# A Bezier Curve Based Free Collision Path Planning of an Articulated Robot 

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#### Abstract

The main objective of this paper is to find a path for the robot arm from its given start point to its desired goal point in an automated manner without collision with the obstacles. This paper investigates the problem of path planning for a 5 axis robot, operating in environment with obstacles whose boundaries are enveloped by cubic shape. The path planning approach presented is developed in the robot joint space and consists of three steps. The first step is to used Bezier curve technique , the second step is to generate a sufficient number of intermediate points in Cartesian space along Bezier curve and the third step is to convert the coordinate of the generated intermediate points from it's Cartesian space into joint space and move the robot arm along the free collision generated path. This work is not limited to theoretical studies or simulations, experiments have been run with various tests, on a LabVolt R5150 robot to assess the real efficiency and usability of the adopted method. The method applies to robots in a fixed and known environment. A number of experiments were carried out to test the ability of the robot arm to reach it's goal without collision. Experiment results verify the effectiveness of the method developed in this paper.


Keywords: Free collision path planning, Articulated robot, Bezier curves, Joint space


الخلاصة
ان الهـف الاساس من هذا البحث هو تخطيط مسار لروبوت ممفصل ذا خمسة محـاور وبشكل مؤتمت وخال من التماس او الاصطدام بالعو ائق الموجودة في محيط العمل بعد تعريف نقطتي بداية ونهاية المسار . اعتمدت الطريقة المقترحة في هذا البحث ثلاثة خطوات لنوليد المسـار , حيث من خلال المرحلة الاولى تم تطبيق مفهوم بيزر لاشتقاق وتوليد المنحنيـات المطلوبـة ( المسـارات) في حين تم تقسيم المنحنيات ضمن المرحلة الثانية الـى عدد ملائم من التّقسيمات استنتّجت مـن خلالهـا النقاط البينية الملائمة والني تقع على المسـار المستنتج, امـا المرحـة الثالثة فمـن خلالها تم تحويل


$$
\begin{aligned}
& \text { الاحداثيات الديكارتية للنقاط اليينية المستنتجة الـى احداثيات مفاصـل الربوت لقيـادة ذراع الروبوت } \\
& \text { ضمن المسار المقترح دون الاصطدام بالعو ائق. }
\end{aligned}
$$

$$
\begin{aligned}
& \text { عملية وباستخدام الروبوت LabVolt R5150 لاختبار كفاءة وفاعلية الطريقة المقترحة حيث تم } \\
& \text { النطبيق في بيئة ثابتة ومعرفة من خلال عدة تجـارب لاختبـار قابليـة وصـول الروبوت للهـدف دون } \\
& \text { الاصطدام بالعو ائق, واثبتت جميع التجارب نجاح وفاعلية الطريقة المقترحة لنوليد المسار. }
\end{aligned}
$$

## INTRODUCTION

A
manipulator without sensors has no ability to avoid obstacles in its workspace and it has to be taught every point on its trajectory, so that the arm may be free from collision as the arm moves along desired path from a start point to a goal point. This path is stored and used each time the manipulator is moved from the start to the goal points. Obviously, this method is only good in cases of repetitive tasks where there is no variation in the position of either the start or the goal point [1].

Bezier curves have several advantages for geometric modeling. The first and last control points are coincident with the endpoints of the curve segment. The curve is also tangent to the first and last edges of the control polygon and the curve generally follows the shape of the control polygon, making it intuitive to modify. Bezier curves can also be strung together, providing automatic continuity between the endpoint of one Bezier curve and the starting point of another Bezier curve [2].

## BEZIER CURVES

Bezier curves have become the foundation of parametric freeform curve and surface geometry in CAD and visualization. A Bezier curve is defined by a series of two or more control points. The control points make up what is called the control polygon. Linear segments that connect the ordered series of control points form the control polygon [3].

Bezier curves have an associated degree. The degree, $m$, of a Bezier curve is:

$$
m=n-1
$$

where $n$ is the number of control points. Thus, a first degree Bezier curve has two control points; a second degree Bezier curve has three control points, etc. Examples of Bezier curves are included in figure 1. The control polygons are shown as lines connecting the control points. The shape of the Bezier curve is completely defined by the location of the control points. By moving the control points, the curve changes in a unique, mathematically defined manner [4].


Figure (1). Bezier Curves.
In this work the property of Bezier curves which that the curve pass only through the start and the end control points and it does not pass through the intermediate control points, has been invested to plan free collision robot path by considering the start and the end control points as the initial and the goal position of the robot path while the other control points are considered as obstacles as clearly defined in the next section.

## OBSTACLE DESCRIPTION

Obstacles may have polygonal or any other shape. However, it is not particularly desirable to have the manipulator pass very close to the obstacle boundaries, and thus the smallest cube which bounds an original obstacle has been used in this work to approximate the obstacle. Every point on the robot arm has to be located outside the cubic obstacle to ensure a collision free trajectory with that obstacle.

## PATH PLANNING PROCEDURE

In order to move the robot arm from the start point to the goal point in the presence of obstacles, a sequence of joint angles along the path have to be determined [5]. The problem of finding a feasible collision free path, from Start to Goal, can be solved by applying Bezier technique, a number of intermediate points will be found and used for path planning.
The adopted path planning method to move the arm through a number of intermediate points to reach the desired goal point is illustrated in the following steps:

Step one : input Cartesian coordinates of Start point, Goal point and obstacles Step two: considering the previous points as a control points of a Bezier curve
Step three: applying Bezier curve techniques $\quad P(t)=\sum_{i=0}^{n} P_{i} B_{i, n}(t)$
Step four: partitioning the generated curve into sufficient intermediate segments and points
Step five: convert the Cartesian space of each generated intermediate point into joint space

Step six: move the robot arm through the generated joints spaces from the start point towered the goal point
Path planning cases:
One of the objectives for path planning in the Cartesian space is to minimize the distance between the starting point and goal point [6]. As Robot path can be represented by dotted line and the position of various nodes can be expressed by $P_{i}$, which means that the robot arm can move along the dotted lines and complete the path. $P_{i}$ represent the intermediate nodes that the robot must pass through. When the robotic arm moves, the path can be represented by a serious of nodes $P_{i}$. So, the primary problem in Cartesian coordinate space path planning is how to generate a series of intermediate nodes between the beginning (Start) and the end (Goal) of path, which is identified by nodes $P_{i}$. In this work the intermediate nodes of the robot path have been generated using Bezier curve techniques by identify it's control points and it's degree equation (1). Mathematically a parametric Bezier curve of nth-degree is defined by:

$$
\begin{equation*}
P(t)=\sum_{i=0}^{n} P_{i} B_{i, n}(t) \tag{1}
\end{equation*}
$$

As an example, given the control points P0.P1, P2, P3, the cubic Bezier curve can be defined as:

$$
\begin{equation*}
P(t)=\sum_{i=0}^{3} P_{i} B_{i, 3}(t) \tag{2}
\end{equation*}
$$

Where

$$
B_{0,3}(t)=(1-t)^{3}, B_{1,3}(t)=3 t(1-t)^{2}, B_{2,3}(t)=3 t^{2}(1-t), B_{3,3}(t)=t^{3}
$$

are the Bernstein polynomials of degree three.
After the generation of the intermediate nodes of the robot's trajectory, then the simplest path between these nodes is the straight line segments to guide the robot arm along the desired free collision path, by considering that the control points as obstacles .

This work is not limited to theoretical studies or simulations, experiments have been run with various tests, on a LabVolt R5150 robot at university of technology to assess the real efficiency and usability of the proposed technique. The method applies to robots in a fixed and known environment. A number of experiments were carried out to test the ability of an stationary revolute robot to reach it's goal without collision.
1-Environment with single obstacle : in this case three nodes [ start, obstacle and goal] as shown in the Table(1), have been modeled to generate $2^{\text {nd }}$ order Bezier curve according to the equation (3) :

$$
\begin{equation*}
P(t)=\sum_{i=0}^{2} \boldsymbol{P}_{i} \boldsymbol{B}_{i, 2}(t) \tag{3}
\end{equation*}
$$

Table (1) Physical environment coordinates of single obstacle case.

|  | START P0 | OBSTACLE P1 | GOAL P2 |
| :---: | :---: | :---: | :---: |
| X-coordinate | 150 | 300 | 150 |
| Y-coordinate | 50 | 250 | 350 |
| Z-coordinate | 100 | 200 | 400 |

According to the mathematical formulae of Bezier curve (3) and the coordinate of the control points Table (1) the $2^{\text {nd }}$ order Bezier curve can be generated and represented as illustrated in Figure (2) using Matlab software.


Figure (2) $\mathbf{2}^{\text {nd }}$ order Bezier curve.
The generated Bezier curve [ $2^{\text {nd }}$ order] can be reshaped by sufficient intermediate nodes Figure (3) with suitable increment to guide the robot arm through these nodes to reach the desired goal.


| Intermediate nodes (generated) |  |  |  |
| :--- | :--- | :--- | :--- |
| $P_{i}$ | X-. | Y- | Z- |
| P1 | 150 | 50 | 100 |
| P2 | 164.25 | 69.75 | 110.25 |
| P3 | 177 | 89 | 121 |
| P4 | 188.25 | 107.75 | 132.25 |
| P5 | 198 | 126 | 144 |
| P6 | 206.25 | 143.75 | 156.25 |
| P7 | 213 | 161 | 169 |
| P8 | 218.25 | 177.75 | 182.25 |
| P9 | 222 | 194 | 196 |
| P10 | 224.25 | 209.75 | 210.25 |
| P11 | 225 | 225 | 225 |
| P12 | 224.25 | 239.75 | 240.25 |
| P13 | 222 | 254 | 256 |
| P14 | 218.25 | 267.75 | 272.25 |
| P15 | 213 | 281 | 289 |
| P16 | 206.25 | 293.75 | 306.25 |
| P17 | 198 | 306 | 324 |
| P18 | 188.25 | 317.75 | 342.25 |
| P19 | 177 | 329 | 361 |
| P20 | 164.25 | 339.75 | 380.25 |
| P21 | 150 | 350 | 400 |
|  |  |  |  |

Figure(3) Intermediate points on the curve.

2-Environment with two obstacles: for two obstacles environment, four nodes [start, $1^{\text {st }}$ obstacle, $2^{\text {nd }}$ obstacle and goal] have been defined and modeled as a control points to generate $3^{\text {rd }}$ order ( $n-1$ ) Bezier curve, the coordinates of the four point are illustrated in the Table (2).

Table (2) Physical environment coordinates of two obstacles case.

|  | START P0 | $1^{\text {st }}$ <br> P1 | OBSTACLE | $2^{\text {nd }}$ <br> P2 |
| :--- | :--- | :--- | :--- | :--- |
| X-coordinate | 150 | 150 | 300 | GOAL P3 |
| Y-coordinate | 50 | 200 | 250 | 150 |
| Z-coordinate | 100 | 300 | 200 | 350 |

According to the mathematical formulae of Bezier curve (2) and the coordinate of the control points (Table 2) the $3^{\text {rd }}$ order Bezier curve can be generated and represented as illustrated in Figure (4) using Matlab software.


Figure (4) $3^{\text {rd }}$ order Bezier curve.

The generated $3^{\text {rd }}$ order Bezier curve can be divided and reshaped by sufficient intermediate nodes Figure (5) with suitable increment to generate a free collision path to move the manipulator to the desired goal,


| Intermediate nodes (generated) |  |  |  |
| :--- | :--- | :--- | :--- |
| $P_{i}$ | X | Y | Z |
| P1 | 150 | 50 | 50 |
| P2 | 151.0688 | 71.76875 | 71.76875 |
| P3 | 154.05 | 92.15 | 92.15 |
| P4 | 158.6063 | 111.2563 | 111.2563 |
| P5 | 164.4 | 129.2 | 129.2 |
| P6 | 171.0938 | 146.0938 | 146.0938 |
| P7 | 178.35 | 162.05 | 162.05 |
| P8 | 185.8313 | 177.1813 | 177.1813 |
| P9 | 193.2 | 191.6 | 191.6 |
| P10 | 200.1188 | 205.4188 | 205.4188 |
| P11 | 206.25 | 218.75 | 218.75 |
| P12 | 211.2563 | 231.7063 | 231.7063 |
| P13 | 214.8 | 244.4 | 244.4 |
| P14 | 216.5438 | 256.9438 | 256.9438 |
| P15 | 216.15 | 269.45 | 269.45 |
| P16 | 213.2813 | 282.0313 | 282.0313 |
| P17 | 207.6 | 294.8 | 294.8 |
| P18 | 198.7688 | 307.8688 | 307.8688 |
| P19 | 186.45 | 321.35 | 321.35 |
| P20 | 170.3063 | 335.3563 | 335.3563 |
| P21 | 150 | 350 | 350 |
|  |  |  |  |

Figure (5) Intermediate points on the curve.

3- Environment with three obstacles: for three obstacles environment, five nodes [start, $1^{\text {st }}$ obstacle, $2^{\text {nd }}$ obstacle, $3^{\text {rd }}$ obstacle and goal] were defined and modeled to generate 4th order ( $\mathrm{n}-1$ ) Bezier curve, the coordinate of the five points are illustrated in the Table ( 3 ).

Table (3) Physical environment coordinates of three obstacles case.

|  | Start P0 | $1^{\text {st }}$ Obstacle P1 | $2^{\text {nd }}$ Obstacle P2 | $3^{\text {rd }}$ Obstacle P3 | Goal P4 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| X-coord. | 150 | 150 | 300 | 300 | 150 |
| Y-coord. | 50 | 200 | 250 | 300 | 350 |
| Z-coord. | 100 | 300 | 200 | 400 | 400 |

According to the mathematical formulae of Bezier curve (4) and the coordinate of the control points (Table 3) the $4^{\text {th }}$ order Bezier curve can be generated and represented as illustrated in Figure (6) using Matlab software.

$$
\begin{equation*}
P(t)=\sum_{i=0}^{4} P_{i} B_{i, 4}(t) \tag{4}
\end{equation*}
$$



Figure (6) $4^{\text {th }}$ order Bezier curve.
The generated $4^{\text {th }}$ order Bezier curve can be divided and reshaped by sufficient intermediate nodes with suitable increment to generate a free collision path to move the manipulator to the desired goal, as shown in Figure (7).

| 崖 | Intermediate nodes (generated) |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Figure(7) Intermediate points on the curve. | $P_{i}$ | X | Y | Z |
|  | P1 | 150 | 50 | 100 |
|  | P2 | 152.0544 | 78.54938 | 135.7931 |
|  | P3 | 157.47 | 104.39 | 164.29 |
|  | P4 | 165.2044 | 127.7994 | 187.0431 |
|  | P5 | 174.32 | 149.04 | 205.44 |
|  | P6 | 183.9844 | 168.3594 | 220.7031 |
|  | P7 | 193.47 | 185.99 | 233.89 |
|  | P8 | 202.1544 | 202.1494 | 245.8931 |
|  | P9 | 209.52 | 217.04 | 257.44 |
|  | P10 | 215.1544 | 230.8494 | 269.0931 |
|  | P11 | 218.75 | 243.75 | 281.25 |
|  | P12 | 220.1044 | 255.8994 | 294.1431 |
|  | P13 | 219.12 | 267.44 | 307.84 |
|  | P14 | 215.8044 | 278.4994 | 322.2431 |
|  | P15 | 210.27 | 289.19 | 337.09 |
|  | P16 | 202.7344 | 299.6094 | 351.9531 |
|  | P17 | 193.52 | 309.84 | 366.24 |
|  | P18 | 183.0544 | 319.9494 | 379.1931 |
|  | P19 | 171.87 | 329.99 | 389.89 |
|  | P20 | 160.6044 | 339.9994 | 397.2431 |

4- Environment with four obstacles: for four obstacles environment, six nodes [start, $1^{\text {st }}$ obstacle, $2^{\text {nd }}$ obstacle, $3^{\text {rd }}$ obstacle, $4^{\text {th }}$ obstacle and goal] were defined and modeled as a control points to generate 5 th order ( $\mathrm{n}-1$ ) Bezier curve , the coordinate of the six points are illustrated in the Table (3) while the mathematical formulation of the $4^{\text {th }}$ order Bezier curve as in equation (5).

Table (4) Physical environment coordinates of three obstacles case.

|  | Start P0 | $1^{\text {st }}$ Obst. P1 | $2^{\text {nd }}$ Obst. P2 | $3^{\text {rd }}$ Obst. P3 | 4th Obst. P4 | Goal P5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| X-coord. | 350 | 100 | 250 | 350 | 150 | 250 |
| Y-coord. | 100 | 150 | 250 | 300 | 350 | 400 |
| Z-coord. | 125 | 250 | 350 | 400 | 300 | 275 |

$P(t)=\sum_{i=0}^{5} P_{i} B_{i, 5}(t)$
using data listed in Table (3) and equation (5) the $5^{\text {th }}$ order Bezier curve can be represented as shown in Figure(8).


Figure（8） $5^{\text {th }}$ order Bezier curve．
The generated $5{ }^{\text {th }}$ order Bezier curve can be divided and reshaped by sufficient intermediate nodes with suitable increment to generate a free collision path to move the arm to the desired goal，as shown in Figure（9）．


Figure（9）Intermediate Points on the curve．Simulation

| Intermediate nodes（ generated） |  |  |  |
| :---: | :---: | :---: | :---: |
| $P_{i}$ | X | Y | Z |
| P1 | 350 | 1. | 125 |
| P2 | 296.944 | lr，7ra | 155.5915 |
| P3 | 260.6065 | 1ra，．vr | 184.7165 |
| P4 | 237.8681 | $1 \leqslant 0, \mathrm{Vrq}$ | 212.1218 |
| P5 | 225.808 | 17ヶ，1ヶ7 | 237.528 |
| P6 | 221.7285 | 1＾．，入09 | 260.6445 |
| P7 | 223.1795 | 191，0入9 | 281.1845 |
| P8 | 227.9827 | Y17，．v9 | 298.8798 |
| P9 | 234.256 | rru，lor | 313.496 |
| P10 | 240.4381 | r¢9，7入१ | 324.8475 |
| P11 | 245.3125 | r70，7r | 332.8125 |
| P12 | 248.0322 | rA．，9r | 337.3478 |
| P13 | 248.144 | r90，7E | 338.504 |
| P14 | 245.6126 | r．9，${ }^{\text {ras }}$ | 336.4405 |
| P15 | 240.8455 | 「Yヶ，¢7） | 331.4405 |
| P16 | 234.7168 |  | 323.9258 |
| P17 | 228.592 | 「¢१，77¢ | 314.472 |
| P18 | 224.3522 | rır，rın | 303.8235 |
| P19 | 224.4185 | rve，quy | 292.9085 |
| P20 | 231.7764 |  | 282.8538 |
| P21 | 250 | \＆．． | 275 |

## Simulation

A number of experiments were simulated to test the ability of an stationary 5 axis revolute robot to reach it's goal without collision with the obstacles presents in it's work space using the generated data of the planned paths as the input to RoboCIM software. The adopted procedure gives good results by avoiding the collision with the obstacles during it's movement from the start point to the goal in all the simulated cases as shown in the Figure (10).

a-Simulation of case one

c-Simulation of case three

b-Simulation of case two

d- Simulation of case four

Figure(10) Simulation cases of the adopted procedure using RoboCIM software.

## EXPERIMENTS

A number of experiments were carried out to test the ability of the robot to reach it's goal without collision with the obstacles. The adopted method has been tested in real environment in the university of technology for four different cases using a Lab-Volt R5150 5 axis robot arm, the adopted model gives good results for all the tested cases as shown in the Figure (11).


Figure (11) Testing of the LabVolt $\mathbf{R 5 1 5 0}$ using the adopted method.

## CONCLUSIONS

In this paper a free collision path planning strategy for revolute robots of five axes has been adopted and tested which makes use of safe knowledge of the infeasible joint space due to the obstacles in the workspace. By applying Bezier techniques, curves with different orders are used to generate a path of a sequence of intermediate points to reach a final goal.

We believe that the solution developed in this paper will make the use of Bezier techniques more useful in applications of robot path planning in known environments (obstacles) with out collision. The generated paths (Bezier Curves) would have to be completed by partitioning it into sufficient segments to generate
intermediate nodes that arm to follow by a sequence of joint angles to reach the goal.
By testing the adopted procedure with several different cases, we found that it is an efficient, accurate, and effective.

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