Eng. &Tech. Journal, Vol.31, Part (A), No.13, 2013

Shear Strength of Reinforced Concrete Beams under Distributed Loading

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Received on: 9/7/2012 & Accepted on: 4/4/2013

ABSTRACT

This work confirms results indicated by previous research that uniformly distributed loading leads to higher reinforced concrete (RC) beam shear capacity-compared to 1 or 2 point loading. Different design methods are compared in this investigation, including those by ACI [(1)] and other Codes.

Twelve beams, without stirrups, are tested in this work to farther investigate the influence of distributed loading. Two design equations are proposed in this work, one includes the size effect and the other without it. The former has proved to be the more accurate.

Of 200 test results obtained from the literature, the proposed design equations lead to a COV value of V_{test} /V_{DES} of 16.8% and 17.1%. These compare favourably with other design methods.

مقاومة القص للعتبات الخرسانية المسلحة تحت تاثير الاحمال المنتشرة

الخلاصه

تشير البحوث الى أن مقاومة القص للعتبات الخرسانية المعرضه لأحمال منتشره تكون مرتفعه بالمقارنه مع التحميل بنقطة أو نقطتين. تم تطبيق النتائج على عدة طرق للمقارنه تتضمن على سبيل المثال مدونه معهد خرسانة الامريكي (ACI) و مدونات اخرى.

تم فحص ١٢ عتبة لدراسة أثر توزيع الاحمال على مقاومة القص , تم اقتراح معادلتين في هذا العمل ,واحده تتضمن تأثير الحجم واخرى لا تتضمن تاثير الحجم .

عند دراسة ٢٠٠ نموذج مأخوذ من البحوث السابقة لمقاومة القص وجد بأن معامل التغاير (COV) للنسبة V_{Test}/V_{DES} ادت الى أن مقدار COV يتراوح بين ١٦,٨% و ١٧,١% للمعادلتين المقترحتين (المفصلة و البسيطة) على التوالي.

INTRODUCTION

Beams are important parts of reinforced concrete structures. Beams resist the loading and distribute these loads to the columns or supports.

The behaviour of reinforced concrete beams at shear failure is distinctly different from their behaviour in flexure. In contrast with beams under uniformly distributed loading, the ones that are loaded under concentrated loading fail abruptly

https://doi.org/10.30684/etj.31.13A.5
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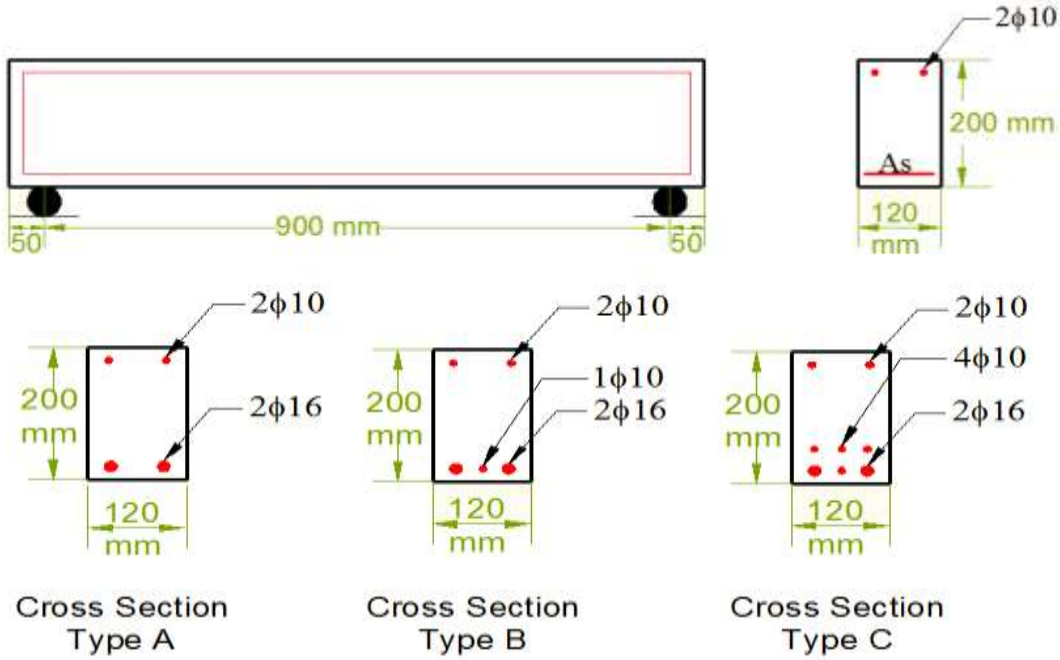
without sufficient/advanced warning, and the diagonal cracks that develop are considerably wider than flexural cracks[2].

BEAMS WITHOUT STIRRUPS

Simply supported beam tests have been carried out to create an understanding of their shear failure under uniformly distributed loads, and to compare their behavior with ones loaded under single or two-point loading. All tested beams failed in shear. The beams were grouped into three main series named as **R1**& **R2** & **R3**. Series **R1** used □ =0.019 and the second series **R2** used ρ= 0.023 and the third series used ρ=0.032, ρ– being the ratio of tensile reinforcement. The variables investigated in this work also included distribution of loading. All beams which had longitudinal reinforcement only were tested to shear failure.

BEAM DETAILS

In this investigation twelve shear tests are reported. Of these tests, three beams have one point load, three have two point loads, and three have four point loads and three have eight point loads. The beams were designed to have extra strength in flexure to ensure shear failure. The beam dimensions are presented in Figure (1).



Series R1 Series R2 Series R3

Figure (1) Beam Reinforcement Details.

In all beam specimens the cross section was 120mm wide and 200mm in depth, the overall length was 1000mm, with clear span of 900mm, and two 10mm diameter bars were used at the top.

The main test variables are:

1. Type of beam loading (one, two, four and eight concentrated loads).
2. Longitudinal reinforcement (ρ=0.019, 0.023, 0.032).
3. Depth effect (d) =172mm in type A and B, 154mm in type C.

Table (1) Beams Details

Group No.	Beams	(a1)* mm	a1/d	ρ_w	Cross Section Type
R1	R1C1	450	2.616	0.019	A
	R1C2	225	1.308	0.019	A
	R1C4	180	1.046	0.019	A
	R1C8	100	0.581	0.019	A
R2	R2C1	450	2.616	0.023	B
	R2C2	225	1.308	0.023	B
	R2C4	180	1.046	0.023	B
	R2C8	100	0.581	0.023	B
R3	R3C1	450	2.922**	0.032	C
	R3C2	225	1.461**	0.032	C
	R3C4	180	1.168**	0.032	C
	R3C8	100	0.649**	0.032	C

***a1=distance from support to nearest loading point
** These beams had d= 154mm. All others had d=172mm.

Notes: R1C1= Series ρ1 + Concentrated one point load.
R1C2= Series ρ1 + Concentrated two point loads.
R1C4= Series ρ1 + Concentrated four point loads.
R1C8= Series ρ1 + Concentrated eight point loads.
R2C1= Series ρ2 + Concentrated one point load.
R2C2= Series ρ2 + Concentrated two point loads.
R2C4= Series ρ2 + Concentrated four point loads.
R2C8= Series ρ2 + Concentrated eight point loads.
R3C1= Series ρ3 + Concentrated one point load.
R3C2= Series ρ3 + Concentrated two point loads.
R3C4= Series ρ3 + Concentrated four point loads.
R3C8= Series ρ3 + Concentrated eight point loads.

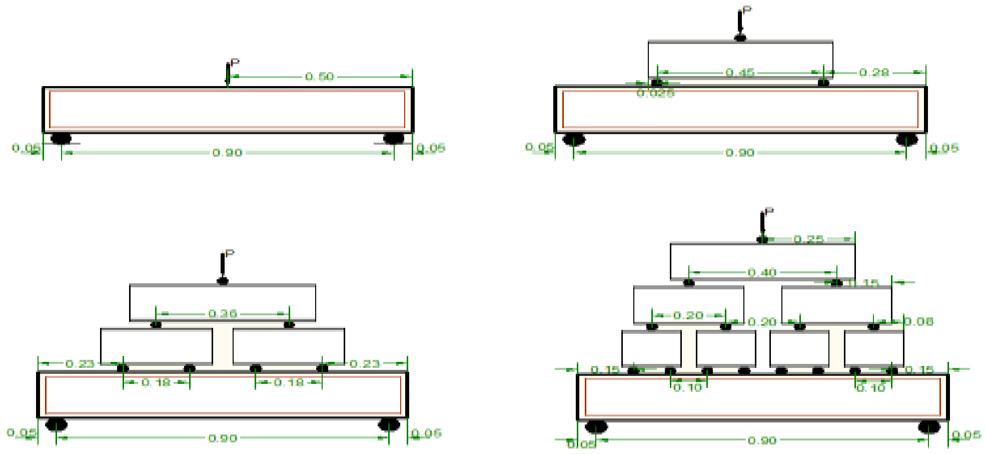


Figure (2)1, 2, 4, 8 Point Loads.

DISCUSSION OF RESULTS

Following are results of the 3 major parameters of variation.

1. Shear Strength Characteristics ($f'c$):

High shear stress on a beam leads to the formation of inclined cracks. Shear failure is difficult to predict accurately. Shear cracking load is considered to be the load at which significant changes in the load-carrying mechanisms occur, resulting in the redistribution of stresses within the beam. While ultimate load is defined as the load at which the failure occurs, a diagonal tension crack is defined as an inclined crack in the shear span extending from the tensile reinforcement toward the nearer concentrated load. The diagonal cracking load is taken as the load at which the diagonal tension crack first crosses the neutral axis of the beam.

Table (2) represents the shear strength at the appearance of a diagonal crack for beams without shear reinforcement along with the ultimate shear failure loads, as measured during testing. In this study the diagonal cracking load is defined as the shear load at the time when the critical diagonal crack (the one that causes failure) formed within the shear span crossing mid-depth of the beam. It can be noted that the values of the diagonal cracking load obtained by using this definition are not very accurate like the ultimate shear failure load because the former are sensitive to the speed of observation.

The twelve beams are divided into three groups. Beams of each group differ in the values of the parameter (a_1/d) considered while the other variables (ρ_w & fc') were kept constant. The experimental results for each parameter are presented in Table (2) below:

Table (2) Results of Cracking and Ultimate Shear Strength for Test Beams.

Group	Beams	ρ_w	a_1/d	f'_c (MPa)	Shear Strength		Vu,test / Vcr,test
					Diagonal Cracking V_{cr} kN	V_u kN	
1	**R1C1**	0.019	2.62	28.36	45.00	70.00^{***}	1.55
	R1C2	0.019	1.31	28.36	*	135.00	---
	R1C4	0.019	1.05	28.36	100.00	145.00	1.45
	R1C8	0.019	0.58	28.36	120.00	147.50	1.23
2	**R2C1**	0.023	2.62	27.92	50.00	57.50	1.15
	R2C2	0.023	1.31	27.92	*	145.00	---
	R2C4	0.023	1.05	27.92	115.00	150.00	1.30
	R2C8	0.023	0.58	27.92	122.50	260.00	2.12
3	**R3C1**	0.032	2.92	30.24	62.50	67.50	1.08
	R3C2	0.032	1.46	30.24	122.50	205.00	1.67
	R3C4	0.032	1.17	30.24	115.00	245.00	2.13
	R3C8	0.032	0.65	30.24	137.50	265.00	1.92

* Not recorded during testing.

** As indicated earlier, a_1 is the distance between the support and the nearest load.

*** Due to probable testing machine error, this beam is not used in the statistical analysis of the results.

2. **Effect of Shear Span to Depth Ratio (a_1/d):**

The diagonal cracking and ultimate shear failure loads of the tested RC beams decrease with increasing (a_1/d) ratio, as shown in Figure (2). However, the response of the ultimate load in all these beams to the change of (a_1/d) ratio is much more pronounced than those of the diagonal cracking loads. This is also evidenced by the wide range within which the ratios of the ultimate to diagonal cracking loads differed in beams of varying (a_1/d) ratios as shown in Table (2).

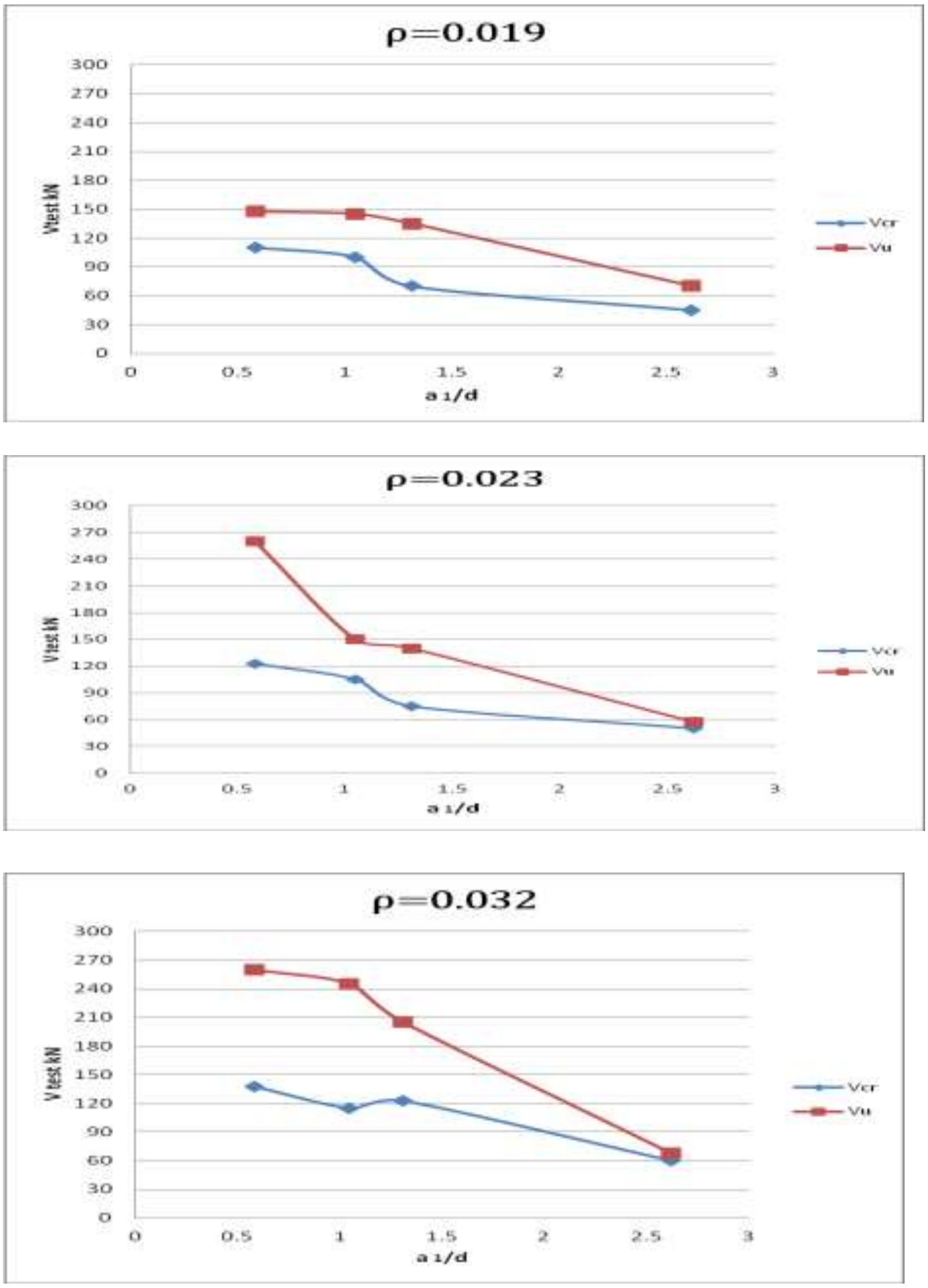


Figure (3) Effect a_1/d Ratio on Shear Strength of RC Beams.

The effect of (a_1/d) ratio on the shear strength can be explained as follows: for the same applied load level, any intended increase in (a_1/d) ratio means larger shear span; this would result in an increase in the flexural stress, which in turn increases the tensile stress component at the depth of the beam. Combined with the shear stress in the shear span, this direct stress increases the principal tensile stress, and hence decreases the diagonal cracking load. On the other hand, increasing the (a_1/d) ratio results in a higher bending moment in the shear span; thus, the depth of penetration of the flexural cracks increases, and hence the flexural stresses near the crack tip increase. By increasing the (a_1/d) ratio, the probability grows that a flexural crack will develop into an inclined one.

3. **Effect of Longitudinal Reinforcement Ratio (ρ_w)**:

The effect of the steel ratio ρ_w on the shear strength of RC beams without stirrups is shown in Fig.(3). It is obvious that by increasing ρ_w, the shear strength of test RC beams is increased. This applies to all cases except for the case of one load with ρ_w of 0.019. For this reason, this is the only result which will not be used in the statistical analysis, which can be confirmed from Table (2).

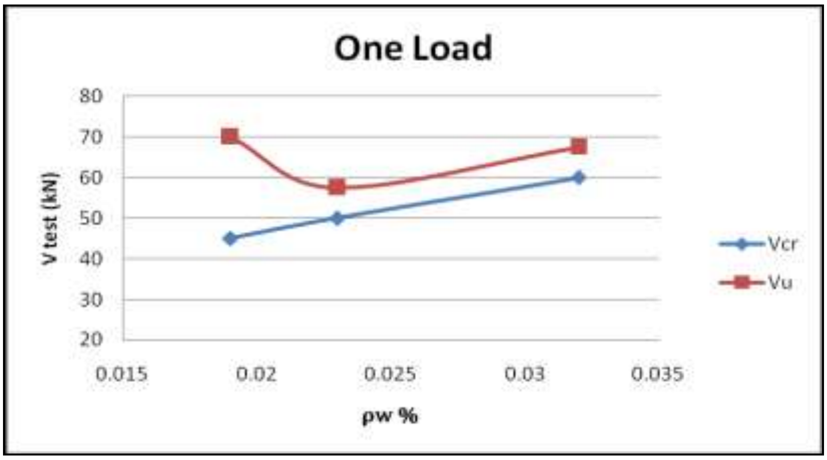
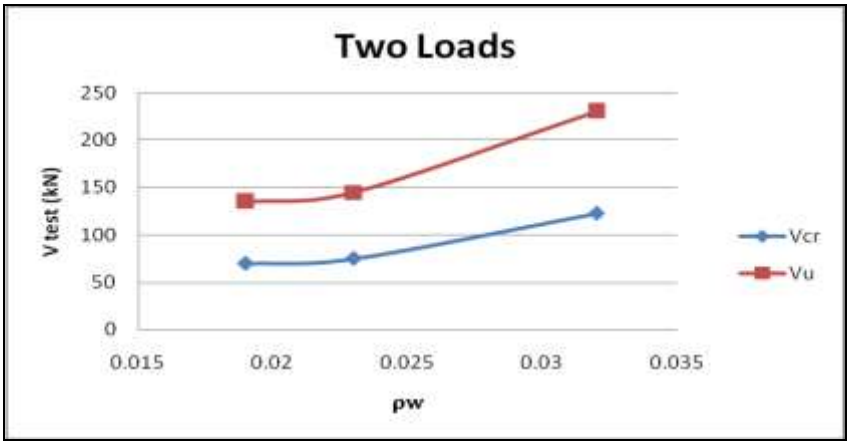
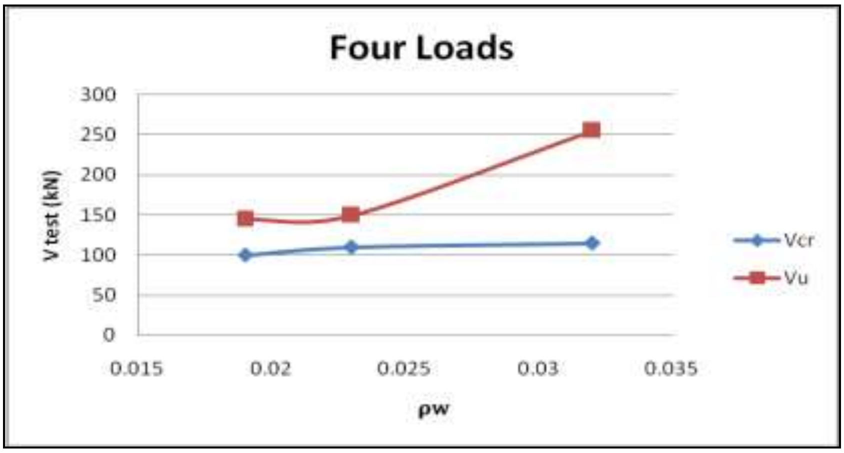


Figure (4) Effect of Steel Ratio ρ_w on Shear Strength of RC Beams under 1,2,4,8 Concentrated Loads.





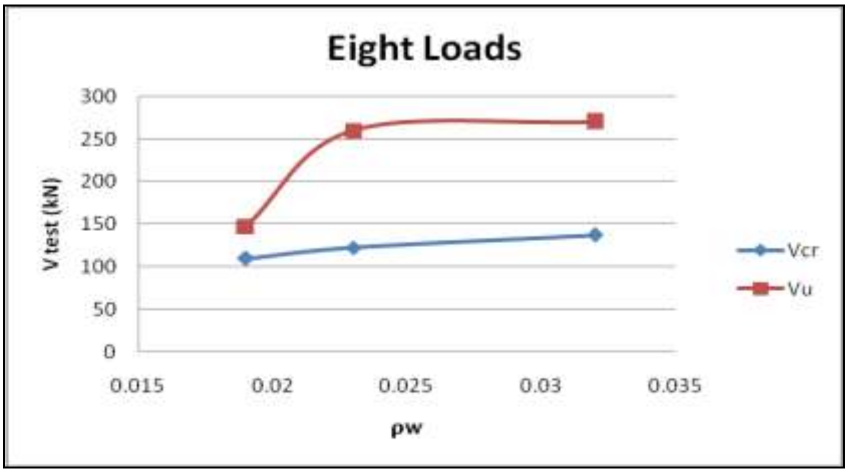


Figure (4) Continued.

The longitudinal reinforcement ratio has a pronounced effect on the basic shear transfer mechanism. An important factor that affects the rate at which a flexural crack develops into an inclined one is the magnitude of shear stresses near the crack tip. The intensity of principal stresses at levels above the flexural crack depends on the depth of penetration of the crack. The greater the value of ρ_w the less is the penetration of the flexural crack. The less the penetration of the flexural crack, the less is the principal stress for a given applied load, and consequently the greater must be the shear to cause the principal stresses that will result in diagonal tension cracking.

Increasing ρ_w also increases the dowel capacity of the member by increasing the dowel area and hence decreasing the tensile stresses induced in the surrounding concrete.

Increasing (ρ_w) also affects the friction or interface shear along the diagonal crack surface capacity. Beams with low (ρ_w) will have wide, long cracks in contrast to the shorter, narrow cracks found in beams with high (ρ_w). Since the friction or interface shear along the diagonal crack surface mechanism depends on the crack width, an increase in the friction or interface shear along the diagonal crack surface force is to be expected with an increase in (ρ_w).

CRACK PATTERNS

Prior to further discussion of test results, it is helpful to discuss the general behaviour of beams failing in shear. Cracks in concrete beams are formed generally at regions where the tensile stresses exist that exceed the specified tensile strength of concrete. Accordingly, two types of cracks may be observed in the tested beams; the flexural cracks resulting from flexural tensile stresses at the regions of the simple beam cross-section below the neutral axis, and the shear cracks which are formed as a result of the inclined or "principal" tensile stresses acting on the beam at regions of combined moment and shear. Plate (1) shows photographs of the crack patterns after the failure of the tested beams.

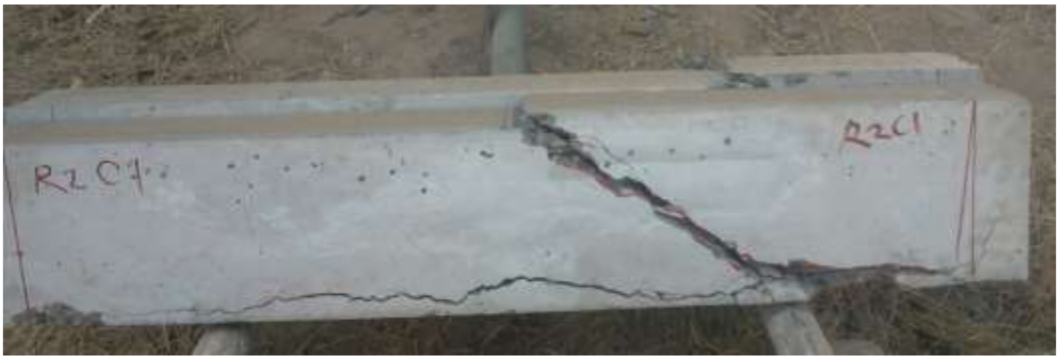
In general, all the tested beams exhibited similar linear behaviour at the initial loading stages up to the occurrence of the first hairline inclined crack at an inclination approximately of 45 degrees to the horizontal axis of the beam.

R1C1 (ρ=0.019, one concentrated load)

R1C2(ρ=0.019, two concentrated loads).

R1C4(ρ=0.019, four concentrated loads).

R1C8(ρ=0.019, eight concentrated loads).
Plate (1)



R2C1(ρ=0.023, one concentrated load)

R2C2(ρ=0.023, two concentrated loads).

R2C4(ρ=0.023, four concentrated loads)
Plate (1) continued.

R2C8(ρ=0.023, eight concentrated loads).

Plate (1)

R3C1(ρ=0.032, one concentrated load)

R3C2(ρ=0.032, two concentrated loads).

R3C4(ρ=0.032, four concentrated loads)

Plate (1)Continued.

R3C8(ρ=0.032, eight concentrated loads)
Plate (1)Continued.

FAILURE OF TESTED BEAMS

The stages in the crack pattern development for the tested beams failing eventually in diagonal tension are essentially as follows:

1. At low loads, hairline cracks are formed in the shear spans between point load and support.
2. With further increase in load, new flexural cracks are formed in the shear spans between point load and support. These latter cracks gradually became inclined as they propagated to regions above the longitudinal reinforcement and curved toward the loading point.
3. With further increase in load, diagonal inclined cracks were generally initiated in the shear span at a position approximately mid-depth of the RC beam. In most of the tested beams these diagonal cracks were formed in both shear spans at the same load level or little different.
4. Cracks propagation continued, the inclined cracks which were formed from the initiating flexural cracks extended toward the point load. The diagonal cracks initiating in the shear span extended toward the point load in the one direction and nearly horizontally at the level of the longitudinal reinforcement towards the support in the other direction.
5. Finally, one of the diagonal cracks extended in the compressive zone towards the point load causing failure. This failure behaviour can be considered as a mode of diagonal tension failure.

TEST RESULTS

In general, all the tested beams exhibited similar linear behaviour from the initial loading up to the load causing cracking.

The general cracking performance and behaviour under load was similar for all specimens. It was not easy to detect initial cracking; but once the first flexural cracks were detected, it was easy to follow the progress of the cracks.

All the beams tested in this work failed in shear, the inclined cracks led to redistribution of internal stresses. Finally, failure occurred in diagonal tension.

The general cracking performance was again similar in all beams. However, the ultimate load was higher in the beams when the concentrated load changed from 1, 2, 4, to 8 point loads as shown in Figure (4) except for the beam whose results are discarded, as explained earlier.

STATISTICAL ANALYSIS

Based on statistical work for the 11 tested beams proposed Eq.(1) is arrived at which gives the lowest COV percentage of 23.87%, Table (3), compared with all other methods.

$$V_{Prop.}=4.5(f_c')^{0.55}(\rho_w)^{0.64}(V_u d/M_u)^{0.78}(390/d)^{0.26} b_w d \quad \ldots(1)$$

A simpler equation (2) is also proposed to predict beam shear strength, where the COV rises to 25.79%.

$$V_{Prop}=2.2(f_c' \rho_w V_u d/M_u)^{0.6} b_w d \quad \ldots(2)$$

In the above equations, it is important to notice that for the 11 considered beams in this work, plus 200 for reference[3].

1. No reduction factors are used.
2. The limits for concrete compressive strength are (6.35-101.85)MPa.
3. The limits of a_1/d are (0.99-8.52).
4. The limits for shear strength of tests (2.14-358.67)kN.
5. The limits for longitudinal reinforcement (ρ_w) of tests are (0.0050-0.0429).
6. In size effect $(390/d)^{0.26}$ used is $d \leq 390$ mm in Eq.(1).

Table (3) gives a comparison for the results of applying the 11 beams to more than one method, based on the ratio of V_{Test}/V_{DES}. From the table it can be seen that the lowest COV percentage is by Eq.(1): 23.87%, and Eq.(2):25.79%. These compare favourably with (33.31%-44.67%) by other existing methods (ACI Committee 318M-11(1), British Code(4), Canadian Code(5), New Zealand Code(6), Zsutty(7), Sarsam and Al-Musawi(8)), see Table (5).

Table (3) Comparison for predicting V_{Test}/V_{DES} based on 7 different methods in 11 beams doing in this work.

NO.	Methods	Mean	SD	COV (%)
1	ACI (11-3)(1)	9.2	4.11	44.67
2	ACI (11-5)(1)	6.43	2.22	34.58
3	BS(4)	6.97	2.84	40.77
4	CAN(5)	7.82	3.49	44.67
5	NZ(6)	4.85	1.86	38.36
6	ZST(7)	4.09	1.36	33.31
7	S&A(8)	6.9	2.69	38.97
8	Eq. (1)	2.52	0.6	23.87
9	Eq. (2)	4.77	1.23	25.79

In order to apply Eq.(1), and Eq.(2) to 200 existing test results from the literature, these values are compared in Table (1-4). It can be seen that the COV of Eq.(1) is the lowest at 16.78%.This with (21.88%-33.63%) by other existing methods. The simplified Eq.(2) is the second lowest at 17.08%.

Table (4) Comparison for predicting V_{Test}/V_{DES} based on 7 different methods for 200 tests without stirrups from the literature.

NO.	Methods	Mean	SD	COV(%)	High	Low	H/L
1	**ACI (11-3)(1)**	1.44	o.46	**31.98**	3.89	0.70	5.56
2	**ACI (11-5)(1)**	1.36	0.37	**27.14**	3.40	0.71	4.79
3	**BS(4)**	1.37	0.41	**29.79**	3.87	0.91	4.25
4	**CAN(5)**	1.22	0.39	**31.98**	3.31	0.59	5.61
5	**NZ(6)**	1.01	0.34	**33.63**	2.86	0.46	6.22
6	**ZST(7)**	1.07	0.24	**22.22**	2.28	0.68	3.35
7	**S&A(8)**	1.5	0.33	**21.88**	3.38	0.97	3.48
8	**Eq. (1)**	1.36	0.229	**16.78**	2.01	1.04	1.93
9	**Eq. (2)**	1.87	0.32	**17.08**	3.00	1.29	2.32

CONCLUSIONS

This section includes the results obtained based on the experimental tests carried out in the present research work on RC beams without shear reinforcement. The following conclusions can be drawn:

1. Diagonal cracking load and ultimate shear load decreased with increasing a_1/d (for example, increasing a_1/d from 0.58 to 2.62 decreased the diagonal cracking load and ultimate shear load from 122.5kN to 50kN, 260kN to 57.5kN, respectively in beams with ρ_w=0.023).
2. ACI code(1), CAN code(5), BS code(4), NZ code(6), ZST(7), and S&A(8) equations lead to higher COV values than those proposed in this work-Eqs.(1) & (2).
3. The test results show that the shear strength values for ACI [Eq.(11-3)](1) and CAN code(5) decreased with rising $f'c$ because these equations give greater effect for $f'c$ on the shear strength. This contrasts with BS(4), Zsutty(7), and Sarsam and Al-Musawi(8) equations, which include other significant factors such as ρ_w and $V_u d/M_u$.
4. For all tests of RC beams, the ratio of the observed ultimate shear strength to diagonal cracking load (V_u/V_{cr}) had a range between (1.08-2.13). This may not be as accurate as other results, since it depends on the observer noticing cracking.
5. From the test results indicated for the ratio of V_{Test}/V_{DES} the following conclusion can be made, as expected. The least accurate methods have the greatest scatter in this relationship, e.g. ACI Eq.(11.3)(1) and the CAN(5) method; both of which rely on $(f'c)^{1/2}$ only. The scatter in descending order for the more detailed methods is: ACI Eq.(11.5)(1), S&A(8), NZ (6)method, BS(4), Eq.(2), ZST, and Eq.(1).
6. In agreement with previous research(1,4,5,6,7,8), where most beams were loaded by 1 and 2 point loading, there is a clear evidence from this work that shear capacity rises as loading distribution is greater.
7. Of the previous design methods, only the BS Code(4) includes a size effect of (400/d) for strengthening beams in shear (V_{DES}). The size effect in proposed Eq.(1) is also used, leading to an improved COV compared to other proposed method.

PROPOSED EMPIRICAL EXPRESSIONS

1. Based on test results obtained from this investigation, two expressions have been proposed to predict the shear strength of RC beams without shear reinforcement, these are:
   $V_{Prop.}=4.5(f'c)^{0.55}(\rho_w)^{0.64}(Vu/Mu)^{0.78}(390/d)^{0.26}b_wd$ …..(1)
   $V_{Prop.}= 2.2(f'c\ \rho_w\ Vud/Mu)^{0.6}b_wd$ …..(2)
2. Comparisons with experimental data indicate that the proposed expressions properly estimate the effects of primary factors, such as concrete compressive strength, longitudinal steel ratio, shear span to effective depth ratio.
3. The two proposed expressions, [Eq.(1) and Eq.(2)], have low COV values of 16.78% and 17.08% respectively, but it is obvious that [Eq.(2)] is a simpler formula than [Eq.(1)]; based on 200 tests from the literature.
4. By testing the proposed shear strength expressions against the experimental result of this study, improvement in the overall prediction accuracy (COV value of the ratio of V_{Test}/V_{DES}) is 21.88% with respect to the expression proposed by S&A(8), and 22.22% with respect to the Zsutty(7) equation.

Table (5)Shear strength models.

Model	Equation
ACI 11-3 (ACI18M-11)(1)	$V_c = 0.17\sqrt{fc'}\,b_w d$
ACI 11-5 (ACI18M-11)(1)	$V_c = \left(0.16\sqrt{fc'} + 17\rho_w \frac{V_u d}{M_u}\right) b_w d \le 0.29\sqrt{fc'}\,b_w d$ Where $V_ud/M_u \le 1.0$
Canadian Code(5)	$V_{CAN} = 0.2(fc')^{0.5} b_w d$
New Zealand Code(6)	$V_{NZ} = (0.07 + 10\rho_w)\sqrt{fc'}\,b_w d$
British Code(4)	$V_{BS} = 0.79(100\rho_w)^{1/3}(f_{cu}/25)^{1/3}(400/d)^{1/4} b_w d/1.25$ Where $(400/d)^{1/4}$ is only applicable if d<400 mm
Zsutty's Method(7)	$V_{ZST} = 2.3(fc'\rho_w \frac{d}{a})^{1/3} b_w d$
Sarsam and Almusawi Method(8)	$V_{S\&A} = 1.8(fc'\rho_w \frac{V_u d}{M_u})^{0.38} b_w d$

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