انها بتوافق جيد مع نتائج طرق اخرى متوفرة بالمصادر.

## INTRODUCTION

A variety of methods are currently available for computing critical stress intensity factors for linear elastic crack problems. The stress intensity factor (SIF) is a measure of the strength of the stress singularity at the



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crack tip, and is useful from a fracture mechanics perspective as it characterize the displacement, stress, and strain in the vicinity of the crack tip. Additionally, the stress intensity concept is important in terms of crack extension as critical values of the SIF govern crack initiation[1]. The calculation of SIFs in finite plates under tensile loading conditions is usually done through numerical approximation.

Typically this is performed using finite elements and boundary element methods [2,3], or recently mesh free methods [4] and extended finite element methods [5]. For modeling of crack extensions, the original crack length must be modified during the loading condition at each loading step. This requires calculating the displacement field near the crack tip at each load step and updating the crack length before the next load step. This procedure is very difficult when using methods such as finite elements and boundary elements because of updating the original mesh of the problem and this demands time and effort consumption from the analyst.

This paper presents a matlab procedure for the above analysis using the approximate and analytical solution for crack problems available in the literature to compute the critical stress intensity factors and updating crack length without the need for re-meshing
or further calculations.

## GOVERNING EQUATIONS

The Westergaard principal stress field equations for mode I stress intensity factor in infinite plate are given in reference [3] as follows:

$$\sigma_1 = \frac{K_I}{(2\pi r)^{1/2}} \cos\left(\frac{\theta}{2}\right)\left[1+\sin\left(\frac{\theta}{2}\right)\right]$$

$$\sigma_2 = \frac{K_I}{(2\pi r)^{1/2}} \cos\left(\frac{\theta}{2}\right)\left[1-\sin\left(\frac{\theta}{2}\right)\right]$$

$$\sigma_3 = \begin{Bmatrix} \upsilon(\sigma_1+\sigma_2).........Plane-strain \\ 0.......................plane-stress \end{Bmatrix} \quad \text{.... (1)}$$

The displacement field for the same mode are also given as follows:

$$u = \left(\frac{K_I}{4\mu}\right)\left(\frac{r}{2\pi}\right)^{1/2}\left[(2\kappa-1)\cos\left(\frac{\theta}{2}\right)-\cos\left(\frac{3\theta}{2}\right)\right]$$

$$v = \left(\frac{K_I}{4\mu}\right)\left(\frac{r}{2\pi}\right)^{1/2}\left[(2\kappa+1)\sin\left(\frac{\theta}{2}\right)-\sin\left(\frac{3\theta}{2}\right)\right] \quad \text{... (2)}$$

where,

$$\mu = \frac{E}{2(1+\upsilon)}$$

$$\kappa = \begin{Bmatrix} (3-\upsilon)/(1+\upsilon).........Plane-stress \\ (3-4\upsilon)..................Plane-strain \end{Bmatrix}$$

The Brown solution for $K_I$ stress intensity factor of finite plate under tension load for different crack configuration, limited to half crack length to plate width ratio

$$K_I = C\sigma\sqrt{\pi a} \qquad \text{.....(3)}$$

Where the compliance function *C = f(a/W)* for:

(i) Central crack Plate

$$C = 1 + 0.256\left(\frac{a}{W}\right) - 1.152\left(\frac{a}{W}\right)^2 + 12.2\left(\frac{a}{W}\right)^3$$

(ii) Single edge crack plate

$$C = 1.12 - 0.231\left(\frac{a}{W}\right) + 10.55\left(\frac{a}{W}\right)^2 - 21.72\left(\frac{a}{W}\right)^3$$

$$+ 30.39\left(\frac{a}{W}\right)^4$$

(iii) Double edge crack plate

$$C = \frac{1.122 - 0.561\left(\frac{a}{W}\right) - 0.205\left(\frac{a}{W}\right)^2 + 0.471\left(\frac{a}{W}\right)^3 - 0.190\left(\frac{a}{W}\right)^4}{\sqrt{1-\left(\frac{a}{W}\right)}}$$

σ is the applied tensile stress, a is half the crack length, and W is the plate width.

The elastic stress field in the vicinity of a crack tip, as given by the previous equations, shows that as r tends to zero the stresses become infinite (i.e. stress singularity at the crack tip). Since many structural materials deform plastically above the yield stress, there will be in reality a plastic zone surrounding the crack tip, and the elastic solution for such situations, may require modification to some of the linear elastic fracture mechanics concepts. The two physically acceptable yield criteria for metals and alloys are the well-known Tresca and von Mises yield criteria. In this work the von Mises criterion will be considered which requires that the distortion energy per unit volume approaches its critical value. In simple tension this criterion can be expressed in terms of principal stresses as follows [7]:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2Y^2 \qquad \text{.....(4)}$$

where Y is the yield stress of material in simple tension.

The summation of the square terms in the above equation termed the equivalent stress ($\sigma_e$) which represents the strength of the plasticity near the crack tip and can be plotted in a contour form to show the boundary of the plastic zone surround the crack tip.

**ANALYSIS PROCEDURE**

The steps for the new procedure of calculating the crack extension are shown in Figure (1). A matlab program is written based upon the the mentioned steps. To validate the procedure and the program a three case studies of cracked plates with three different configuration are chosen for this purpose and the results are shown in the next section.

**RESULTS AND DISCUSSION**

An Aluminum plate of dimension 50 mm x 20 mm and thickness 1 mm, with initial crack length of 6 mm, given in Reference [8], with modulus of elasticity 70 GN/m$^2$, and possion's ratio 0.33 and Yield stress of 175 MN/m$^2$ is considered according to Reference [9], (yield stress for aluminum is 150-200 MPa), for the analysis of different crack configuration as shown below:

**(a)- Central-crack plate**

The plate considered to demonstrate the analysis is a central cracked plate of width *W* and half crack length *a*. Using the developed matlab program, the analysis starts by considering an *a/W* ratio of 0.1 and a 25 stress increments starting at a stress value which just causes yielding. The results for this analysis are as follows:

Figure (2) shows the calculated stress intensity factor $K_I$ for each stress increment. It is obvious from this figure that the $K_I$ value increases with the increasing of the load increment.

Figure (3) illustrate the calculated crack length for each stress increment starting with initial half crack length of 2. It is clear from this figure that the behavior of the crack extension is not linear. Figure (4) presented the calculated principal and equivalent stresses and the yield stress for each crack length. This figure is very important because it shows the value of the critical crack length which is the point of intersection between the equivalent stress and the yield stress.

The crack extentions $\delta_a$ have been calculated for each load step and presented against the stresses in Figure (5), which shows the point of intersection where the critical crack extension occurres. Figure (6) shows the behavior of the calculated $K_I$ values with the increase in crack length during the analysis. It is clear that the behavior is not linear and the calculated values are increasing with the increase of crack length. Finally, for this case, the normalized crack extension values against different *a/w* values are presented in Figure (7). This figure is very important for designers since its described the behavior of the crack extension with different crack length to width ratio .

**(b)- Other crack configuration**

The other crack configuration considered for the analysis are the single-edge notch plate and the double edge notched plate, in which their results are compared to the previous crack configuration as follows:

Figure (8) shows the crack extension for the thress crack configuration of the plate (i.e. the cental crack plate (CC plate), the single-edge notch plate (SE plate), and the double-edge notch plate (DE plate)), calculated at the point of intersection at each *a/W* ratio. It is clear from the Figure that the DE plate configuration exhibit the largest extension among the other configurations. Finally, Figure (9) represents the crack extension design curve for the three configurations. Its shows that the crack extension ratio for the DE plate remain nearly constant with the ratio of *a/W* , while the other configurations behave differently as its clear from the figure. This figure is very essential for designers and engineers since its demonstrate the behavior of the crack extension for different crack configurations.

## CONCLUSIONS

From the above analysis, the following conclusions can be drawn:

1- The new crack extension technique presented in this paper proved to be very effective and essential for calculating critical crack extensions for cracked plate with different crack configurations.

2- The normalized design curve found in this work, seem to be very essential for designers and engineers.

3- The developed procedure reduced the need for numerical analyses such as finite elements or boundary elements, which require more time and effort, to calculate the crack extension.

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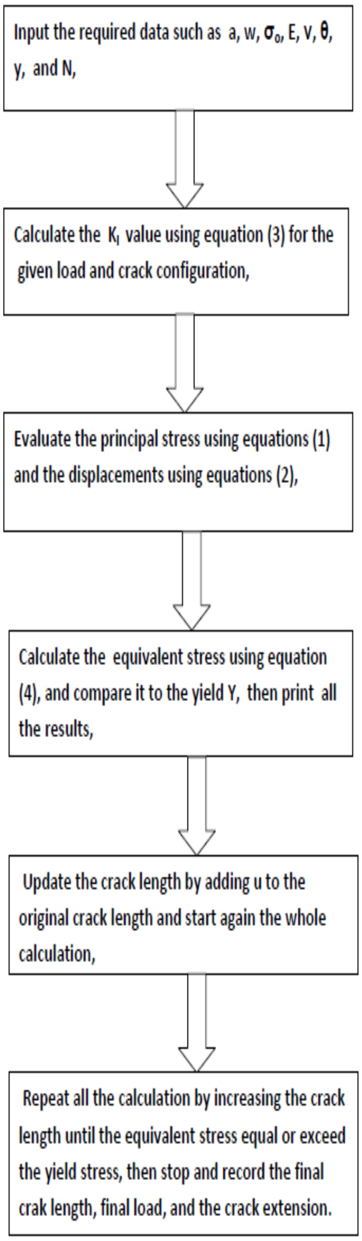



**Figure.(1) Procedure steps for the new crack extension technique.**

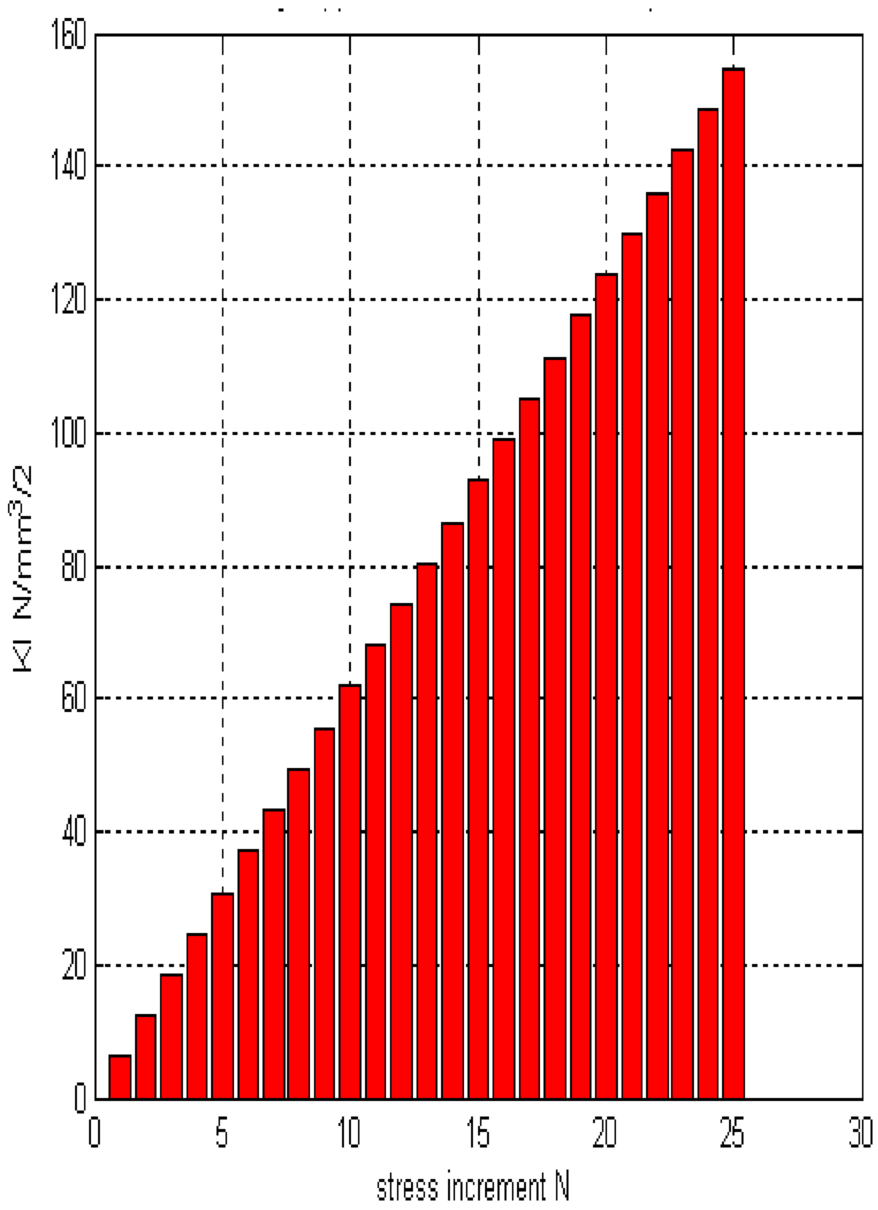



**Figure (2) KI versus stress increment (N).**

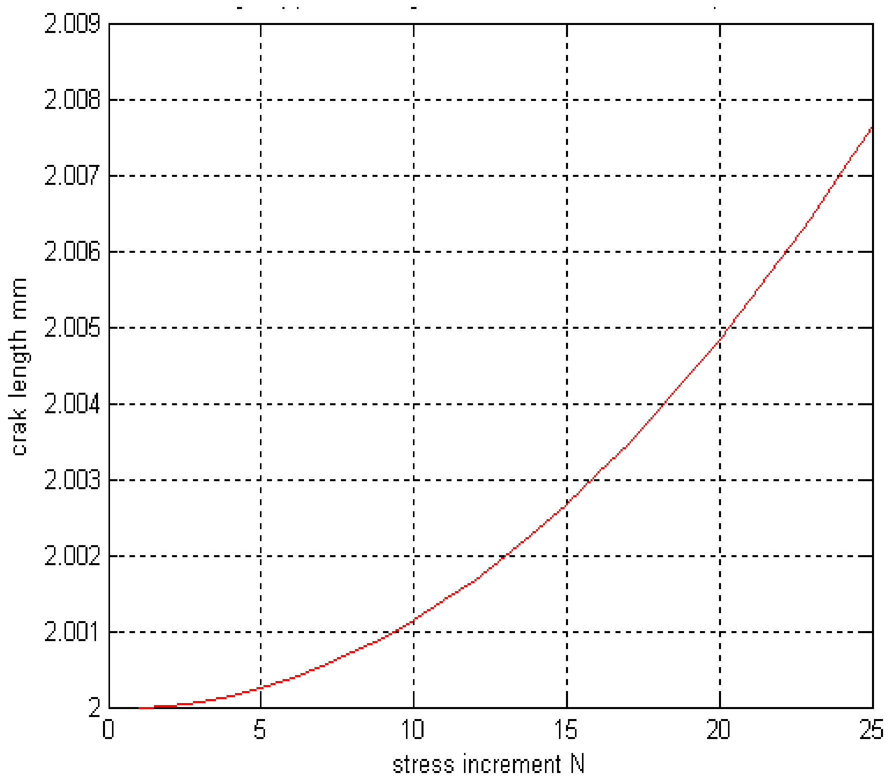
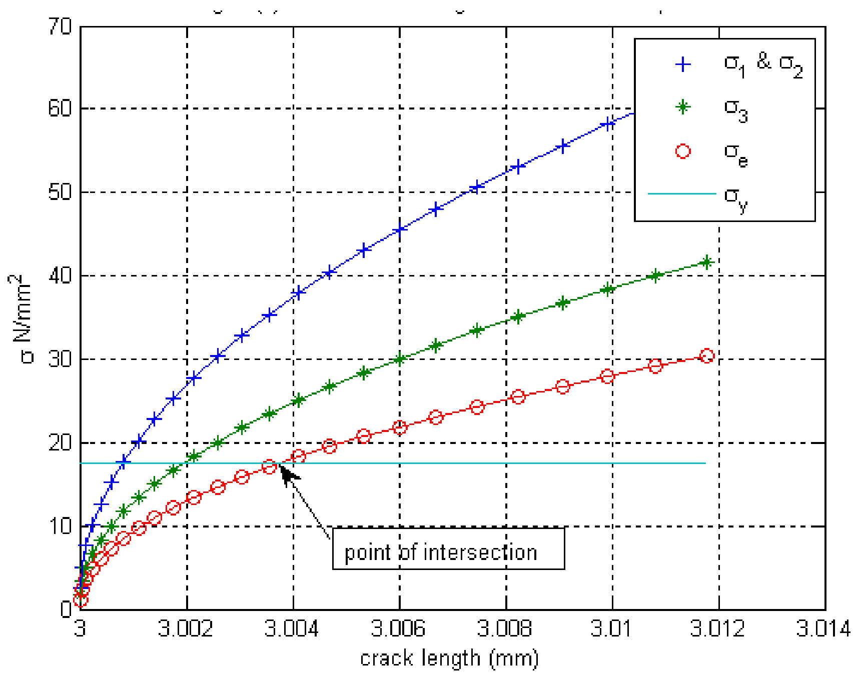



**Figure (3) Crack length versus N for central crack plate.**




**F**

**igure (4) Variation of critical crack length for central crack plate with σ (N/mm²).**

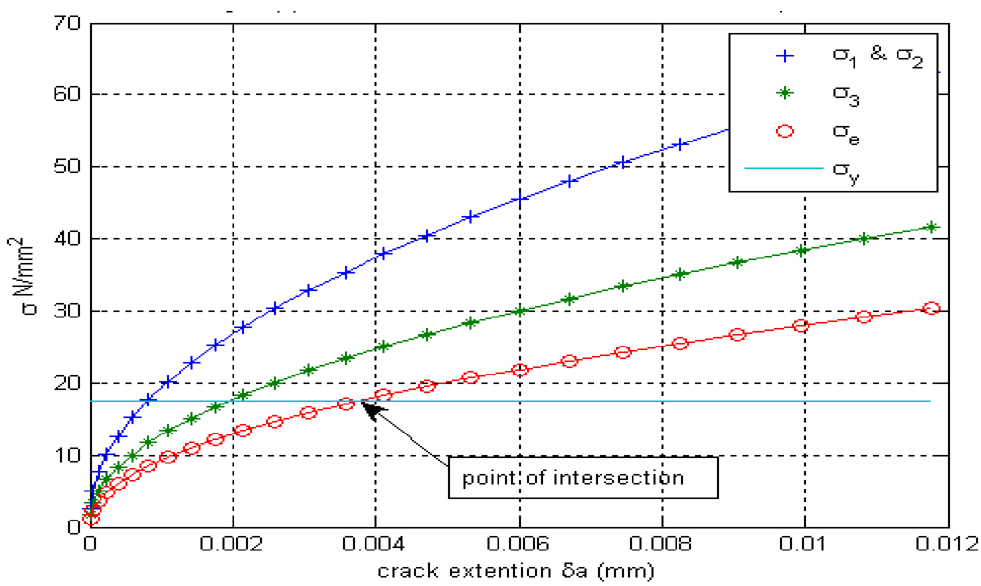
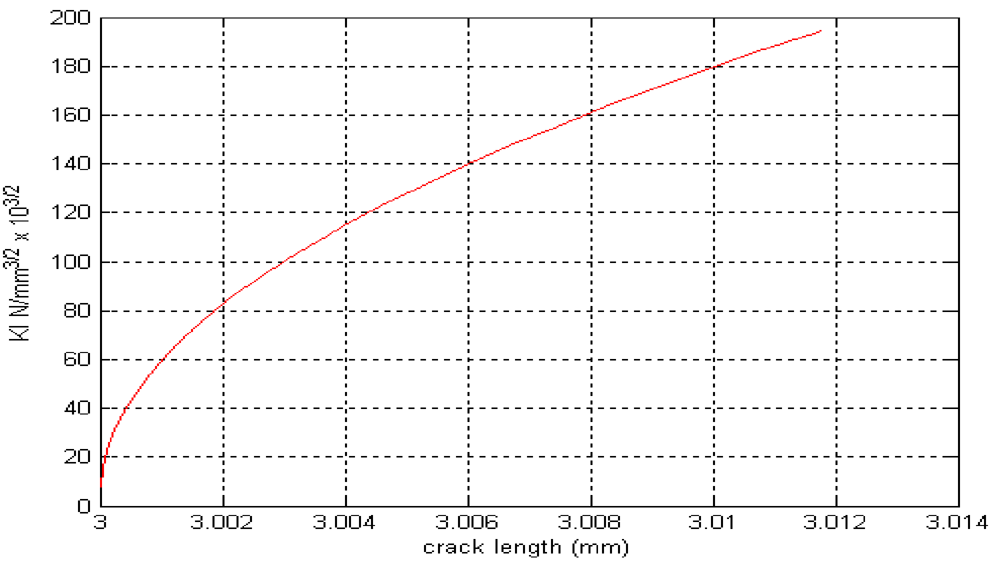
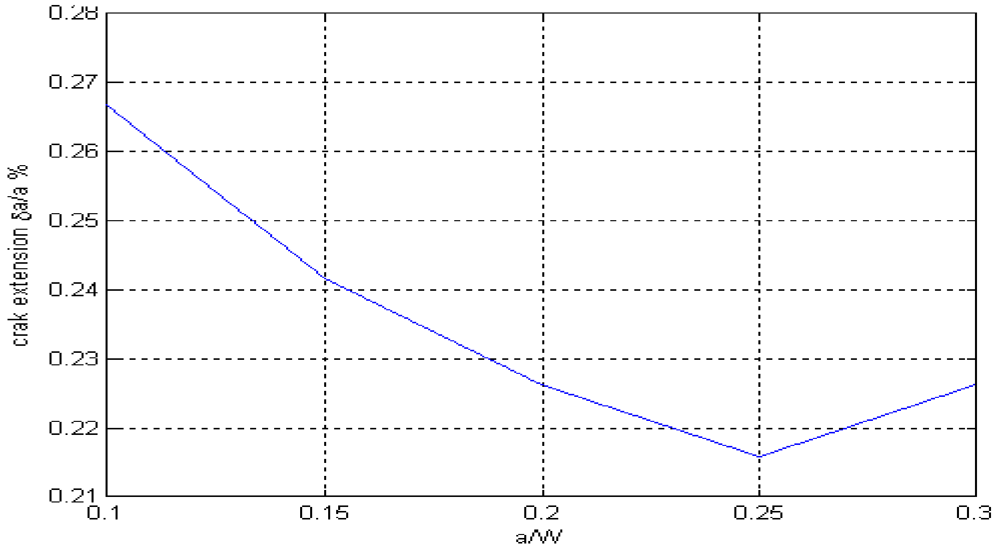


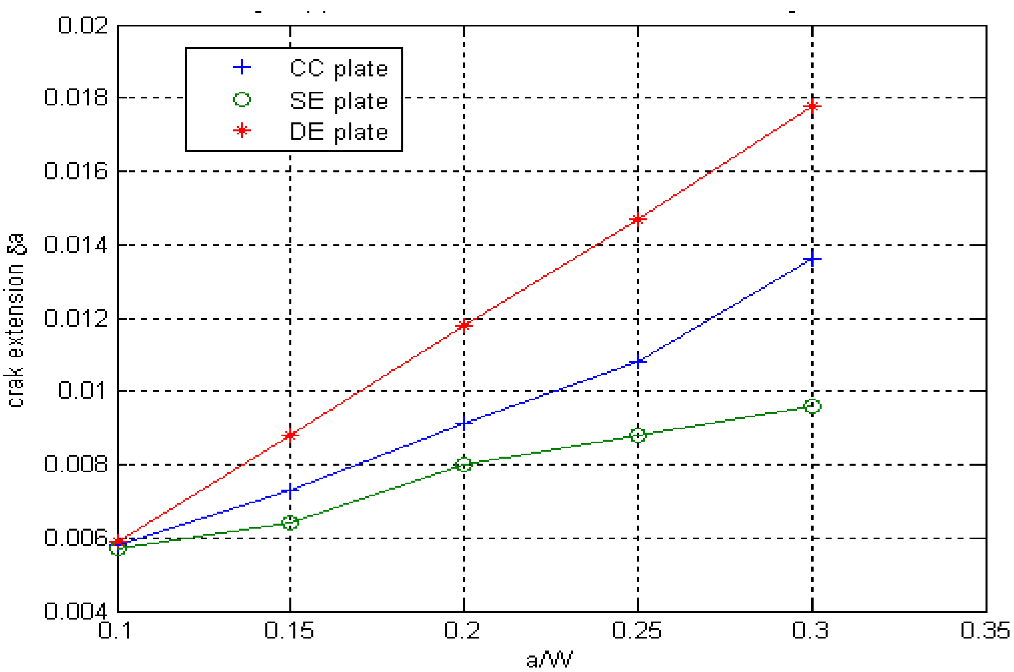

**Figure (5) Variation of critical crack length for central crack plate with σ (N/mm$^2$).**



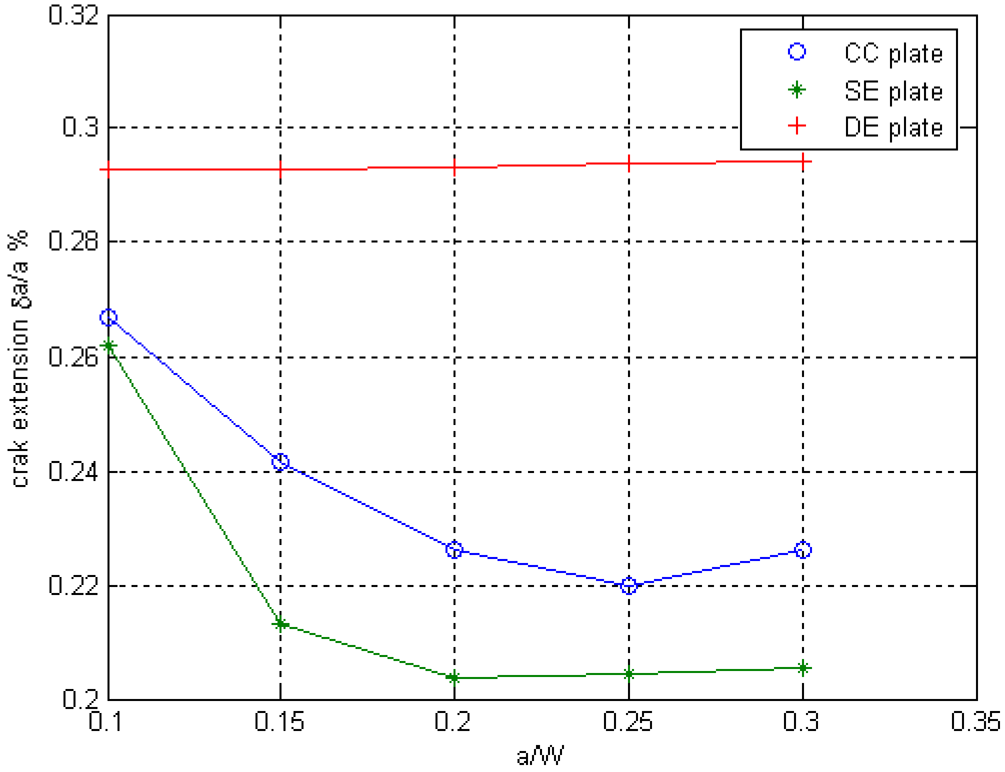

**Figure(6) K versus crack length for central crack plate.**




**Figure(7)Crack Extention Design Curve for Cental Cracked Plate.**




**Figure(8) Crack Extention for different Crack onfiguration Plate.**




**Figure(9)Crack Extention Design Curve for different crack configuration.**