Modeling and Buckling Analysis of Polymeric Composite Columns

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Abstract

This research studied the buckling of composite columns made from glass fiber reinforced unsaturated polyester. The composite specimens were prepared by hand lay-up technique with different fiber volume fraction $V_f$, aspect ratio and angle of fibers when reinforced with coarse and fine woven fibers. Mathematical models were done by using statistical analysis which show the critical load of the composite column as a function of volume fraction, fiber angle and aspect ratio.

The results show that the maximum value of the critical load can be observed at volume fraction $V_f = 11\%$, aspect ratio $L/T = 3.5$ and fiber angle $\theta = (0^\circ/90^\circ)$ for fine fiber was (500.72 N). Also its found the maximum critical load for coarse fiber can be observed at $V_f = 8\%$, $L/T = 3.5$ and $\theta = (0^\circ/90^\circ)$ was (400.4 N).

Keywords: Polymer composite, Glass fiber, Buckling test , Model.

نمذجة وتحليل الانبعاج للأعمدة المتراكبة البوليميرية

الخلاصة

في هذا البحث تم دراسة الانبعاج للأعمدة المركبة تتكون من البولي أم этиكر غير مقوى بالليف زجاجي. حضرت العينات المركبة بالطريقة اليدوية عند توفير كل من الكسر الحجمي والنسبية البدائية وزاوية ليف لنوعين من النقاومة الصغرى الخشنة والنااعمة. تم عمل نماذج رياضية باستخدام التحليل الأحصائي الذي بين الحمل الشوكي للعمود المركب كدالة للكسر الحجمي وزاوية ليف والنسبية البدائية.

بينت النتائج أن أقصى قيمة للحمل الشوكي حدث عند الكسر الحجمي ($V_f = 11\%$) والنسبية البدائية ($L/T = 3.5$) وزاوية ليف ($\theta = 0^\circ/90^\circ$) عند النقاومة الصغرى الخشنة ($500.72$ N). أيضا وجد بأن أقصى قيمة للحمل الشوكي عند النقاومة بالحصيرة الخشنة حدث عند الكسر الحجمي ($V_f = 8\%$) والنسبية البدائية ($L/T = 3.5$) وزاوية ليف ($\theta = 0^\circ/90^\circ$) كانت ($400.4$ N).

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INTRODUCTION

In recent years the improved design, fabrication and mechanical performance of low-cost composites has led to increase in the use of composites for large patrol boats, hovercraft, mine hunters and corvettes [1].

A beam under axial compressive load can become unstable and collapse. This occurs when the beam is long and its internal resistance to bending moment is insufficient to keep stable. At the critical load the beam became unstable and buckled that a large deflection occurs due to small increase in force, this force called critical load or (buckling load). The equations of the buckling of the column have been derived many years ago and readily variable to design engineers. Columns made of polymer matrix fiber reinforced composite materials are increasingly used in automotive, aerospace, structural and mechanical engineering industries. And buckling loads are important parameter in the design and development of high – performance composite [2].

Zureick & et al. in (1992) reported preliminary results of a buckling test conducted on GFRP box section having cross section dimension of 76.2x76.2 mm², a wall thickness of 6.3 mm, and a length of 2,570 mm. In this study, the columns had a pinned-pinned support and the effective slenderness ratio of 89. Also, three additional axially loaded pultruded box sections having slenderness ratios of (87, 76, and 66). It was concluded that the Southwell plot could be used to estimate the buckling loads of the columns [3].

Seangatith in (2000) studied the buckling of GFRP box columns with different supports. He was presents the experimental results of the axially loaded GFRP box columns with pinned-pinned and pinned-fixed support and comparing the obtained results with column design equations. It was found that the design equations fit practically well with the test results [4].

Fatahi & et al in (2010) illustrated buckling and post buckling analysis of beam composite having studied numerically by using generalized differential quadrature method (GDQM). Also investigated the effects of shear deformation and the bending. Stretching coupling were added by incorporating a shear deformation beam theory. The results show that the GDQ technique can be used as a powerful, reliable, accurate and efficient numerical tool in assessing the buckling and post buckling responses of delaminated composite structures [5].

Alwan & et al in (2011) conducted a study involve calculated the tensile and buckling load of composite beam This research focuses on the preparation of polymer matrix composite material reinforced by natural jute fiber with different volume fractions. The experimental work and finite element techniques were used. The results appeared the critical load increased with increase the fiber volume fraction [6].

Jamel & et al in (2012) studied the buckling behavior of composite laminated plates subject to uniform and nonuniform compressive load. This study focused to analytical analysis (for laminates with cutouts) and numerical analysis by finite element method (for laminates with and without cutouts) for different type of loads, in addition to many design parameters of the laminates such as aspect ratio, thickness and lamination angle and parameters of the cutout such as shape, size, position, direction and radii rounding [7].
Thuc & et al in (2012) illustrated vibration and buckling of cross-ply composite beams using refined shear deformation theory. Numerical results are obtained for composite beams to investigate critical buckling loads [8].

The aim of this research was to study the influence of volume fraction and orientation of glass fibers on critical load of composite column made from of unsaturated polyester reinforced by woven fine and coarse fibers.

Theoretical Part

Column fails by buckling when the axial compressive load exceeds some critical load. The critical load of the composite column can be calculated from the Euler equation as follows [9]:-

\[ P_{cr} = C \frac{\pi^2 E I}{L^2} \]  

Where:

- \( C \): is the end condition number, when both end are free to pivot use \( C=1 \).
- \( L \): is the length of the column (m).
- \( A \): Area of cross section (m²).
- \( I \): is moment of inertia (m⁴).
- \( E \): Modulus of elasticity (Gpa).

The compressive stress can be well below the material yield strength at the time of buckling the factor that determines if a column is short or long is it is slenderness ratio (S) [10]:

Where:

\[ S = \frac{L}{r} \]  

and

\[ r = \sqrt{\frac{1}{A}} \]  

That:

- \( r \): is radius of gyration (m).
- \( s \): Slenderness ratio.
- \( I \): is moment of inertia (m⁴).

A short column is usually defined as one whose slenderness ratio is less than (10).

Mathematical Model

Response surface method (RSM) is a collection of a mathematical and statistical technique that are useful for modeling, analysis and optimizing the process in which response of interest is influenced by several variables and the objective. In many engineering fields, there is a relationship between an output variable of interest \( z \) and a set of controllable variables \( \{x, y, \ldots, N_n\} \). In some systems, the nature of relationship between \( y \) and \( x \) values might be known. Then, a model can be written in the form [11]:

\[ z = f(x, y, \ldots, N) + \varepsilon \]  

The variables \( x, y \) and \( N \) are independent variables where the response \( z \) depends on them. The experimental error term, denoted as \( \varepsilon \).

In this study, multiple polynomial (least square fitting) regression analysis is used to establish a mathematical model among the experimentally obtained parameters. Multiple regression analysis techniques are applied to relate the volume fraction and angle of glass fibers for two type (fine and coarse fiber), the best form of
the of the relationship between the property, volume fraction and angle of fiber parameters is chosen in the form of [12]:-

\[ z = a_0 + b_1 x + c_1 y + d_1 x^2 + e_1 y^2 + f_1 x y \]  

--------- (5)

The analysis of variance (ANOVA) is also called coefficient of multiple determination referred to (R) measure the proportionate reduction of total variation in associated with use of the set of predictors in the model (is used to check the validity of the model) it is defined in terms of SST, SSR, and SSE as [13] :-

\[ R^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST} \]  

--------- (6)

where:-

SSE:- the sum of squared error.
SSR:- the regression sum of squares.
SST:-the total correct sum of squares.

For the three factors, the second polynomial (regression) could be expressed as:-

\[ Z = b_0 + b_1 x + b_2 y + b_3 z + b_4 x^2 + b_5 y^2 + b_6 z^2 + b_7 x y + b_8 x z + b_9 y z \]  

--------- (7)

where:-

\[ b_0, b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9 \] are coefficients linear terms.

Experimental Work

Basically two main tasks were carried out to achieve the objectives of study. The first task was the preparation of composite columns by combining the unsaturated polyester and woven fiber glass (illustrated in Figure (1)) with different fibers volume fraction (0, 3, 5 & 8 % Vf) for coarse fibers and (0, 4, 7 & 11% Vf) for fine fibers, and with different angle of fiber (0°/90°) and (45°/-45°). Then it was continued by performing the buckling test carried out to determine the characteristics of the studied composite. The usage of unsaturated polyester resin as a matrix was chosen because it is the standard economic resin commonly used, preferred material in industry and besides, it yields highly rigid products with a low heat resistance property. The unsaturated polyester resin is provided from the Saudi Arabia Company in the form of transparent viscous liquid at room temperature. The resin was prepared by mixing unsaturated polyester with 2% hardener. The hardener type used is the Methyl Ethyl Keton Peroxide (MEKP) is provided from the Saudi Arabia Company.

Preparation of Composites

The composite specimens were fabricated by using hand lay-up technique. Composites having different fibers content were prepared by varying the type, volume fraction, aspect ratio (L/T) for composite column and angle of fibers for fine and coarse woven fibers. In the first process of preparing the composite specimen's preparation process is to set the percentage of fibers content in the composite. The amount of resin needed for each category of composite laminate was calculated after that. Then the resin was mixed uniformly with hardener, the mixture was poured carefully into the moulds, and left in the mould for 24 hours. After the composites were fully dried, they were separated off from the moulds, and then put the specimens in oven at (55˚C) for (1 hrs) [14].
The mould used in this study was glass sheet in dimension (30 cm×20 cm ×0.4cm). Specimens are prepared after the composites are ready as shown in Figure (2). Buckling test is done by using universal testing machine type (LARYE) with capacity (50 KN) applied load. The specimen (composite column) was loaded in axial compression using a uniaxial tensile testing machine. The specimen was mounted with the boundary condition selected in the tensile test machine along the top and bottom edges and kept free at the other two ends (pin-pin). For axial loading, the test specimen with boundary condition was placed between the two extremely stiff machine heads, of which the upper heads was fixed during the test, whereas the lower head was moved upwards by servo hydraulic cylinder. The dial gauge type (MITUTOYO) was mounted at the center of the specimen to detect the deflection in order to measure and draw the load-deflection curve.

Results & Discussion

Load-Deflection Curves

Figures (3-14) show the relationship between buckling load and deflection for both fine and coarse fiber at different volume fraction, L/T and \( \theta = (0^\circ/90^\circ) \) & \( (45^\circ/-45^\circ) \). It can clearly evident that when the applied loading was increased the buckling deformation also increased. Also prominent reduction in strength has also been observed when the composite column become more slender. Test results indicate that the deformation is smallest in \( (0^\circ/90^\circ) \) fiber orientation. This is clearly illustrated by the fact that \( (0^\circ/90^\circ) \) orientation is stiffer, since fiber direction and loading direction are separated by a small angle. As the fiber orientation \( (45^\circ/-45^\circ) \) the fiber direction becomes almost transverse to the loading direction and hence the stiffness in load direction \( (45^\circ/-45^\circ) \) becomes less. The critical load was determined by applying projection of the intersection on to load-deflection curve. It can be seen from these figures when the column at L/T = 3.5 and \( \theta = (0^\circ/90^\circ) \) for both fine and coarse fiber having higher buckling load with small deflection. If making comparison between the figures of fine fibers and coarse fibers, it were found the composite specimens reinforced with fine fibers gives higher critical load values that of composite specimens when reinforced with coarse fibers.

Effect of Fiber Volume Fraction and Aspect Ratio of Composite Column on Critical Load for Fine & coarse Fiber

Figures (15-18) show the critical load versus fiber volume fraction for both fine and coarse glass fiber at different slenderness ratio of column. It can be seen from these figures the critical load increased as the fiber volume fraction increased because of the glass fiber have stiffness and this lead to increase the stiffness of composite specimens and improved buckling resistance. Also it can be seen from these figures when the column length is increased, the buckling resistance is decrease. It is seen that the critical loads change according to the fiber orientation, its found the maximum critical load can be observed at \( V_f = (11\%) \), L/T = (3.5) and \( \theta = (0^\circ/90^\circ) \) for fine fiber was (500.72 N), while critical load for coarse fiber at \( V_f = (8\%) \), L/T = (3.5) and \( \theta = (0^\circ/90^\circ) \) was (400.4N). The minimum value of critical load for fine fiber can be observed at \( V_f = (4\%) \), L/T = (4.5) and \( \theta = (45^\circ/-45^\circ) \) was (105.775 N), while the minimum value for coarse fiber can be seen at \( V_f = (3\%) \), L/T = (4.5) and \( \theta = (45^\circ/-45^\circ) \) was (80.7 N).
Effect of Fiber Orientation & Volume Fraction on Critical Load for Fine & Coarse Fiber

Figures (19-24) show the variation of critical load with angle of fiber at different volume fraction and aspect ratio of composite column. It can be seen that the critical load increased depend on fiber direction respective to direction of the load. This behavior of the composite column may be attributed to the fact that (0º/90º) fiber orientation produced on increased resistance to buckling, while in (45º/-45º) fiber orientation, the resistance of buckling is lower. This behavior may be attributed to the variation of fiber orientation, as the fiber orientation angle increased the stiffness of composite specimens also increased [1]. It was found that (0º/90º) fiber orientation was the best and has the maximum load bearing capacity and strength than other (45º/-45º) orientation.

Mathematical Model Results for Buckling Test

The experimental results are modeled using RSM. Table (1) shows the summary of models and coefficient multiple determinations (R²) of the properties as function of (x= volume fraction of fiber) and (y=angle of fiber). It can be seen from these models that the volume fraction have greater effect than the angle of fiber on the properties.

Development of Mathematical Model Results of Buckling Test

The validity of regression models developed is further tested by drawing scatter diagrams. Typical scatter diagrams for all the models are presented in table (2). Where (X= volume fraction, Y=Angle of fiber & M= aspect ratio (L/T). The observed values and predicted values of the responses are scattered close to the (45º) line, indicating an almost perfect fit of the developed empirical models [15].

CONCLUSIONS

The main conclusion of buckling result was:-
1- The critical load of composite columns will increased with increased fiber volume fraction for two type of glass fiber, where the maximum value of the critical load can be observed at volume fraction for fine fiber \( V_f =11\% \) was (500.72 N) while the maximum value of the critical load can be observed at volume fraction for coarse fiber \( V_f =8\% \) was (400.4 N).
2- The fiber orientation (0º/90º) have higher critical load than fiber orientation (45º/-45º) for two type of fiber.
3- The value of critical load decreases with increase length of composite column.
4- Mathematical models represent the volume fraction have greater effect than angle of fiber on the critical load.
5- Development of mathematical models represent the observed values and predicated values of the responses scattered close to the (45º) line.

REFERENCES:

Table (1): Mathematical Model Results.

<table>
<thead>
<tr>
<th>Property</th>
<th>Mathematical model and coefficient of multiple determination</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical load for fine fiber at L/T=3.5</td>
<td>$Z=(79.453)+(45.645)\times x+(-2.836655)\times y+(-0.776637)\times x^2+(0.0315184)\times y^2+(-5.15083e-12)\times x\times y$</td>
<td><img src="image1.png" alt="Figure 1" /></td>
</tr>
<tr>
<td></td>
<td>$R^2 = 0.99139$</td>
<td></td>
</tr>
<tr>
<td>Critical load for fine fiber at L/T=4</td>
<td>$Z=(60.90383)+(37.26406)\times x+(-2.6118)\times y+(-0.895488)\times x^2+(0.02902)\times y^2+(8.74478e-11)\times x\times y$</td>
<td><img src="image2.png" alt="Figure 2" /></td>
</tr>
<tr>
<td></td>
<td>$R^2 = 0.98695$</td>
<td></td>
</tr>
<tr>
<td>Critical load for fine fiber at L/T=4.5</td>
<td>$Z=(53.2378)+(25.4859)\times x+(-2.261944)\times y+(-0.362101)\times x^2+(0.0251327)\times y^2+(-3.198763e-11)\times x\times y$</td>
<td><img src="image3.png" alt="Figure 3" /></td>
</tr>
<tr>
<td></td>
<td>$R^2 = 0.97968$</td>
<td></td>
</tr>
<tr>
<td>Critical load for coarse fiber at L/T=3.5</td>
<td>$Z=(77.90735)+(42.59163)\times x+(-2.563333)\times y+(-0.3244444)\times x^2+(0.0284815)\times y^2+(1.038227e-12)\times x\times y$</td>
<td><img src="image4.png" alt="Figure 4" /></td>
</tr>
<tr>
<td></td>
<td>$R^2 = 0.99016$</td>
<td></td>
</tr>
<tr>
<td>Critical load for coarse fiber at L/T=4</td>
<td>$Z=(61.57736)+(22.6955)\times x+(-2.841366)\times y+(2.004955)\times x^2+(0.0315707)\times y^2+(2.190684e-10)\times x\times y$</td>
<td><img src="image5.png" alt="Figure 5" /></td>
</tr>
<tr>
<td></td>
<td>$R^2 = 0.9765959$</td>
<td></td>
</tr>
<tr>
<td>Critical load for coarse fiber at L/T=4.5</td>
<td>$Z=(43.99073)+(26.9614)\times x+(-1.4848)\times y+(-0.3249555)\times x^2+(0.0164978)\times y^2+(-1.7337e-10)\times x\times y$</td>
<td><img src="image6.png" alt="Figure 6" /></td>
</tr>
<tr>
<td></td>
<td>$R^2 = 0.984546$</td>
<td></td>
</tr>
</tbody>
</table>
### Table (2): Development of Mathematical Model Results of Buckling Test.

<table>
<thead>
<tr>
<th>Property</th>
<th>Development of mathematical model and coefficient of multiple determination</th>
<th>Scattering</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fine fiber</td>
<td>$Z = (1016.2) + (103.823)Y + (-2.998)M + (-0.678)X^2 + (-0.295)Y^2 + (54.2292)M^2 + (-0.34)X<em>Y + (-16.6238)X</em>M + (0.145984)Y*M$</td>
<td>$R^2 = 0.98928$</td>
</tr>
<tr>
<td>Coarse fiber</td>
<td>$Z = (-41.443) + (93.287)X + (-2.29)Y + (78.208)M + (0.45185)X^2 + (0.025517)Y^2 + (-13.004)M^2 + (-15.6343)X*M$</td>
<td>$R^2 = 0.97784$</td>
</tr>
</tbody>
</table>

![Figure (1): Glass fiber used in this research.](image)
Figure (2) Specimens of Buckling Test.
A) Specimens with L/T = 3.5  
B) Specimens with L/T = 4  
C) Specimens with L/T = 4.5

Figure (3): Plots between load & deflection for fine fiber at θ= (0°/90°) & (L/T) = (3.5).
Figure (4): Plots between load & deflection for fine fiber at $\theta = (0^\circ/90^\circ)$ & $(L/T) = (4)$.

Figure (5): Plots between load & deflection for fine fiber at $\theta = (0^\circ/90^\circ)$ & $(L/T) = (4.5)$.

Figure (6): Plots between load & deflection for fine fiber at $\theta = (45^\circ/-45^\circ)$ & $(L/T) = (3.5)$. 
Figure (7): Plots between load & deflection for fine fiber at $\theta = (45^\circ/-45^\circ)$ & $(L/T) = (4)$.

Figure (8): Plots between load & deflection for fine fiber at $\theta = (45^\circ/-45^\circ)$ & $(L/T) = (4.5)$.

Figure (9): Plots between load & deflection for coarse fiber at $\theta = (0^\circ/90^\circ)$ & $(L/T) = (3.5)$. 
Figure (10): Plots between load & deflection for coarse fiber at θ = (0°/90°) & (L/T) = (4).

Figure (11): Plots between load & deflection for coarse fiber at θ = (0°/90°) & (L/T) = (4).

Figure (12): Plots between load & deflection for coarse fiber at θ = (45°/-45°) & (L/T) = (3.5).
Figure (13): Plots between load & deflection for coarse fiber at $\theta = (45^\circ/-45^\circ)$ & $(L/T) = (4)$.

Figure (14): Plots between load & deflection for coarse fiber at $\theta = (45^\circ/-45^\circ)$ & $(L/T) = (4.5)$.

Figure (15): Shows the relationship between volume fraction & critical load for fine fiber at $\theta = (0^\circ/90^\circ)$. 
Figure (16): Shows the relationship between volume fraction & critical load for fine fiber at $\theta = (45^{\circ}/-45^{\circ})$.

Figure (17): Shows the relationship between volume fraction & critical load for coarse fiber at $\theta = (0^{\circ}/90^{\circ})$.

Figure (18): Shows the relationship between volume fraction & critical load for fine fiber at $\theta = (45^{\circ}/-45^{\circ})$. 
Figure (19): Shows the effect of fiber orientation on critical load for fine fiber at $L/T = (3.5)$.

Figure (20): Shows the effect of fiber orientation on critical load for fine fiber at $L/T = (4)$.

Figure (21): Shows the effect of fiber orientation on critical load for fine fiber at $L/T = (4.5)$. 
Figure (22): Shows the effect of fiber orientation on critical load for coarse fiber at $L/T = (3.5)$.

Figure (23): Shows the effect of fiber orientation on critical load for coarse fiber at $L/T = (4)$.

Figure (24): Shows the effect of fiber orientation on critical load for coarse fiber at $L/T = (4.5)$. 