

Robust PI-PD Controller Design for Magnetic Levitation System

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ABSTRACT

This paper presents a design of robust PI-PD position controller for Magnetic Levitation Ball system. The Magnetic Levitation system considered in this work is taken as a ferromagnetic ball suspended in a voltage controlled magnetic field. The Magnetic Levitation system is unstable, because electromagnetic force is very sensitive and there is a noise that creates acceleration forces on the steel ball resulting the instability due to existence of positive poles causing the steel ball to move into the unbalanced region. The robust controller is aimed to keep a steel ball suspended in the air in the desired position by maintaining the balance between the magnetic force and ball's weight. The Particle Swarm Optimization (PSO) method is used to tune the gains of the PI-PD controller. The simulation and experimental results show the effectiveness of the designed controller.

تصميم المسيطر التناسبي التكاملي- التناسبي التفاضلي المتين للسيطرة على منظومة التعليق المغناطيسي

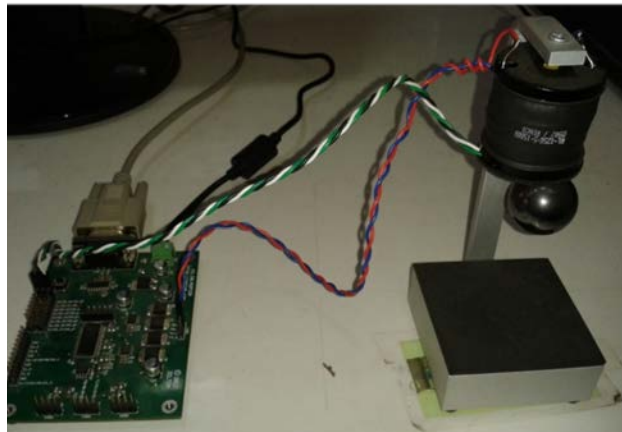
الخلاصة

يعرض هذا البحث تصميم للمسيطر التناسبي-التكاملي التفاضلي المتين للسيطرة على نظام التعليق المغناطيسي. ان نظام التعليق المغناطيسي المعتمد في هذا البحث هو كرة معلقة بالمجال المغناطيسي ويتم السيطرة عليها عن طريق الجهد الكهربائي. هذا النظام يعتبر غير مستقر وذلك لان القوة الكهرومغناطيسية حساسة جدا وبسبب وجود الضوضاء التي تولد قوى تسارع تؤدي الى عدم الاستقرار. ان هذا المسيطر المتين المصمم في هذا البحث يهدف الى جعل الكرة معلقة في الهواء وفي الموقع المطلوب عن طريق الحفاظ على التوازن بين القوة المغناطيسية ووزن الكرة. طريقة الحشد الجزيئي استخدمت في هذا البحث لأيجاد القيم المثلى للمسيطر المقترح. نتائج المحاكاة والنتائج العملية تثبت كفاءة المسيطر المقترح.

INTRODUCTION

A lot of research effort in control system field has been focused on the control of Magnetic Levitation system (MLS). They are widely used in various fields such as frictionless bearings, high-speed Magnetic Levitation passenger trains, levitation of wind tunnel models, vibration isolation of sensitive machinery, levitation of molten in induction furnaces, levitation of slabs during manufacture etc. MLS are generally highly nonlinear and open loop unstable systems. This unstable aspect of MLS and its inherent nonlinearities make the modeling and control problems very challenging. Several dynamic models of magnetic force have been proposed over the past years and with these models various control strategies have been used [1, 2].

MLS demonstrates a classic magnetic levitation control experiment, that of suspending a steel ball in space as shown in Figure(1).



Figure(1): MLS with the hardware interface components.

The MLS controls the magnetic field generated by an electromagnet to levitate a small permanent magnet in midair. With an appropriate controller in the loop, the small magnet levitates in the air indefinitely without any disturbance. The vertical position of the levitating magnet is measured using a linear Hall effect sensor and the current in the electromagnet is actively controlled to achieve stable levitation. The control circuitry consists of a set of power supplies and amplifiers connected to a control computer. The control computer is a dedicated digital signal processor (DSP), programmed through the computer. The power supplies and amplifiers receive the signal from the position sensor and send power to the actuator. An input / output box wired to the analog to digital and digital to analog converters on the computer allows the power electronics and the computer to talk to one another. In the photograph above, the ball rests on a micrometer / force measurement apparatus used in calibrating the system [3].

The objective of this work is to keep a metal ball suspended in mid-air by adjusting the field strength of an electromagnet. The electromagnet current may be increased

until the magnetic force produced is equal to or greater than, the gravitational force acting on the ball. Variations in the electromagnet current cause the ball to either fall (when current is decreasing) or are attached to the electromagnet (when current is increasing). The proposed controller aims to stabilize the ball when current disturbance occurs, by finding the best parameters of a PI-PD controller.

Magnetic levitation System Model

The first step in controlling a system is deriving an accurate model for the system. Each of the system elements' behaviors can be derived from basic physics. In our system, many of the equations are left in terms of constants and these constants are dependent on materials and geometry, and are thus specific to the hardware. Figure 2 shows the MLS system free body diagram [3].

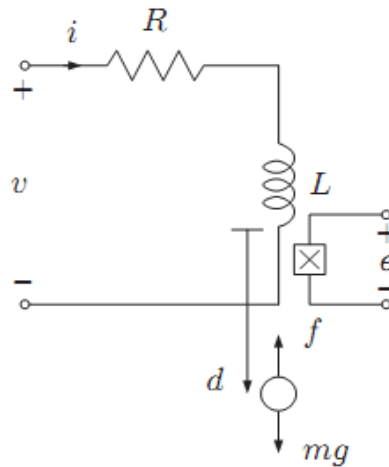


Figure (2): MLS system free body diagram.

Where:

R is the resistance of the coil, L is the inductance of the coil, v is the voltage across the electromagnet, i is the current through the electromagnet, m is the mass of the levitating magnet, g is the acceleration due to gravity, d is the vertical position of the levitating magnet measured from the bottom of the coil, f is the force on the levitating magnet generated by the electromagnet and e is the voltage across the Hall effect sensor. The Hall effect sensor is connected to one of the analog input of a hardware-in-the-loop real-time control platform for Matlab/Simulink (Hilink) control board and the electromagnet is driven by one of the H-bridges of the same board. As shown in Figure (2), two forces act on the steel ball: gravity and the electro-magnetic force from the coils.

The nonlinear model is derived by analyzing the mechanical and electromagnetic subsystems. On the basis of the electro-mechanical modeling nonlinear model of MLS can be expressed in terms of the following differential equations [1, 4]:

$$v = \frac{dx}{dt} \quad \dots(1)$$

$$m\ddot{d} = mg - k \frac{i}{d^3} \quad \dots(2)$$

$$u(t) = V_R + V_L = iR + L \frac{di}{dt} \quad \dots (3)$$

Letting $x = [x_1 \ x_2 \ x_3] = [d \ \dot{d} \ i]$ be the state of the system, $z = d$ be the controlled output, $y = e$ be the measured output, $u = v$ be the control input and $w = n$ be the disturbance/noise input, the standard state equation description of the system can be written as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ g - \frac{k}{m} \frac{x_3}{x_1^3} \\ -\frac{R}{L} x_3 + \frac{1}{L} u \end{bmatrix}$$

$$z = x_1$$

$$y = \beta \frac{1}{x_1^2} + \gamma x_3 + w + \alpha \quad \dots(4)$$

Where:

α , β and γ are constants which depend on the Hall effect sensor and the geometry of the system. On the other hand, in order to control the ball position to a desired position x_{01} , the equilibrium point of the system is at [1, 3]:

$$\begin{bmatrix} x_{1e} \\ x_{2e} \\ x_{3e} \end{bmatrix} = \begin{bmatrix} \left(\frac{ku_e}{gmR}\right)^{1/3} \\ 0 \\ \frac{u_e}{R} \end{bmatrix} \quad \dots(5)$$

Where:

u_e is the required equilibrium coil voltage to suspend the levitating magnet at $x_{1e} = d_e$. Note that there is a unique equilibrium point.

The Jacobian linearization of the system about the equilibrium point is [3]:

$$\begin{aligned} \delta \dot{x} &= A\delta x + B_1\delta w + B_2\delta u \\ \delta z &= C_1\delta x + D_{11}\delta w + D_{12}\delta u \\ \delta y &= C_2\delta x + D_{21}\delta w + D_{22}\delta u \end{aligned} \quad \dots (6)$$

Where:

$$\delta x = x - x_e, \delta w = w - w_e, \delta u = u - u_e, \delta z = z - z_e, \text{ and } \delta y = y - y_e$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 3g \left(\frac{gmR}{ku_e}\right)^{1/3} & 0 & -\frac{gR}{u_e} \\ 0 & 0 & -\frac{R}{L} \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix},$$

$$C_1 = [1 \ 0 \ 0], \quad C_2 = [-\frac{2\beta gmR}{ku_e} \ 0 \ \gamma]$$

$$D_{11} = [0], D_{12} = [0], D_{21} = [1], \quad D_{22} = [0] \quad \dots(7)$$

Note that $w_e = 0$, $z_e = x_{1e}$ and $y_e = \beta \frac{1}{x_{1e}^2} + \gamma x_{3e} + \alpha$.

In this derivation, the back *emf* induced by the moving levitating magnet is ignored as it is very small. If the Hall effect sensor is located below the levitating magnet, then γ is also very small and it can also be neglected. Letting the desired d_e be (20mm) and using measurements obtained by the Hilink platform, the parameters of the MLS are given in Table (I).

Table (I): The physical parameters of MLS [3].

Parameter	m	g	R	L	k	X_{01}	X_{02}	X_{03}	α	γ	u_e	z_e	y_e	w_e	β
Value	0.0413	9.81	1.71	0.0151	3.1×10^{-6}	20	0	1.05	2.48	0.31	1.79	20	3.87	0	4.25×10^{-4}
Unit	Kg	m/s ²	Ω	mH	Kg.m ⁵ /s ² /A	mm	m/s	A	V	V/A	V	mm	V	V	V.m ²

Stabilization using PI-PD Controller

The modified form of PI-PD controller structure is shown in Figure (3). The components of this figure can be defined as [5]:

$$G_p(s) = \frac{N_p(s)}{D_p(s)} \quad \dots(8)$$

$$K_{PI}(s) = K_p + \frac{K_i}{s} = \frac{K_p s + K_i}{s} \quad \dots(9)$$

$$K_{PD}(s) = K_f + K_d s \quad \dots(10)$$

Where:

N_p, D_p are the polynomials of the numerator and denominator of the system to be controlled. K_p, K_i are the PI controller parameters. K_f, K_d are the PD controller parameters.

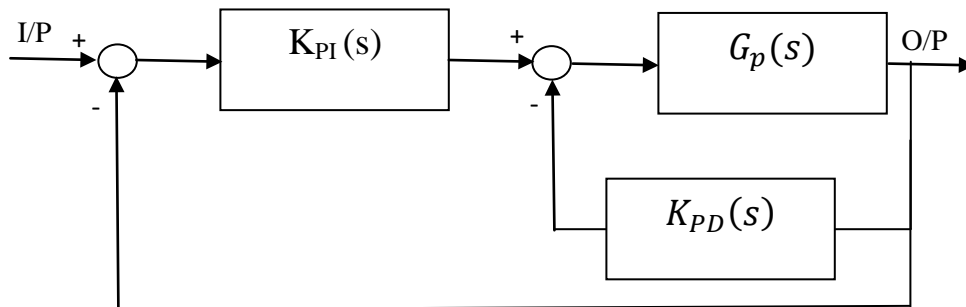


Figure (3): PI-PD controller structure.

The internal PD feedback loop can convert an open loop unstable process to an open loop stable process and for resonant or integrating processes can ensure appropriate locations of the open loop stable process poles. Therefore, the PI-PD controller structure has advantages over the conventional PID controller [6]. On the other hand, conventional design procedure is based on a plant with fixed parameters, although most practical systems models have uncertainties. Therefore, the design of a satisfactory control scheme requires the consideration of robustness to parameter uncertainties, to stability and performance [7]. In this work, the first method used to determine the four parameters of PI-PD controller for MLS is the method proposed reference [5]. This method is based on plotting the stability boundary loci in the (K_d, K_f) and (K_p, K_i) planes. The stability boundary locus is dependent on the controller parameters and frequency. Thus, a very fast way of calculating all the stabilizing parameters of PI-PD controller for a given control system is applied. This method is also used for specified gain and phase margins. The method is combined with the Kharitonov theorem to deal with uncertain parameters. Consequently, the PD controller parameters are determined by [5]:

$$K_f = -\frac{w^2 N_{po} + D_{pe} N_{pe}}{w^2 N_{po}^2 + N_{pe}^2} \dots(11)$$

$$K_d = \frac{D_{pe} N_{po} - D_{pe} N_{pe}}{w^2 N_{po}^2 + N_{pe}^2} \dots(12)$$

Where N_{po}, N_{pe} are odd and even parts of the numerator. N_{po}, D_{pe} are odd and even parts of the denominator Dp . With a gain-phase margin tester, $(Ae^{-j\phi})$ this is connected in forward path together with the equivalent closed loop transfer function of the inner loop as shown in Figure 4 [5].

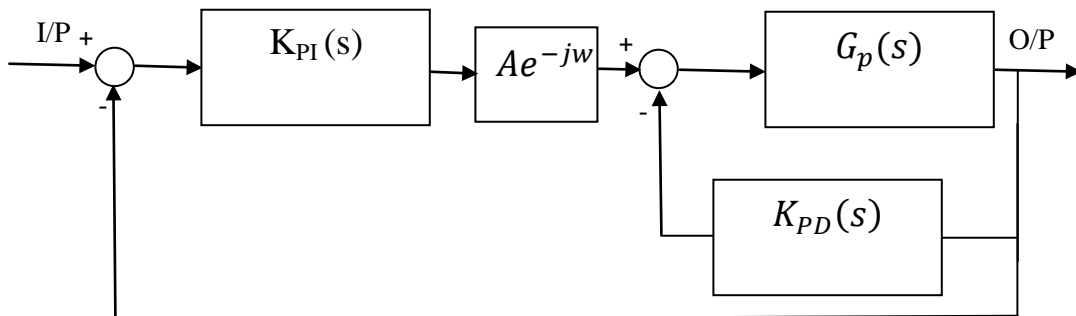


Figure (4): Equivalent block diagram of Figure 3 with gain phase margin tester.

Where:

$$G(s) = \frac{G_p(s)}{1+K_{PD}G_p(s)} = \frac{N_p(s)}{D_p+(K_f+K_d s)N_p(s)} \quad \dots(13)$$

$$G(j\omega) = \frac{N_e+j\omega N_o}{D_e+j\omega D_o} \quad \dots(14)$$

Where:

$$\begin{aligned} N_e &= N_{pe} \quad , \quad N_o = N_{po} \quad , \\ D_e &= D_{pe} + K_f N_{pe} - \omega^2 K_d N_{po} \quad , \\ D_o &= D_{pe} + K_f N_{po} + K_d N_{pe} \end{aligned}$$

Using the procedure given for the computation of the parameters of, $K_{PD}(s)$ one obtains:

$$K_p = \frac{X \cos(\phi) + Y \sin(\phi)}{-A(N_e^2 + \omega^2 N_o^2)} \quad \dots(15)$$

$$K_i = \frac{V \cos(\phi) - Z \sin(\phi)}{-A(N_e^2 + \omega^2 N_o^2)} \quad \dots(16)$$

Where:

$$\begin{aligned} X &= \omega^2 N_o D_o + N_e D_e \quad , \quad Y = \omega(N_o D_e - N_e D_o), V \\ &= \omega^2 (N_o D_e - N_e D_o) \quad \text{and} \\ Z &= \omega(N_e D_e - \omega^2 N_o D_o) \end{aligned}$$

Setting, $A=1$ and $\phi=0$ in equations (15) and (16), all stabilizing K_p and K_i values for fixed parameter of $K_{PD}(s)$ can be found.

The following steps represent the design procedure of PI-PD controller:

Step 1: compute the stability region which includes all the stabilizing values of k_d and k_f using equations (11) and (12).

Step 2: from stability region obtained in step 1, find the values of k_d and k_f for which the dominant closed loop poles of inner loop have damping ratio (ζ) around 0.65-0.7.

Step 3: using k_d and k_f values found in step 2, obtain the stability region which include all the stabilizing values of k_i and k_p using equations (15) and (16).

Step 4: from the stability region obtained in step 3, find the values of k_i and k_p and obtain step response of the system. If the result is not satisfactory then try for different values of k_i and k_p .

PI-PD Controller Design using PSO

This section is dedicated to present the second method used to determine the four parameters of the PI-PD controller for MLS which is the PSO method. The PSO method is one of the powerful optimization methods with high efficiency in comparison to other methods. It is quick in the convergence speed, few in the parameters, simple in operations, therefore, it is suitable to solve optimization problems. The PSO concept involves, at each time step, changing the velocity of each

particle towards its global best and local best locations. The particles are manipulated according to the following equations of motion [8, 9]:

$$\dots (17) v_i^{k+1} = w \times v_i^k + c_1 \times rand \times (x_i^b - x_i^k) + c_2 \times rand \times (x_i^g - x_i^k)$$

$$x_i^{k+1} = x_i^k + v_i^{k+1} \quad \dots (18)$$

Where v_i^k is the particle velocity, x_i^k is the current particle position, w is the inertia weight, x_i^b and x_i^g are the best value and the global best value, $rand$ is a random function between 0 and 1, c_1 and c_2 are learning factors. The PSO requires only a few lines of computer code to realize PSO algorithm. Also it is a simple concept, easy to implement, and computationally efficient algorithm [10].

In this work the PSO is used to obtain the optimal values of the PI-PD controller parameters that ensure a controlled system with a robust stability and performance. The cost function (objective function) to be minimized using PSO method is the integral of time multiplied by absolute error (ITAE) which is expressed by [11]:

$$J = \int_0^\infty |e| dt \quad \dots (19)$$

Where e represents the error signal of the system. Figure (5) shows the block diagram of PSO Based PI-PD controller. The flowchart of using PSO method is shown in Figure (6). The PSO parameters that have been used for carrying out the design of PI-PD controller are: population size=10, inertia factor $h=2$, $c_1=c_2=2$, maximum number of iterations is set to 500 and the number of function evaluations is 5000.

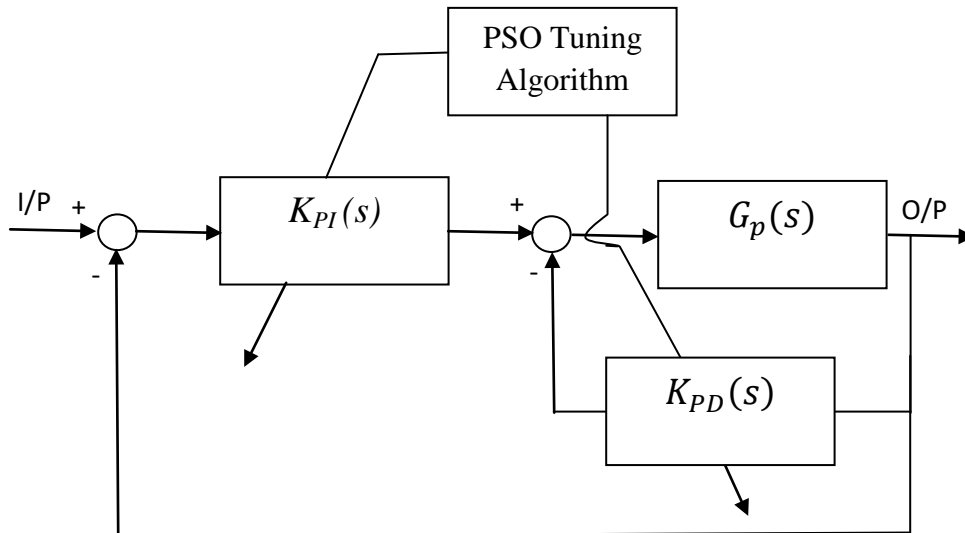


Figure (5) Block diagram of PSO Based PI-PD controller.

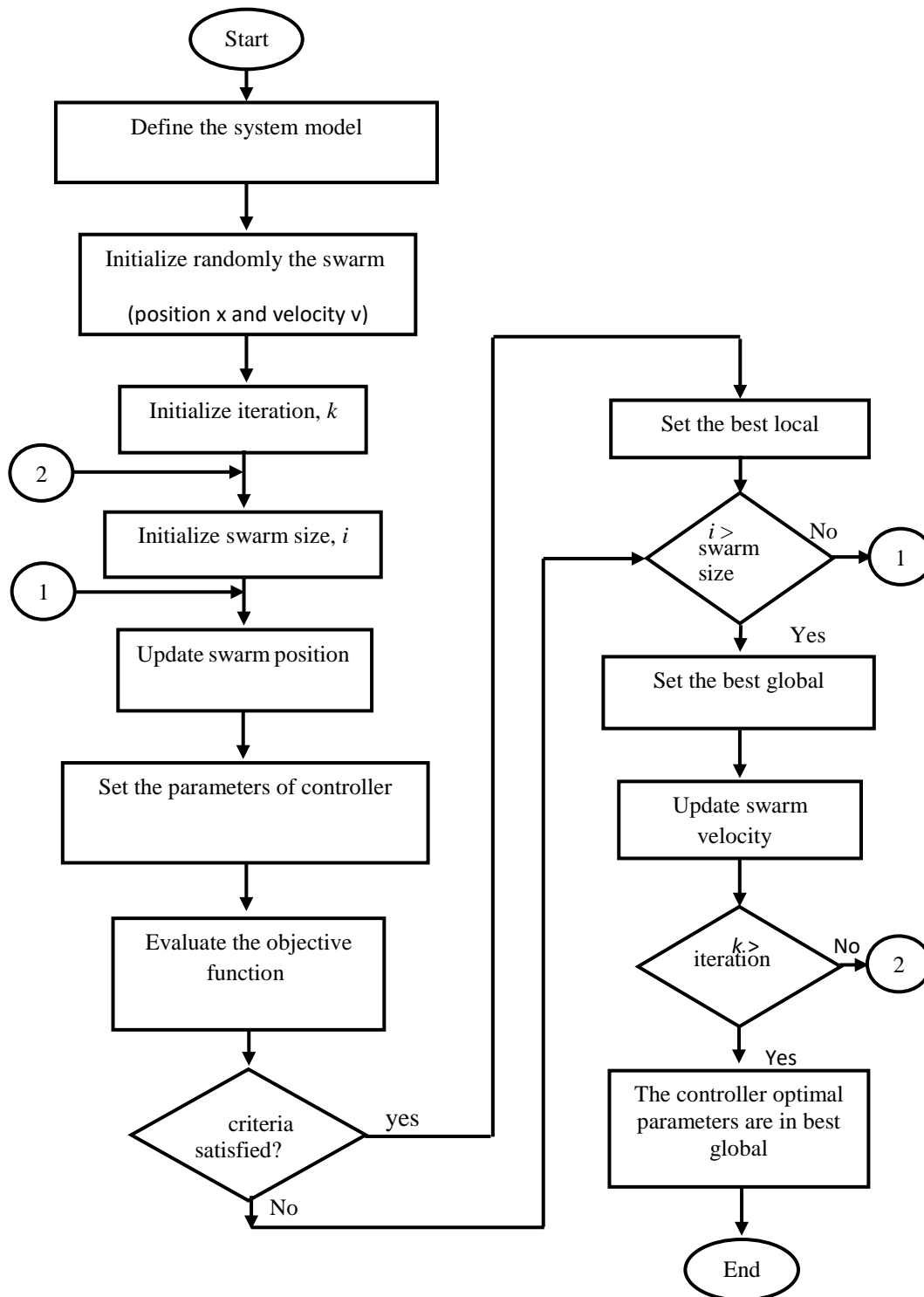


Figure (6) PSO algorithm flowchart.

RESULTS AND DISCUSSION

This section presents the simulation and real time implementation of PI-PD controller for the MLS system using Matlab/Simulink package as a software part. Hilink used for computer interfacing as a hardware part.

Figures (7a and 7b) show the open loop and closed loop system time responses without controller. From these figures it is very clear that the design of controller is required to stabilize the system and achieve a suitable performance. By applying the first method which was proposed by [16] on the MLS, the obtained four-parameters of PI-PD controller are: $K_f = 5.291$, $K_d = 0.0467$, $K_p = 9.7588e - 18$, $K_i = 0.0463$. The magnitude of the applied desired position in all simulation and experimental results in this work is (4.12 volt) reflected to cm. The closed loop response with the determined parameters can be shown in Figure(8). It is noticed that the controller can stabilize the MLS with the following time response specifications: $t_r = 280 sec.$ and $t_s = 600 sec.$. This means that the determined PI-PD controller parameters using the first method cannot achieve a desirable performance, therefore, the PSO method has been used to obtain the optimal parameters of the PI-PD controller that can achieve a more desirable time response specifications. The PSO method for tuning the controller parameters is applied with different number of iterations. Figures (9 and 10) show the most desirable, acceptable system time response and control action to be implemented experimentally obtained using PSO method with (500) iterations. The achieved time response specifications are: $t_r = 0.2 sec.$ and $t_s = 0.3 sec.$ using the following obtained optimal parameters: $K_d = 0.2283$, $K_f = 4.9782$, $K_p = 1.7990$ and $K_i = 3.2638$. The experimental results obtained by applying the designed controller using PSO method is shown in Figure (11).

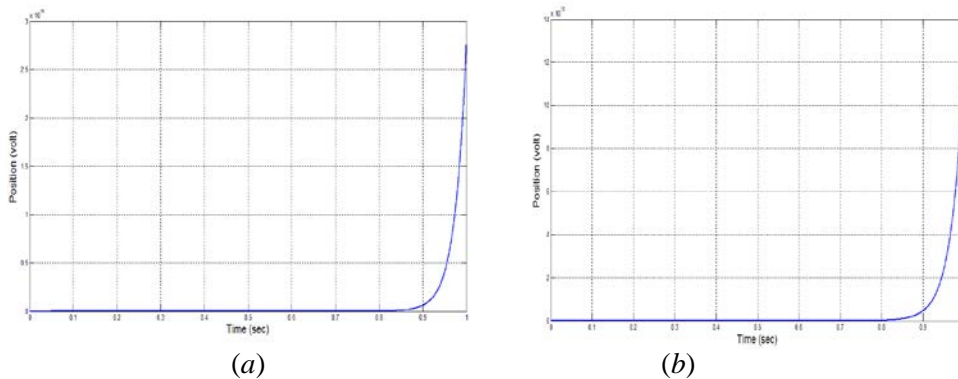


Figure (7) Open loop and closed loop system response without controller.

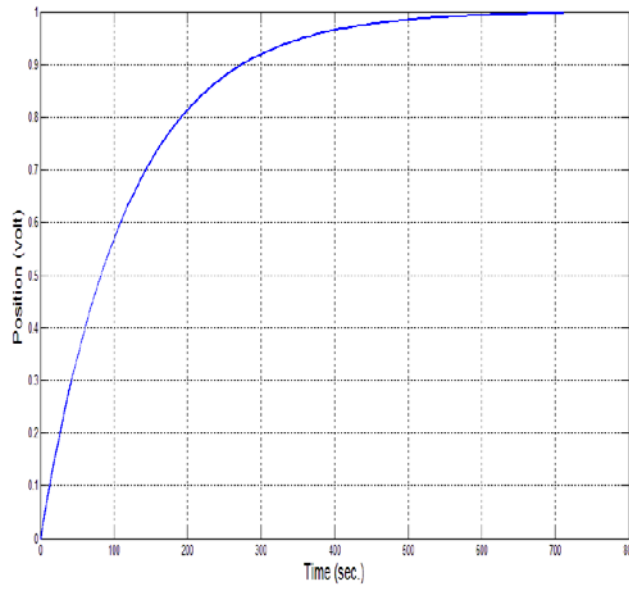


Figure (8) Closed loop response of MLS using conventional PI-PD controller.

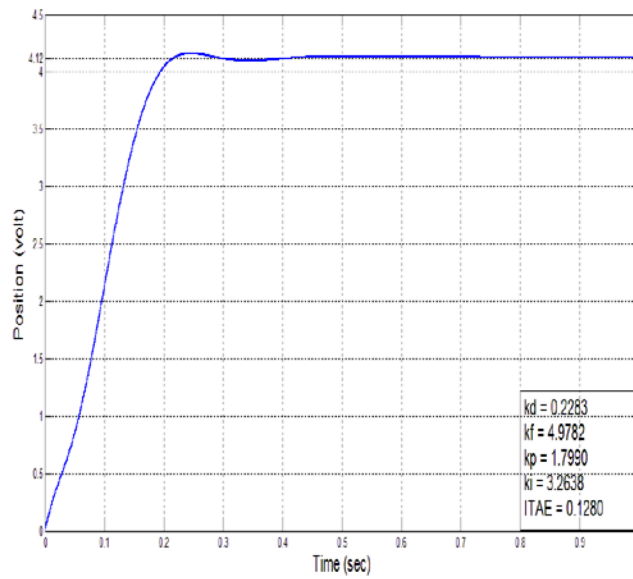


Figure (9) Closed loop response of MLS using PSO based PI-PD controller.

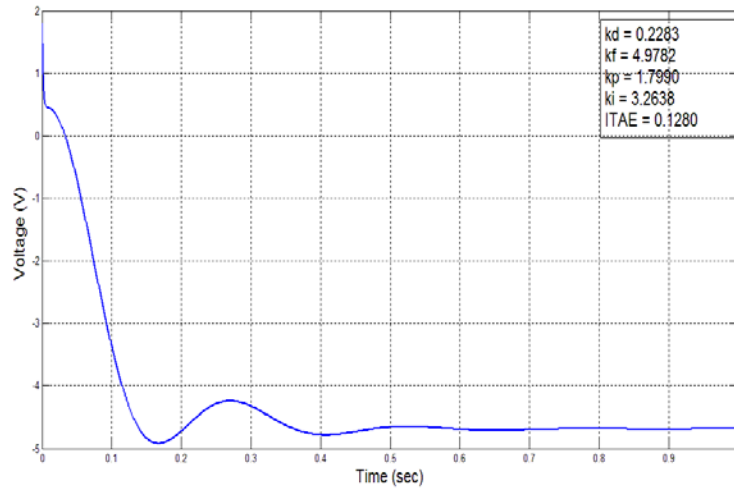


Figure 10: Control action of the controlled system .

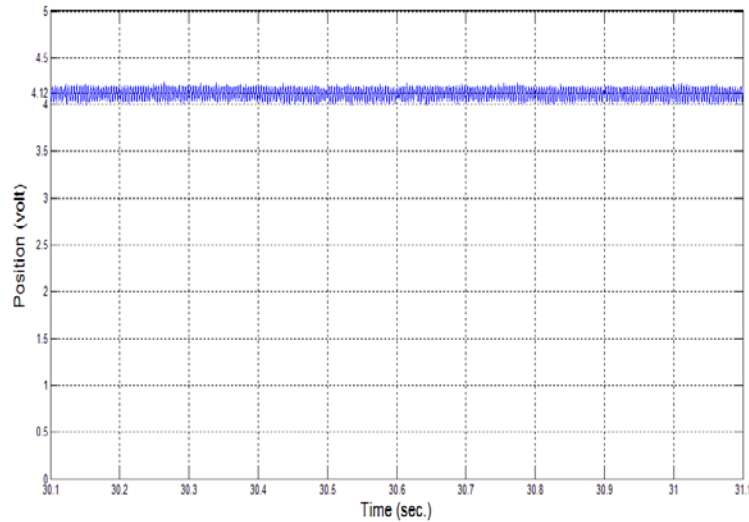


Figure 11: Experimental closed loop response of MLS with PSO Based PI-PD controller.

Table II: Determined and optimized controller parameters and the resulting time response specifications

Controller	Controller parameters				Time response specifications	
	K_p	K_i	K_f	K_d	$t_r(sec.)$	$t_s(sec.)$
Conventional PI-PD	9.7588 $\times 10^{-18}$	0.0463	5.291	0.0467	280	600
PSO based PI-PD	1.7990	3.2638	4.9782	0.2283	0.2	0.3

CONCLUSIONS

The PI-PD controller is a modified form of PID controller and successfully designed to control the unstable and uncertain magnetic levitation system. This form can provide an excellent four parameters controller for control of such as systems to set point changes, in comparison to the conventional PID controller which has limitation in controlling such systems. The PSO method has been used to obtain the optimal parameters of the PI-PD controller for MLS. The obtained results using PSO have a superiority in comparison to those obtained using the method proposed by [5].

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