Nonlinear Finite Element Analysis for Punching Shear Resistance of Steel Fibers High Strength Reinforced Concrete Slabs

Dr. Eyad K. Sayhood  
Building and Construction Engineering Department, University of Technology/Baghdad  
Dr. Samer P. Yaakoub  
Building and Construction Engineering Department, University of Technology/Baghdad  
Hussien Fadhil Hussien  
Building and Construction Engineering Department, University of Technology/Baghdad  
Email: hfh_52@yahoo.com

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ABSTRACT
This study is devoted to investigate the punching shear resistance of high strength reinforced concrete slabs with steel fibers by using the well-known (P3DNFEA), a non-linear finite element program for three-dimensional analysis of reinforced concrete structure.

Nine high strength reinforced concrete slabs with steel fibers and one without steel fibers, have been analyzed in the present study. The finite element solutions are compared with the available experimental data. In general, accepted agreement between the numerical results and the experimental results has been obtained. Parametric studies have been carried out to investigate the effect of concrete compressive strength, steel fiber content, amount of steel rebars, slab depth and column dimensions on the behavior and ultimate strength of reinforced concrete slabs.

The numerical analysis indicated that the increase in the concrete compressive strength \( (f'_c) \) from 40 to 80 MPa has led to an increase in the strength by 69% and 84% for slabs without and with 0.5% steel fibers respectively. The numerical analysis indicated that by using 2.0% steel fibers, the ultimate capacity is increased by 81.7%, compared to a slab without fibers. Also, the finite element solution revealed that increasing the longitudinal reinforcement ratio in the slab from 1% to 2% led to an increase in the ultimate shear strength of about 57%.

Keywords: Punching Shear ,Slab, Steel Fiber ,Finite Element ,Nonlinear
التحليل اللاخطي بالعناصر المحدودة لمقاومة القص الثاقب لل블ادات الخرسانية المسلحة عالياً المقاومة والمعززة باللياف الفولاذية

الخصائص

في هذا البحث تم دراسة مقاومة القص الثاقب للبلاتات الخرسانية المسلحة عالية المقاومة والمعززة باللياف الفولاذية بوساطة التحليل اللاخطي ثلاثي الابعاد بطريقة العناصر المحدودة باستخدام البرنامج الشائع (P3DNFEA). في هذه الدراسة تم تحليل تسعة بلاتات خرسانية عالية المقاومة والمعززة باللياف الفولاذية وبلطة واحدة خالية من اللياف الفولاذية. قُربت النتائج المستحصلة من طريقة العناصر المحددة مع النتائج العملية. بشكل عام، حصل توافق مقبول بين النتائج التحليلية والعملية. المغبرات التي تم دراسها هي تأثير مقاومة الانضغاط، معالجة الألياف الفولاذية، كمية حدث التسليح، عمق البتة وابعاد الأعمدة على تصرف وتحمل البلاتات الخرسانية.

استمرت النتائج التحليلية إلى زيادة مقاومة الانضغاط للخرسانة (40) إلى (80) ميا، بإحكام أدى إلى زيادة البتة بنسبة (69%)، (84%) للبلاتات الخالية من الألياف والبلاتات التي تحتوي على نسبة (0.5%) من الألياف على التوالي.

إن استخدام نسبة (2%) من الألياف الفولاذية أدى إلى زيادة مقاومة القص القصوى للبلاتات بنسبة (81.7%) مقارنة بللابلات التي لا تحتوي على الألياف، بينما زيادة نسبة حديد التسليح الطولي من (1%) إلى (2%) أدى إلى زيادة مقاومة القص القصوى بحدود (57%).

Notation

- $d_f$ Diameter of fiber
- $f'_c$ Uniaxial compressive strength of concrete
- $f'_{cf}$ Uniaxial compressive strength of fibrous concrete
- $f_t$ Uniaxial tensile strength of fibrous concrete
- $L_f$ Fiber length
- $N_f$ Number of fibers per unit area
- $V_f$ Volume fraction of fibers
- $\alpha_1, \alpha_2$ Tension-stiffening parameter
- $\beta$ Shear retention factor or material constant.
- $\lambda$ Compressive strength reduction factor of concrete
- $\sigma_0$ Effective stress at onset of plastic deformation
- $\varepsilon_{cr}$ Cracking tensile strain in concrete
- $\varepsilon_t$ Tensile strain at peak tensile stress
- $\varepsilon_{ufc}$ Fibrous Concrete Ultimate strain in compression
INTRODUCTION

A flat-plate slab system can be defined as a slab with uniform thickness supported on the columns without beams, drop panels or column capitals. Conventionally, the reinforced concrete flat-plate systems are generally used for lightly loaded structures, such as residential or office buildings with relatively short spans. Flat-plates have been widely used due to the reduced construction cost associated with the simple formwork and simple arrangement of flexural reinforcement. An additional advantage of a flat-plate is a reduced building story height that results in more usable space in building for a given or limited height (1).

The undesirable suddenness and the catastrophic nature of the failure can be formed at the connection between the slab and the column which is called punching shear failure. A typical punching shear failure in reinforced concrete begins with the formation of flexural hinges (yield lines) around the perimeter of the loaded area, these hinges develop as a result of the moment caused in the slab by the applied load. This moment then begins to produce radial cracks that extend outward from the area of loading to the perimeter of the slab. Failure of the slab occurs when the diagonal tension cracks intersect the radial cracks and produce a cone of failure (2).

Two characteristics, however, have been limited the use of high strength concrete (HSC): it is brittle and weak in tension. Recently, for improving these deficiencies, instead of reinforcement bars, different kind of fibers are used (3). Research findings in the last decade clearly establish that the ductility of certain structural members can be greatly enhanced with the use of fibers. In addition, fibers are generally favor improvements in first crack and ultimate member strength, impact resistance, and shear resistance. The deficiencies of ordinary reinforced concrete in the form of micro-cracks, which cannot be corrected by bar reinforcement, can be remedied to a significant extent by fiber reinforcement.

Addition of randomly oriented fiber in plain concrete helps to bridge and arrest the cracks formed in the brittle concrete under applied stresses, and enhances the ductility and energy absorption properties of the composite (4).

The ability of steel fibers to enhance shear strength of concrete is attributed to the possible transfer of tensile stresses across crack surfaces, that is known as crack-bridging, when steel fibers are incorporated. This phenomenon permits fiber reinforced concrete to have more ductile failure than normal concrete (5). When fibers are added in a slab-column connection, the residual strength of the slab-column connection after the formation of punching shear crack increases significantly, due to the enhanced structural integrity between the slab and the column (6).

Finite Element Model

In the present research work, a full three-dimensional finite element idealization has been used. This idealization gives accurate simulation for geometry, type of failure and location of reinforcing bars. The 20-node quadratic brick element shown in Fig. (1) is adopted to represent concrete in the present study.
The reinforcement representation that is used in this study is the embedded representation, Fig. (1). The reinforcing bar is considered to be an axial member built into the concrete element. Perfect bond was assumed to occur between the concrete and the reinforcing bars. The reinforcing bars were assumed to be capable of transmitting axial force only.

The numerical integration is generally carried out using the 27(3x3x3) point Gaussian type integration rule. The nonlinear equations of equilibrium have been solved using an incremental-iterative technique operating under load control. The nonlinear solution algorithm that is used in this research work is the modified Newton –Raphson method in which the stiffness matrix is updated at the 2nd, 12th, 22nd, ...etc. iterations of each increment of loading.

![Figure (1) Twenty-node isoparametric brick element. (11)](image)

**Material Model Adopted in the Analysis**

**Behavior in Compression:**

In compression, the behavior of concrete is simulated by an elastic-plastic work hardening model followed by a perfectly plastic response, which is terminated at the onset of crushing. The growth of subsequent loading surfaces is described by an isotropic hardening rule. A parabolic equivalent uniaxial stress-strain curve has been used to represent the work hardening stage of behavior and the plastic straining is controlled by an associated flow rule. The concrete strength under multidimensional state of stress is a function of the state of stress and cannot be predicted by simple tensile, compressive and shearing stress independent of each other. So the state of stress must be scaled by an appropriate yield criterion to convert it to equivalent stress that could be obtained from simple experimental test. The yield criterion that has been used successfully by many investigators (7),(8) can be expressed as:

$$
\text{Material Model Adopted in the Analysis}
$$

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\[ f\left(\{\sigma\}\right) = \left(\alpha I_1 + 3\beta J_2\right)^{0.5} = \sigma_0 \quad \ldots (1) \]

where \( \alpha \) and \( \beta \) are material parameters which are dependent on the type of concrete, mainly on the volume fraction of fiber \( V_f \), and their values are shown in Table (1) (9),(10).\( I_1 \) is the first stress invariant and \( J_2 \) is the second deviatoric stress invariant. \( \sigma_0 \) is an equivalent effective stress at the onset of plastic deformation which can be determined from a uniaxial compression test.

**Table (1) Values of \( \alpha \) and \( \beta \) for different fiber ratios (11)**

<table>
<thead>
<tr>
<th>Fiber content( V_f )%</th>
<th>( \alpha )</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.3546798 ( \sigma_0 )</td>
<td>1.3546798</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0993042 ( \sigma_0 )</td>
<td>2.0993042</td>
</tr>
<tr>
<td>1.0</td>
<td>1.4900526 ( \sigma_0 )</td>
<td>2.4900526</td>
</tr>
<tr>
<td>1.5</td>
<td>1.760526 ( \sigma_0 )</td>
<td>2.7960526</td>
</tr>
<tr>
<td>2</td>
<td>2.278595 ( \sigma_0 )</td>
<td>3.0108</td>
</tr>
</tbody>
</table>

In a reinforced concrete member, a significant degradation in compressive strength can result due to the presence of transverse tensile straining after cracking. In the present study, Vecchio et. al. models are used for high strength concrete (HSC) (9) members, which illustrates the use of the reduction factor, \( \lambda \). The compressive reduction factor, \( \lambda \), for HSC is given as:

\[ \lambda = \frac{1}{1 + K_c K_f} \quad \ldots (2) \]

where \( K_c \) is a factor representing the effect of the transverse cracking and straining and \( K_f \) is a factor representing the effect of concrete compressive strength \( f'_c \).

\[ K_c = 0.35 \left(\frac{\varepsilon_1}{\varepsilon_3} - 0.28\right)^{0.8} \quad \ldots (3) \]

and

\[ K_f = 0.1825 \sqrt{f'_c} \geq 1.0 \quad \ldots (4) \]
where $\varepsilon_1$ is the tensile strain in the direction normal to the crack and $\varepsilon_3$ is the compressive strain in the direction parallel to the crack.

**Behavior in Tension:**

In tension, linear elastic behavior prior to cracking is assumed. Cracking is governed by the attainment of a maximum principal stress criterion. A fixed smeared crack model has been used to simulate the behavior of concrete in tension with a tension-stiffening model to represent the retained post-cracking tensile stresses. The degradation of the shear strength of concrete due to cracking is accounted for by employing a shear-retention model. A smeared crack model with fixed orthogonal cracks is assumed to represent the cracked sampling point. The post-cracking tensile stress-strain relations, Fig. (2), (11), (12) and the reduction in shear modulus with increasing tensile strain Fig. (3), (13) have been adopted in the present work. The tensile strain at peak tensile stress $\varepsilon_{pf}$ is given by:

$$
\varepsilon_{pf} = \varepsilon_t (1 + 0.35 N_f d_f L_f) 
$$

... (5)

Where $N_f$ is the number of fiber per unit area; given by:

$$
N_f = \frac{4V_f}{\pi d_f^2} 
$$

... (6)

**Behavior of Steel Fiber Reinforced Concrete**

In compression, an empirical equation for peak strain value in uniaxial compression of high strength fiber reinforced concrete ($\varepsilon_{pf}$) suggested by Al- Azzawi (14) is adopted in the present study as:

$$
\varepsilon_{pf} = 0.00212 + 0.001 * V_f * \frac{L_f}{d_f} 
$$

... (7)

In this study, an empirical equations for ($f'_{cf}$) suggested by Bunni (15) is adopted and is given by:

$$
f'_{cf} = 1.43 \left( f'_c \right)^{0.941} \left( \frac{L_f}{d_f} \right) V_f K_f \left( \frac{L_f}{d_f} \right)^{0.054} 
$$

... (8)

According to test results reported in reference (16) the concrete peak value of the tensile stress of high strength fiber reinforced concrete ($f'_{tf}$) is proposed in terms of compressive strength of normal concrete ($f'_c$) and fiber volume fraction ($V_f$ in percent) as follows:

$$
f'_{tf} = 0.58 \sqrt{f'_c} + 302V_f 
$$

... (9)
\[ E_c = 3320 \sqrt{f'_c} + 6900 \text{ for } 21 \text{ MPa} < f'_c < 81 \text{ MPa} \quad \ldots (10) \]
\[ \varepsilon_{cuf} = (3011 + 2295V_f) \times 10^{-6} \quad \ldots (11) \]

![Diagram of stress-strain relationship for cracked concrete](image)

**Figure (2)** Post-cracking models for cracked concrete.

**Figure (3)** Shear retention model for cracked concrete (11).
Numerical Example 

Description of Test Specimens: 

Abdulhameed (3) tested experimentally fourteen high strength steel fiber reinforced concrete slabs. Eight of these slabs were chosen for the finite element analysis. The slabs were simply supported along all four edges by using a special manufactured frame with the corners free to rise.

All the slabs were 800 mm square, 60 mm thick and loaded through a central column 100x100 mm. They were provided with a two-way flexural reinforcement consisting of eighteen, 6 mm-diameter deformed steel bars in each direction. Column stubs were reinforced with eight deformed steel bars of 6 mm diameter, cut to length 10 mm less than the height of the column, and two, 6 mm diameter, deformed steel ties 60 mm apart. The overall dimensions of the specimens and the arrangement of the reinforcement are given in Fig.(4) and Fig.(5).

The same type of fibers was used throughout the test program. The fibers were straight plain, 40 mm in length and 0.3 mm in diameter making an aspect ratio \((L_f/d_f)\) of 133. The steel fibers had an ultimate tensile strength of 512 MPa. Different amounts of steel fibers \(V_{(f)}\) were used in the test program.

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Figure (4) Dimensions of test slabs and details of reinforcement.

Figure (5) Reinforcement of column stubs.
Finite Element Idealization and Material Properties:

By taking into consideration the advantage of loading and geometric symmetry, only one-quarter of the specimen has been used in the finite element analysis. The selected quarter was discretized into 51 quadratic brick elements. The external concentrated load was modeled as nodal forces applied on the column stub. The finite element mesh, boundary conditions, and loading arrangement are shown in Fig. (6). Material properties and the adopted additional material parameters are listed in Table (2).

The finite element analysis has been carried out using the 27-point integration rule, with a convergence tolerance of (5%). The modified Newton-Raphson method, in which the stiffness matrix is updated at 2\textsuperscript{nd}, 12\textsuperscript{th}, 22\textsuperscript{nd} … etc. iterations of each increment of loading, was adopted as a nonlinear solution algorithm. Non-uniform increments of applied loads have been used. Large increments of loading were used at early stages of loading, and then smaller increments of loading were used at stages close to the ultimate load.

Figure (6) Finite element idealization, symmetry and boundary conditions used for Abdulhameed slabs.
Results of Analysis:

The experimental and numerical load-deflection curves for slabs S11 to S24 are shown in Fig. (7) and Fig. (8). Generally, the numerical solutions are in acceptable agreement with the experimental results throughout the entire range of the behavior for each tested slab.

The predicted response is approximately similar to the experimental response throughout the pre-cracking stages of behavior. While, it becomes slightly stiffer throughout the post-cracking stages of behavior. However, the ultimate shear capacity obtained from the numerical analysis is almost equal to the experimental ultimate shear capacity. The computed failure loads for all slabs are close to the corresponding experimental collapse load as listed in Table (3).

Parametric Studies

To investigate the effect of some important material and solution parameters on the nonlinear finite element analysis of high strength steel fiber reinforced concrete slabs which fail by punching, slab S12 has been chosen to carry out the parametric study. This study helps to clarify the effect of various parameters that have been considered on the behavior and ultimate load of the analyzed slabs.

Influence of Grade of Concrete

In order to study the influence of using different values of concrete compressive strength ($f'_c$) on the behavior of reinforced concrete slabs with and without steel fibers, slab S21 without steel fibers was analyzed for different values of ($f'_c$). The selected values of the concrete compressive strength used in this study were 40, 50, 60, 70 and 80 MPa. Fig. (9) shows the effect of grade of concrete on the response of slab S21 represented by the load-deflection curves. This figure reveals that as the concrete compressive strength increases, the post-cracking stiffness and the predicted ultimate shear capacity are significantly increased.

While to investigate the effect of compressive strength of fibrous concrete ($f'_{cf}$), slab S12 ($V_f = 0.5$) was analyzed by using the same different values of ($f'_c$) used for slab S21 after applying equation (8), suggested by Bunni (15), to obtain the values of ($f'_{cf}$), as shown in Table (4). Fig. (10) shows the effect of grade of fibrous concrete on the response of slab S12 represented by the load-deflection curves. This figure reveals that as the concrete compressive strength increases, the post-cracking stiffness and the predicted ultimate shear capacity are significantly increased. The numerical ultimate load capacity for slabs S21 and S12 obtained from this study are listed in Tables (5) and (6), respectively.
Table (2) Material properties and material parameters used for Abdulhameed slabs.

<table>
<thead>
<tr>
<th>Slab</th>
<th>S11</th>
<th>S12</th>
<th>S13</th>
<th>S14</th>
<th>S21</th>
<th>S22</th>
<th>S23</th>
<th>S24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volume fraction of fibers $V_f$ (%)</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.0</td>
<td>0.25</td>
<td>0.75</td>
<td>1</td>
</tr>
<tr>
<td>Compressive strength $f'_{cf}$ (MPa)</td>
<td>35.36</td>
<td>49.14</td>
<td>55.06</td>
<td>65.13</td>
<td>42.19</td>
<td>48.76</td>
<td>52.38</td>
<td>53.26</td>
</tr>
<tr>
<td>Young modulus $E_c$ (GPa)</td>
<td>26.64</td>
<td>30.17</td>
<td>31.54</td>
<td>33.69</td>
<td>28.46</td>
<td>30.08</td>
<td>30.93</td>
<td>31.13</td>
</tr>
<tr>
<td>Tensile strength $f_{tf}$ (MPa)</td>
<td>4.96</td>
<td>5.57</td>
<td>5.81</td>
<td>6.19</td>
<td>3.77</td>
<td>4.81</td>
<td>6.46</td>
<td>7.25</td>
</tr>
<tr>
<td>Poisson's ratio $\nu$</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Uniaxial crushing strain $\varepsilon_{cuf}$</td>
<td>0.0004159</td>
<td>0.0004159</td>
<td>0.0004159</td>
<td>0.0004159</td>
<td>0.0003011</td>
<td>0.000585</td>
<td>0.000732</td>
<td>0.0005306</td>
</tr>
<tr>
<td>Steel</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yield stress $f_y$ (MPa)</td>
<td>670</td>
<td>670</td>
<td>670</td>
<td>670</td>
<td>670</td>
<td>670</td>
<td>670</td>
<td>670</td>
</tr>
<tr>
<td>Young modulus $E_s$ (GPa)</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>Steel ratio $\rho$ (%)</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Column size $c$ (mm x mm)</td>
<td>100x100</td>
<td>100x100</td>
<td>100x100</td>
<td>100x100</td>
<td>100x100</td>
<td>100x100</td>
<td>100x100</td>
<td>100x100</td>
</tr>
</tbody>
</table>

**Tension-stiffening parameters**

| $\alpha_1$ | 10 | 10 | 10 | 10 | 8 | 8 | 13 | 15 |
| $\alpha_2$ | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |

† Assumed values

* $E_c = 3320 \sqrt{f'_{cf}} + 6900$ For 21 MPa < $f'_{cf}$ < 81 MPa  
** $f_{tf} = 0.58 \sqrt{f'_{cf}} + 302V_f$  
*** $\varepsilon_{cuf} = (3011 + 2295V_f) \times 10^{-6}$

$\varepsilon_{cuf}$ calculated using $E_c = 3320 \sqrt{f'_{cf}} + 6900$ For 21 MPa < $f'_{cf}$ < 81 MPa  

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\[ \varepsilon_{cuf} = (3011 + 2295V_f) \times 10^{-6} \]
Table (3) Comparison between the experimental and numerical load capacities for Abdulhameed tested slabs.

<table>
<thead>
<tr>
<th>Slab</th>
<th>$V_f$ %</th>
<th>$V_u$ Exp. (kN)</th>
<th>$V_u$ FEM (kN)</th>
<th>$V_u$ FEM/$V_u$ Exp. %</th>
</tr>
</thead>
<tbody>
<tr>
<td>S11</td>
<td>0.5</td>
<td>89.5</td>
<td>77.044</td>
<td>86.1</td>
</tr>
<tr>
<td>S12</td>
<td>0.5</td>
<td>102.5</td>
<td>91.91</td>
<td>89.7</td>
</tr>
<tr>
<td>S13</td>
<td>0.5</td>
<td>129.5</td>
<td>120.24</td>
<td>92.8</td>
</tr>
<tr>
<td>S14</td>
<td>0.5</td>
<td>141</td>
<td>129.7</td>
<td>91.9</td>
</tr>
<tr>
<td>S21</td>
<td>0.0</td>
<td>93.25</td>
<td>84.34</td>
<td>90.4</td>
</tr>
<tr>
<td>S22</td>
<td>0.25</td>
<td>98</td>
<td>92.84</td>
<td>94.7</td>
</tr>
<tr>
<td>S23</td>
<td>0.75</td>
<td>125.5</td>
<td>116.34</td>
<td>92.7</td>
</tr>
<tr>
<td>S24</td>
<td>1</td>
<td>138</td>
<td>129.16</td>
<td>93.6</td>
</tr>
</tbody>
</table>

Table (4) Values of compressive strength of fibrous concrete used for analyzing slab S12

<table>
<thead>
<tr>
<th>$f_c$,MPa</th>
<th>40.0</th>
<th>50.0</th>
<th>60.0</th>
<th>70.0</th>
<th>8.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{cf}$,MPa</td>
<td>43.36</td>
<td>53.49</td>
<td>63.50</td>
<td>73.42</td>
<td>83.25</td>
</tr>
</tbody>
</table>

\[ f_{cf} = 1.43 \left(f'_c\right)^{0.941} \cdot \left(\left(L_f/d_f\right) V_f . K_f \right)^{0.054} \quad \ldots(8) \]

Table (5) Numerical ultimate loads of slab S21 ($V_f=0.0\%$) for various grades of concrete ($f'_c$).

<table>
<thead>
<tr>
<th>Concrete compressive strength (MPa)</th>
<th>FEM</th>
<th>Exp.</th>
<th>FEM</th>
<th>Exp.</th>
<th>FEM</th>
<th>Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>42.19</td>
<td>50</td>
<td>60</td>
<td>70</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>Ultimate load (kN)</td>
<td>76.35</td>
<td>93.25</td>
<td>87.5</td>
<td>103.23</td>
<td>115.34</td>
<td>129.48</td>
</tr>
<tr>
<td>Final deflection (mm)</td>
<td>8.53</td>
<td>10.05</td>
<td>9.12</td>
<td>10.08</td>
<td>10.45</td>
<td>11.58</td>
</tr>
</tbody>
</table>
Table (6) Numerical ultimate loads of slab S12 ($V_f = 0.5\%$) for various grades of concrete ($f'_c$).

<table>
<thead>
<tr>
<th>Concrete compressive strength (MPa)</th>
<th>FEM</th>
<th>Exp.</th>
<th>FEM</th>
<th>Exp.</th>
<th>FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>43.36</td>
<td>49.14</td>
<td>53.49</td>
<td>63.50</td>
<td>73.42</td>
<td>83.25</td>
</tr>
<tr>
<td>Ultimate load (kN)</td>
<td>82.54</td>
<td>102.5</td>
<td>101.5</td>
<td>114.47</td>
<td>135.2</td>
</tr>
<tr>
<td>Final deflection (mm)</td>
<td>8.13</td>
<td>10.20</td>
<td>9.65</td>
<td>10.82</td>
<td>13.05</td>
</tr>
</tbody>
</table>

Slab S11

![Graph showing Load vs. Central Deflection for Slab S11](image1)

Slab S12

![Graph showing Load vs. Central Deflection for Slab S12](image2)

Figure (7). Continuation
Nonlinear Finite Element Analysis for Punching Shear Resistance of Steel Fibers High Strength Reinforced Concrete Slabs

Figure (7). Continuation

Slab S13

Slab S14

Figure (7). Continuation
Figure (7) Abdulhameed slabs, experimental and numerical load- deflection curves.
Figure (8) Abdulhameed slabs, experimental and numerical load-deflection curves.
Figure (9) Influence of concrete compressive strength ($f'_{c}$) on the load-deflection behavior of slab S21 ($V_f=0.0\%$).

Figure (10) Influence of concrete compressive strength ($f'_{cf}$) on the load-deflection behavior of slab S12 ($V_f=0.5\%$).
Influence of Fiber Content

The presence of fibers enhances the stiffness, ductility and ultimate shear capacity of high strength steel fiber reinforced concrete slabs. In order to do this verification, slab S21 (without fibers) was used for this study. The slab has been analyzed by using different amounts of steel fibers of $V_f = 0.0, 0.5, 1.0, 1.5,$ and $2.0\%$. The numerical results of this analysis are listed in Table (7). The finite element analysis for slab S21 reveals that the stiffness, ductility and ultimate shear capacity are significantly increased due to presence of steel fibers, as shown in Fig. (11).

Influence of Amount of Steel

Fig. (12) shows the effect of using different amounts of reinforcement on the load-deflection behavior of slab (S12). Selected steel ratios were 0.26, 1, 1.5 and 2% . The finite element solutions indicated that the post-cracking stiffness and the ultimate shear capacity are increased with the increase in the amount of steel. It can be noted that the deflection of the slab at the ultimate load was decreased with increasing the amount of steel. The numerical results of this analysis are listed in Table (8). The finite element analysis indicated that the ultimate load capacity is increased by about (57%) when the steel ratio is increased by (50%).

Influence of Slab Depth

To study the influence of using different slab thicknesses on the behavior of high strength steel fiber reinforced concrete slabs, Slab S12 was numerically analyzed by FEM with three different values of slab thickness. The chosen thicknesses were 45, 60 and 75 mm. The load-deflection curves obtained from the finite element analysis together with the experimental result (slab thickness equals to 60 mm) are shown in Fig. (13). The finite element results reveal that a considerable increase in the ultimate load capacity has been achieved by increasing the slab thickness. The FEM results of this analysis are listed in Table (9). It can be noted that increasing the slab thickness by 25% leads to an increase the ultimate load by about 19%, while decreasing the thickness to 25% leads to a decrease in the ultimate load by about 22%.

| Table (7) Influence of fiber content on the ultimate load of slab S21 |
|-------------------|-------|-----|-----|-----|-----|
| Fiber content $V_f$ % | 0.0   | 0.5 | 1.0 | 1.5 | 2.0 |
| FEM ultimate load $V_u$ (kN) | 84.34 | 93.42 | 112.23 | 128.56 | 153.21 |
| % Increase in ultimate load | Reference | 10.77 | 33.07 | 52.43 | 81.65 |
| Final deflection (mm) | 9.21 | 9.5 | 10.63 | 12.31 | 10.45 |
Table (8) Influence of amount of steel on the ultimate load of slab S12

<table>
<thead>
<tr>
<th>Steel ratio %</th>
<th>FEM</th>
<th>Exp.</th>
<th>FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.26</td>
<td>1</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>1.5</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ultimate load (kN)</td>
<td>39.97</td>
<td>75.26</td>
<td>102.5</td>
</tr>
<tr>
<td>Final deflection (mm)</td>
<td>13.74</td>
<td>11.03</td>
<td>10.20</td>
</tr>
</tbody>
</table>

Table (9) Influence of depth of slab on the ultimate load of slab S12

<table>
<thead>
<tr>
<th>Depth of slab (mm)</th>
<th>FEM</th>
<th>Exp.</th>
<th>FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
<td>60</td>
<td>60</td>
<td>75</td>
</tr>
<tr>
<td>Ultimate load (kN)</td>
<td>71.68</td>
<td>102.5</td>
<td>91.91</td>
</tr>
<tr>
<td>Final deflection (mm)</td>
<td>11.82</td>
<td>10.20</td>
<td>8.88</td>
</tr>
</tbody>
</table>

Figure (11) Influence of fiber content on the load-deflection behavior of slab S21.
Figure (12) Influence of amount of steel on the load-deflection behavior of slab S12.

Figure (13) Influence of depth of slab on the load-deflection behavior of slab S12.
CONCLUSIONS
1- The three-dimensional nonlinear finite element model used in the present research work is able to simulate the behavior of high strength steel fiber reinforced concrete slabs. The finite element analysis showed that the predicted behavior and the ultimate load capacity are in accepted agreement with the experimental results.
2- The predicted ultimate load capacity of slabs was found to be depending on the grade of concrete. The finite element analyses carried out for compressive strength ranging from 40 to 80 MPa indicated that the percentage increase in ultimate load capacity is about 69% and 84% for slabs without and with 0.5% steel fibers, respectively.
3- The ultimate load capacity of the analyzed slabs is significantly increased with the addition of steel fibers compared to slabs without steel fibers, the ultimate load capacity increased by about 10.77%, 33.07%, 52.43% and 81.65% for fiber contents of 0.5%, 1.0%, 1.5% and 2.0%, respectively.
4- The finite element solutions reveal that the ultimate load of slabs is increased with the increase of the amount of reinforcement, when the steel ratio is increased by about 50%, an increase in the ultimate load capacity by 57% is achieved.
5- The finite element analyses results show that the increasing of slab thickness from 60 to 75 mm leads to an increase in the ultimate load by about 19%, while decreasing the thickness of slab from 60 to 45 mm leads to a decrease in the ultimate load by about 22%.

REFERENCES


