# Kinematics Analysis of 5250 Lab-Volt 5-DoF Robot Arm 

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#### Abstract

This paper presents for the first time a complete kinematic analysis of 5250 LabVolt 5-Dof robot which include both forward kinematics and inverse kinematics for this robot arm. The kinematics problem is defined as the transformation from the Cartesian space to the joint space and vice versa. The forward and inverse kinematics for any type of robots are very important in both trajectory planning and position control for the robot arm,the Denavit-Harbenterg (D-H) representation is used to model robot links and joints in this paper. The inverse kinematics have been solved using analytical solution and programmed using MATLAB to move the robot.


Keywords: - Forward Kinematics, Inverse Kinematics, Denavit-Harbenterg

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\begin{aligned}
& \text { التحليل الحركي لذراع روبوت لاب فولت } 5250 \text { ذا خمسة درجات حرية للحركة } \\
& \text { الخلاصة: } \\
& \text { يقدم هنا البحث وللمرة الاولى تحليل كامللمركة لروبوت لاب فولت } 5250 \text { الذي لايه خمس درجات } \\
& \text { حرية للحركة, ويشمل هنا النحليل تحليل كل من الحركة الامامية والعكسية لهيا النوع من الروبوتات. } \\
& \text { وتعرف مشكلة الحركة بانها التحويل من الاحداثياتاللايكارتية الى احداثيات مفاصل الروبات اليوت وبالعكس. } \\
& \text { تحليل الحركة الامامية والعكسية لأي نوع من انواع الروبونات مهم جدا في تخطيط مسار الروبوت } \\
& \text { والسيطرة على موقع الروبوت, وقد استعل في هاءا البحث نحويل Denavit-Harbenterg لنمذجة } \\
& \text { مفاصل ووصلات الروبوت. وقا استخدم في التحليل العكسي الحل التحليلي و تمت برمجته باستخذام } \\
& \text { برنامج الماتلاب لنحريك الروبوت. }
\end{aligned}
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## INTRODUCTION

R
obotics is a relatively young field of modern technology that crosses traditional engineering boundaries. Understanding the complexity of robots and their applications require knowledge of (Electrical, Mechanical Systems and Industrial)Engineering, Computer science, Economics, and Mathematics. New disciplines of engineering, such as manufacturing engineering, applications engineering, and knowledge engineering have emerged to deal with the complexity of the field of robotics and factory automation. [1]

Robotics is one of the main disciplines in the industry which can be used in the development of new technologies. The synergy of robotics with the different applications like submarine task, car assembly operation; vision systems and artificial intelligence allows the innovation and reduces the manufacture costs. For this purpose, it is important that the robot programmers are able to visualize and test the behavior of the robots in different circumstances and with different parameters. [2]

The transformation between the joint space and the Cartesian space of the robot is very important. Robots are operated with their servo motors in the joint space, where as tasks are defined and objects are manipulated in the Cartesian space [3].

The kinematics solution of any robot arm consist two main problems: forward and inverse kinematics. The forward kinematics determine the robot end effect or position where will be in the Cartesian space if its joint angles are known, while the inverse kinematics calculate the joint angles of the arm what will beif the desired position and orientation of the end effect or are determined.

In this research the Denavit-Harbenterg [4] approach has been used for driving the kinematics of the Lab-Volt 5250 robot arm as shown in figure (1).


Figure (1). Lab-Volt 5250 robot arm [5]

## DESCRIPTION

Lab-Volt 5250 robot arm has five degree of freedom (DoF) plus gripper motion. It is also similar to human arm from the number of joints point of view. These joints provide shoulder rotation, shoulder back and forth motion, elbow motion, wrist up and down motion, wrist rotation and gripper motion.
Lab-Volt 5250 robot arm has five rotational joints and a moving grip. Joint1 represents the Base and its axis of motion is $z_{0}$. This jointprovides a rotational $\theta_{1}$ angular motion around $z_{0}$ axis in $x_{0} y_{0}$ plane. Joint 2 is identified as the shoulder and its axis is perpendicular to Joint 1 axis. It provides an angular motion $\theta_{2}$ in $x_{1} y_{1}$ plane. z axes of Joint 3 (Elbow) and Joint 4 (Wrist)are parallel to Joint 1 z axis, They provide $\theta_{3}$ and $\theta_{4}$ angularmotions in $x_{2} y_{2}$ and $x_{3} y_{3}$ planes respectively. Joint 5 is identified as the gripper. Its $z_{4}$ axis is vertical to $z_{3}$ axis and itprovides $\theta 5$ angular motion in $x_{4} y_{4}$ plane see figure 2. [3]


Figure (2). Coordinate Frames of 5250 Lab-Volt Robotic Arm

Lab-Volt 5250 robot arm rotational joints and the gripper are controlled by dedicated servo motors. These motors are connected to a serial servo controller unit (5221) to control the Lab-Volt 5250 robot arm from a computer through the serial port.

## KINEMATICS

Kinematics is the science of motion that treats the subject without regard to the forces that cause it. Within the science of kinematics, one studies the position, velocity, acceleration, and all higher order derivatives of the position variables (with respect to time or any other variable(s)). Hence, the study of the kinematics of manipulators refers to all the geometrical and time-based properties of the motion [6].

Many methods can be used in the direct kinematics calculation. The DenavitHartenberg analyses is one of the most used, in this method the direct kinematics is determinate from some parameters that have to be defined, depending on each mechanism. However, it was chosen to use the homogeneous transformation matrix. This transformation specifies the location (position and orientation) of the hand in space with respect to the base of the robot, but it does not tell us which configuration of the arm is required to achieve this location [1].

## FORWARD KINEMATICS

The forward kinematics is a set of equations that calculates the position and orientation of the end- effect or in terms of given joint angles. This set of equations is generated by using the Denavit-Harbenterg(D-H)parameters obtained from the frame assignation[7].

The parameters for the Lab-Volt 5250 arm are listed in Table 1 which derived from figure 2, where $\theta_{i}$ representsrotation about the Z -axis, $\alpha_{i}$ rotation about the X -axis, $d_{i}$ translation along the Z-axis, and $a_{i}$ translationalong the X-axis.

Table (1) The D-H parameters of the Lab-Volt 5250 arm

| Frame | $\mathrm{a}_{\mathrm{i}}(\mathrm{mm})$ | $\alpha_{\mathrm{i}}($ degree $)$ | $\mathrm{d}_{\mathrm{i}}(\mathrm{mm})$ | $\theta_{i}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 90 | 380 | $\theta_{1}$ |
| 2 | 380 | 0 | 0 | $\theta_{2}$ |
| 3 | 230 | 0 | 0 | $\theta_{3}$ |
|  | 0 | 90 | 0 | $\theta_{4}$ |
| 5 | 0 | 0 | 150 | $\theta_{5}$ |

Where
mm: millimeter
The forward kinematics describe the transformation from one frame to another, starting at the base and ending at the end-effect or. The transformation matrix $A_{\mathrm{i}}$ between two neighboring frames $o_{\mathrm{i}-1}$ and $o_{\mathrm{i}}$ is expressed as [4]
$A_{\mathrm{i}}=\operatorname{Rot}_{\mathrm{z}, \mathrm{\theta}_{i}} \operatorname{Trans}_{\mathrm{z}, \mathrm{d}_{i}} \operatorname{Trans}_{\mathrm{x}, \mathrm{a}_{i}} \operatorname{Rot}_{\mathrm{x}, \alpha_{i}}$
$=\left[\begin{array}{cccc}\mathrm{C} \theta_{\mathrm{i}} & -\mathrm{S} \theta_{\mathrm{i}} & 0 & 0 \\ \mathrm{~S} \theta_{\mathrm{i}} & \mathrm{C} \theta_{\mathrm{i}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1\end{array}\right]\left[\begin{array}{cccc}1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \mathrm{~d}_{\mathrm{i}} \\ 0 & 0 & 0 & 1\end{array}\right]\left[\begin{array}{cccc}1 & 0 & 0 & \mathrm{a}_{\mathrm{i}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1\end{array}\right]\left[\begin{array}{cccc}1 & 0 & 0 & 0 \\ 0 & \mathrm{C} \alpha_{\mathrm{i}} & -\mathrm{S} \alpha_{\mathrm{i}} & 0 \\ 0 & \mathrm{~S} \alpha_{\mathrm{i}} & \mathrm{C} \alpha_{\mathrm{i}} & 0 \\ 0 & 0 & 0 & 1\end{array}\right]$
$=\left[\begin{array}{cccc}C \theta_{i} & -S \theta_{i} C \alpha_{i} & S \theta_{i} S \alpha_{i} & a_{i} C \theta_{i} \\ S \theta_{i} & C \theta_{i} C \alpha_{i} & -C \theta_{i} S \alpha_{i} & a_{i} S \theta_{i} \\ 0 & S \alpha_{i} & C \alpha_{i} & d_{i} \\ 0 & 0 & 0 & 1\end{array}\right]$
By substituting the D-H parameters from Table 1 into equation (1), we can obtain the individual transformation matrices $A_{1}$ to $A_{5}$, and a global matrix of transformation $T_{5}^{0}$ as below:

$$
\begin{align*}
& A_{1}=\left[\begin{array}{lrrr}
\mathrm{C}_{1} & 0 & \mathrm{~S}_{1} & 0 \\
\mathrm{~S}_{1} & 0 & -\mathrm{C}_{1} & 0 \\
0 & 1 & 0 & \mathrm{~d}_{1} \\
0 & 0 & 0 & 1
\end{array}\right] \\
& A_{2}=\left[\begin{array}{crrr}
\mathrm{C}_{2} & -\mathrm{S}_{2} & 0 & \mathrm{a}_{2} \mathrm{C}_{2} \\
\mathrm{~S}_{2} & \mathrm{C}_{2} & 0 & \mathrm{a}_{2} \mathrm{~S}_{2} \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right] \tag{3}
\end{align*}
$$

$$
\begin{align*}
& A_{3}=\left[\begin{array}{cccc}
\mathrm{C}_{3} & -\mathrm{S}_{3} & 0 & \mathrm{a}_{3} \mathrm{C}_{3} \\
\mathrm{~S}_{3} & \mathrm{C}_{3} & 0 & \mathrm{a}_{3} \mathrm{~S}_{3} \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]  \tag{4}\\
& \mathrm{A}_{4}=\left[\begin{array}{cccc}
\mathrm{C}_{4} & 0 & \mathrm{~S}_{4} & 0 \\
\mathrm{~S}_{4} & 0 & -\mathrm{C}_{4} & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]  \tag{5}\\
& \mathrm{A}_{5}=\left[\begin{array}{cccc}
\mathrm{C}_{5} & -\mathrm{S}_{5} & 0 & 0 \\
\mathrm{~S}_{5} & \mathrm{C}_{5} & 0 & 0 \\
0 & 0 & 1 & \mathrm{~d}_{5} \\
0 & 0 & 0 & 1
\end{array}\right]  \tag{6}\\
& T_{5}^{0}=A_{1} A_{2} A_{3} A_{4} A_{5}=\left[\begin{array}{cccc}
n_{x} & o_{x} & a_{x} & p_{x} \\
n_{y} & o_{y} & a_{y} & p_{y} \\
n_{z} & o_{z} & a_{z} & p_{z} \\
0 & 0 & 0 & 1
\end{array}\right] \tag{7}
\end{align*}
$$

Where $\left(p_{x}, p_{y}, p_{z}\right)$ represents the position, $\operatorname{and}\left(\left\{n_{x}, n_{y}, n_{z}\right\},\left\{o_{x}, o_{y}, o_{z}\right\},\left\{a_{x}, a_{y}, a_{z}\right\}\right)$ the orientationof the end-effecter. The orientation and position of the end-effect or can be calculated in terms of joint angles and the D-H parameters of the manipulator, as shown in equations (8) to (19).

$$
\begin{align*}
& n_{x}=S_{1} S_{5}+C_{5} C_{1} C_{234}  \tag{8}\\
& n_{y}=S_{1} C_{5} C_{234}-C_{1} S_{5}  \tag{9}\\
& n_{z}=S_{234} C_{5}  \tag{10}\\
& o_{x}=S_{1} C_{5}-C_{1} S_{5} C_{234}  \tag{11}\\
& o_{y}=-C_{1} C_{5}-S_{1} S_{5} C_{234}  \tag{12}\\
& o_{z}=-S_{234} S_{5}  \tag{13}\\
& a_{x}=C_{1} S_{234}  \tag{14}\\
& a_{y}=S_{1} S_{234}  \tag{15}\\
& a_{z}=-C_{234}  \tag{16}\\
& p_{x}=C_{1}\left(a_{1}+a_{2} C_{2}+a_{3} C_{23}+d_{5} S_{234}\right)  \tag{17}\\
& p_{y}=S_{1}\left(a_{1}+a_{2} C_{2}+a_{3} C_{23}+d_{5} S_{234}\right)  \tag{18}\\
& p_{z}=d_{1}+a_{2} S_{2}+a_{3} S_{23}-d_{5} C_{234}  \tag{19}\\
& \text { Where }
\end{align*}
$$

$$
C_{i}=\cos \left(\theta_{i}\right), S_{i}=\sin \left(\theta_{i}\right), C_{23}=\cos \left(\theta_{2}+\theta_{3}\right), \text { and } S_{23}=\sin \left(\theta_{2}+\theta_{3}\right) .
$$

## INVERSE KINEMATICS

Inverse Kinematics analysis determines the joint angles for desired position and orientation in Cartesian space. Total Trans formation matrix Equation will be used to calculate inverse kinematics equations. Its solution, however, is much more
complex than direct kinematics since there is no unique analytical solution. Each manipulator needs a particular method considering the system structure and restrictions [1].

In the inverse kinematics the user specifies the desired goal position of the end effect or in Cartesian space as ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ ) where z is the height of the end effect or. From figure (3) $\theta_{1}$ can be calculated from the following equation $\theta_{1}=\operatorname{Atan} 2(p y, p x)$


Figure (3). Geometric analysis
The lengths d 1 , a 3 , a4 and d 5 correspond to the base height, upper arm length, forearm length and gripper length, respectively, and are constant. The angles $\theta_{1}, \theta_{2}, \theta_{3}, \theta_{4}$ and $\theta_{5}$ correspond to shoulder rotation, upper arm, andforearm, wrist, and End effectors, respectively. These anglesare updated as the specified position in space changes. The geometric approach used to solve for these angles, looking at figure 4 concluding that the relationship between $\theta_{2}, \theta_{3}, \theta_{4}$ and $\gamma$ as shown below.


Figure (4). Arm sagittal planar view of robot
$\gamma=\theta_{2}+\theta_{3}+\theta_{4}$
Also the calculation of $\gamma$ can be calculated from the equation 7 as shown below $\cos (\gamma)=T_{5}^{0}(3,3)$

$$
\begin{align*}
& \sin (\gamma)=\sqrt{1-\cos (\gamma)^{2}}  \tag{23}\\
& \gamma=\operatorname{atan} 2\left(\sqrt{1-\cos (\gamma)^{2}},-\cos (\gamma)\right)  \tag{24}\\
& \mathrm{rw}=\mathrm{rg}-\mathrm{d} 5 \cos (\gamma)  \tag{25}\\
& \mathrm{zw}=\mathrm{zg}-\mathrm{d} 5 \sin (\gamma)  \tag{26}\\
& \mathrm{Or} \\
& \mathrm{rw}=\mathrm{a} 2 \cos \left(\theta_{2}\right)+\mathrm{a} 3 \cos \left(\theta_{2}+\theta_{3}\right)  \tag{27}\\
& \mathrm{zw}=\mathrm{a} 2 \sin \left(\theta_{2}\right)+\mathrm{a} 3 \sin \left(\theta_{2}+\theta_{3}\right)  \tag{28}\\
& \cos \left(\theta_{3}\right)=\frac{(z w-d 1)^{2}+r w^{2}-a 2^{2}-a 3^{2}}{2 a 2 a 3}  \tag{29}\\
& \sin \left(\theta_{3}\right)=\sqrt{1-\cos \left(\theta_{3}\right)^{2}}  \tag{30}\\
& \theta_{3}=\operatorname{atan} 2\left(\sin \left(\theta_{3}\right), \cos \left(\theta_{3}\right)\right)  \tag{31}\\
& \theta_{2}=\operatorname{atan} 2((z w-d 1), r w)-\operatorname{atan} 2\left(a 3 \sin \left(\theta_{3}\right), a 2+a 3 \cos \left(\theta_{3}\right)\right) \tag{32}
\end{align*}
$$

## PROGRAM

MATLAB has been used to perform the forward and the inverse kinematics, where three main functions have been used:
i- Function used to determine the transformation matrix for each joint using Denavit-Harbenterg convention as in equation (1).
ii- Function used to determine the forward kinematics for the robot and evaluate the equations from equation (2) to equation (7) and find its solution as shown in equations (8) to (19).
iii- Function used to determine the inverse kinematics for the robot using by performing equations from (20) to (33) and find the joint angles.

## DISCUSSION

Mathematical modeling and kinematic analysis of Lab-Volt5250 robot arm was carried out in this study. Robotarm was mathematically modeled with Denavit -Hartenberg(D-H) method. Forward and Inverse Kinematics solutions are generated and implemented by MATLAB. Ananalysis technique was introduced to reduce the multiplesolutions in inverse kinematics part. A typical example, calculated with the generated software, was included here for the user.

Table (2).D-H parameters of robot arm

| Frame | $a_{i}(m)$ | $\alpha_{i}($ degree $)$ | $d_{i}(m)$ | $\theta_{i}($ degree $)$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 90 | 0.380 | 45 |
| 2 | 0.380 | 0 | 0 | 30 |
| 3 | 0.230 | 0 | 0 | 30 |
| 4 | 0 | 90 | 0 | 45 |
| 5 | 0 | 0 | 0.150 | 30 |

Where $\theta$ values in table 2 are the desired values, the transformation matrices for each link with these desired values as shown in equations below
$\mathrm{A}_{1}=\left[\begin{array}{llcc}0.7071 & 0 & 0.7071 & 0 \\ 0.7071 & 0 & -0.7071 & 0 \\ 0 & 1 & 0 & 0.380 \\ 0 & 0 & 0 & 1\end{array}\right]$
$\mathrm{A}_{2}=\left[\begin{array}{cllc}0.8660 & -0.5 & 0 & 0.3291 \\ 0.5 & 0.8660 & 0 & 0.1900 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1\end{array}\right]$
$\mathrm{A}_{3}=\left[\begin{array}{cllc}0.8660 & -0.5 & 0 & 0.1992 \\ 0.5 & 0.8660 & 0 & 0.1150 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1\end{array}\right]$
$\mathrm{A}_{4}=\left[\begin{array}{llcl}0.7071 & 0 & 0.7071 & 0 \\ 0.7071 & 0 & -0.7071 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1\end{array}\right]$
$\mathrm{A}_{5}=\left[\begin{array}{cllc}0.8660 & -0.5 & 0 & 0 \\ 0.5 & 0.8660 & 0 & 0 \\ 0 & 0 & 1 & 0.150 \\ 0 & 0 & 0 & 1\end{array}\right]$

$$
\left.\begin{array}{c}
0  \tag{38}\\
0 \\
150 \\
1
\end{array}\right]
$$

The total transformation matrix between the base and the gripper is given below
$\mathrm{T}_{5}^{0}=\left[\begin{array}{cccc}0.1951 & 0.7039 & 0.6830 & 0.4165 \\ -0.5120 & -0.5209 & 0.6830 & 0.4165 \\ 0.8365 & -0.4830 & -0.2588 & 0.8080 \\ 0 & 0 & 0 & 1\end{array}\right]$
$T_{5}^{0}$ was determined by using MATLAB and it is the final forward kinematics solution of the robot arm. Matrix values are checked against the physical positions of the robot arm in Table 3.

## Table (3).Differences between calculated and physical values of Lab-Volt 5250

 robot arm| Position values | $\mathrm{T}_{5}^{0}$ Values <br> $\mathbf{( m )}$ | Measured <br> Values <br> $(\mathbf{m})$ |
| :--- | :--- | :--- |
| $\mathrm{P}_{\mathrm{x}}$ | 0.4165 | 0.415 |
| $\mathrm{P}_{\mathrm{y}}$ | 0.4165 | 0.415 |
| $\mathrm{P}_{\mathrm{z}}$ | 0.8080 | 0.804 |

When calculated xyz coordinates of the target position are compared with the measured coordinates as in table 3, it is observed that the values were very close to each other with an error about $0.5 \%$ while in [3] was about $4 \%$.On the other hand, inverse kinematics equations will be used to determine the target position and its orientation for the robot arm.

## CONCLUSION

In this research, kinematics analyses of Lab-Volt 5250 robot arm were accomplished for the first time. MATLAB program has been used to perform this analysis and find the forward and inverse kinematics by making two dedicated functions for each one respectively and give the command to the robot. These are very important for the trajectory tracking and position control of the robot arm.
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